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1. Basic Theory and Terminology

Before beginning a study of pawn endings, it will be useful for the reader to make the acquaintance of its basic theory, and also of its basic concepts and definitions. But we must straight away point out that the terminology of pawn endings is as yet insufficiently developed; among the authors of works on these endings there is no unity, and considerable confusion exists over concepts and definitions.

Some theorists use only the concept of critical squares, while others talk of key squares, when by this they frequently mean one and the same thing. Some try to manage altogether without these terms. There are some old-fashioned authors who regard teaching about the opposition to be the main thing, while others regard corresponding squares as the most important. Without especially wishing to criticize, we have tried to generalize the concepts and definitions used by theorists, and, using the study of key squares as a basis, present the modern state of pawn endgame theory.

To the informed reader, certain of the conclusions drawn by us may seem trivial, but one should not be surprised by this. First, they are, as it were, links in the chain of our reasoning, and, second, without them the understanding of more complex examples may prove difficult.

It is well known that the aim in the endgame, and in a pawn endgame in particular, is to promote a pawn to a queen. To start with, using elementary examples, we will try to disclose the conditions under which this aim can be achieved. The first condition is the presence of a passed pawn. But this is not enough. There is also a second condition—it is necessary that the opponent’s king should not be able to stop the pawn.

1. Here is a typical elementary position, in which the pawn inexorably becomes a queen. Irrespective of where the opponent’s king is placed, it is powerless to prevent this. Note the position of the white king—from e7 it simultaneously both defends the pawn and controls its promotion square. Incidentally, the e7 and d7 squares have the same property, although the latter is an intermediate one: in order to make way for the pawn, all the same the king will have to move either to e7, or to c7.

As we see, the seizure by the king of one of these squares is very important: by this the aim of the endgame is automatically achieved—the pawn promotes to a queen.

_In endgame theory the three squares in front of a pawn on the 6th rank are called the key squares of this pawn._

First conclusion: in order that a pawn should
promote to a queen, it is sufficient to occupy one of its key squares with the king.

It is natural that the defending side should try to prevent their seizure. Consider another simple position.

2. Here everything depends on who it is to move. If it is White, he plays 1 d7 and puts his opponent in zugzwang, and after the forced 1 ... Ke7 2 Kc7, seizing one of the key squares, White wins.

It is a quite different matter if it is Black’s move. He, of course, plays 1 ... Kc8, maintaining control over c7. Now 2 d7+ no longer wins: after 2 ... Kd8 White himself is in zugzwang, and the forced 3 Kd6 leads to stalemate.

Second conclusion: in such situations the struggle for the achievement of the ultimate aim—the queening of the pawn—reduces to a struggle for the key squares. The player with the pawn will try to invade with his king on one of these squares; the defender, by manoeuvring with his king close to the key squares, will aim not to allow the opponent’s king onto them.

In the next chapter we will make a detailed acquaintance of the entire system of key squares of a passed pawn, but now we will examine a schematic position, where a king attacks the opponent’s passed pawn.

3. So that the attack should be successful, the king must attack the pawn from the front or from the side. If there is no obstacle in the pawn’s path, the attack from the rear is senseless, since the pawn will simply run away from its attacker.

Thus the king can attack the pawn from three surrounding squares—in this case from c6, d5 and e6. If the black king is some distance away, the penetration of the white king onto any one of these squares may place Black in a critical position, since it will lead to the loss of the pawn. Such squares around a pawn, from which the opponent’s king can attack it, it is logical to call critical.

Thus we obtain a third conclusion: a passed pawn has a definite system of squares surrounding it. Some of these are key squares; if its own king can reach any one of these, the pawn will queen. Others are critical squares; the penetration of the opponent’s king onto these squares may lead to the loss of the pawn.

Let us now turn to an elementary example, where there are blocked pawns.

4. White’s plan consists of two stages. First he must win the opponent’s pawn, and then try to queen his own. It is obvious that after winning the pawn his task will reduce to one considered earlier—to the struggle for the key squares.

White’s king has already penetrated onto
one of the critical squares of the enemy pawn. It is not difficult to see that here this factor is decisive: if it is Black's turn to move, he immediately loses his pawn. While if it is White's move, he maintains the zugzwang situation by 1 Ke7. It is easy to show that, compared with a passed pawn, the system of critical squares for a blocked pawn is expanded. In the given case the critical squares of the d6 pawn will be e6 and c7, the symmetric squares c6 and c7, and the intermediate square d7.

5. White’s king is threatening to attack the opponent's pawn, but its opposite number is intending to do exactly the same. What then should happen here? It turns out that the direct invasion of the critical e6 square is a terrible mistake. After 1 Ke6? Kc5 we reach a position of mutual zugzwang, in which the side to play first (White) loses. It is correct first to step onto the other critical square—e7. After 1 Ke7! Kc5 2 Ke6 it is Black who ends up in zugzwang.

But if it is Black to move, he wins in similar fashion: 1 ... Kc4! 2 Ke6 Kc5, and it is White who is in zugzwang.

Fifth conclusion: critical squares do not always become key squares for the other side. This depends on the relative placing of the kings, and on the turn to move. In other words, to determine whether a critical square is a key square, in certain cases a preliminary calculation is needed, in order to ascertain the resulting positions of mutual zugzwang.

6. If White begins, he can immediately attack the opponent's pawn, but such activity is incorrect and leads to defeat. On 1 Kb5? there follows 1 ... Ka7, when White ends up in zugzwang and loses his pawn. Therefore White plays 1 Kb4. Now it is Black who must not attack the pawn: after 1 ... Ka7? 2 Kb5 he himself ends up in zugzwang. The correct continuation is 1 ... Kb8 2 Kc4 Ka8! with a draw.
Pawns such as those we have just been examining are picturesquely termed ‘untouchable’ pawns: whoever is the first to attack such a pawn—he loses. As we will see later, in positions with ‘untouchable’ pawns an exact calculation of the reserve moves is very important, since in the resulting position of mutual zugzwang every tempo counts.

Our conclusions regarding key squares have been deduced from positions with a minimal number of pawns, but in principle these conclusions are also applicable to positions with a greater number. In each case, however the pawns are deployed, both the critical and the key squares can be established. At this point I should like to emphasize that these squares are not simply theoretical concepts. With their help it is easier to analyze many complex endings. They enable the correct plan of play to be quickly and faultlessly found.

And another important conclusion. The basic device in the struggle for key or critical squares is the creation of a zugzwang position, in which one of the sides is forced to take unfavourable action. In double-edged situations the zugzwang may be mutual, and then everything will depend on whose turn it is to move.

Let us now examine in more detail how the struggle for the key squares proceeds.

7. We know that g6 is a key square, and therefore Black must not allow the opponent’s king onto it. But it turns out that h6 will also be a key square: by reaching it, the white king also penetrates by force to g6. This means that Black has two moves which maintain the balance—1 ... Kg7 and 1 ... Kh7, defending both key squares.

Now let us move the position two files to the left.

8. Here, apart from the familiar squares e6 and f6, the g6 square also turns out to be a key one. In fact, if Black plays 1 ... Ke7, there follows 2 Kg6 Ke8 3 Kf6 Kd7 4 Kf7 Kd8 5 Ke6 Kc7 6 Ke7, and White wins.

The correct method of defence is 1 ... Kf7,
standing opposite the opponent’s king, so as not to let it onto any of the three squares e6, f6 and g6. Such a placing of the kings is called the opposition. In the given instance it is vertical opposition.

The following example demonstrates horizontal opposition.

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9. Black maintains the balance by 1 ... Kf7, placing his king in horizontal opposition and then retaining it. E.g. 2 Kh6 Kf6! The opposition is a well known term in the theory of pawn endings. Here we should mention that the opposition is the only means of struggle for three adjoining key squares, situated on a rank or on a file.

Of course the kings can also be deployed opposite each other along a diagonal, but it is important to emphasize that such an opposition is a transitional one, and it can always be transformed into horizontal or vertical opposition.

In our example White can continue 3 Kh5, so as on 3 ... Kg7 to have the decisive reply 4 Kg5. In this case Black is saved by 3 ... Kf7!, standing in opposition along the diagonal. Then if 4 Kg5 Kg7, transposing into vertical opposition, while if 4 Kh6 Kf6, reverting to horizontal opposition.

So that for the moment we don’t have to go carefully into the subtleties associated with a specific pawn arrangement, we will examine a schematic position, in which the struggle for three key squares along a file takes place on an empty board.

10. In defending the key squares, Black’s king must each time stand in opposition to the opponent’s king: 1 Kf7 Kd7 2 Kf6 Kd6 3 Kf5 Kd5. This is clear. It is important to note, however, that the main opposition will nevertheless be Kf6/Kd6. Why? Because only in this case, when the opponent conceives the opposition, can White’s king break immediately onto one of the key squares by a by-pass manoeuvre. E.g. if it is Black to move in the position Kf7/Kd7: 1 ... Kd6 2 Kf6! (but not 2 Ke8, since then Black maintains the balance by transposing into vertical opposition by 2 ... Ke6!) 2 ... Kd5 (or 2 ... Kd7 3 Ke5) 3 Ke7, and the aim is achieved.

Conclusion: the main, decisive opposition will be that on the middle line.

Let us now move the kings to h8 and b8 respectively, and see how they approach each other in the struggle for the key squares.

11. Suppose that it is Black to move: 1 ... Kc7 2 Kg7! Kc6 3 Kg6! Kc5 4 Kf7! Kd6 5 Kf6, and the white king inevitably reaches one of the key squares. If on 3 Kg6 Black had replied 3 ... Kc7, there would have followed 4 Kf5 Kd6 5 Kf6. This distinctive king manoeuvre,
making a 'herring-bone pattern' in creating the threat of a by-pass, is typical of the struggle for three key squares, and will be frequently encountered.

But if it is White to move, then, by maintaining a symmetric king position with respect to the e-file, Black succeeds in not allowing the penetration of the opponent's king onto the key squares. E.g. 1 Kh7 KB7 2 Kg6 Kc6 3 Kg5 Kc5 4 Kf5 Kd5, or 1 Kg7 Ke7 2 Kf7 Kd7 etc. Such a placing is also essentially an opposition, only it is called distant opposition (but distant opposition can always be transformed into close opposition).

A broader concept than opposition is that of corresponding squares.

Grigoriev, 1921

12. Let us begin by determining the key squares. The first of these is easily found—it is b3, the critical square of the black pawn. But here there are also two other key squares—c2 and f2. On reaching there, White easily drives back the opponent's king, advances his d-pawn, and wins the opponent's pawn. Black saves the game, only if he does not allow the white king onto any of these squares.

It is very tempting to stand in opposition by playing 1 ... Ke3, but it is precisely this move that is a mistake. After 2 Kd1 Kf3 (alas, control must be maintained over the e2 square) 3 Kc1 Ke3 4 Kb1 Kd4 5 Ka2 Kc5 6 Kd3 White achieves his aim. The correct continuation is 1 ... Kf3! 2 Kd1 Ke3 3 Kc1 Kd4 4 Kb1 Kc5 5 Ka2 Kb4, and the black king arrives in time. 2 Kf1 Ke3 3 Kg2 also achieves nothing due to 3 ... Kd2 4 d4 K×c2, when the queens appear simultaneously.

Let us investigate what is happening. As the white king moves from e1 to a2, by a series of specific moves the black king succeeds in passing from f3 to b4 and in not allowing the seizure of the key squares. The routes of the kings are as though linked. To each move of the white king there is a single corresponding move of the black king. It can be said the square corresponding to e1 is f3, that to d1 is e3, c1—d4, b1—c5, and a2—b4. In the theory of pawn endings, such a mutual connection of squares is called correspondence, and the squares themselves have received the name of corresponding squares.

So as not to allow White to seize any of the key squares, the black king must move precisely along the corresponding squares.

Conclusion: correspondence is the most general means of struggle for key squares.

In the above example the black king managed to maintain the correspondence, and the ending was a draw. This is by no means always the result. The following example shows that, if the correspondence cannot be maintained, defeat is unavoidable.
You have made the acquaintance of one of the simplest cases of corresponding squares, so-called triangulation (d5, c4, d4). By manoeuvring with his king in this triangle, White breaks the correspondence to his advantage. In order to make two steps forward, the white king first takes one step back.

Grigoriev, 1920

13. White has an extra pawn, and for it to be promoted to a queen, his king must break through to d7—the key square of this pawn. There is also another way—to win the a6 pawn, for which he must take his king to b6—the critical square of this pawn. At first sight it appears impossible to win: on 1 Kc5, with the threat of penetrating to b6, Black has a single but adequate reply in 1...Kc7, while if 1 Kd6 Kd8.

It is not difficult to guess that we again have here a case of corresponding squares: c5-c7, d6-d8 and d5-c8.

We come to the conclusion that, on the approaches to the key squares, Black succeeds in maintaining the correspondence. Let us now try stepping back with the king, say, to d4, exploiting the fact that the black king is restricted in its movements by the edge of the board. It is clear that he cannot play 1...Kc7 due to 2 Kc5. This means that he must play 1...Kd8 or 1...Kb8, i.e. corresponding to d4 there are two squares—b8 and d8. But what if a further waiting move, 2 Kc4, is made? Corresponding to this square, which is adjacent to c5 and d5, are the same two squares—b8 and d8. But here's the trouble—the king cannot jump from one of these to the other. That means that in this case Black can no longer maintain the correspondence: 2...Kc8 is decisively met by 3 Kd5! Kd8 4 Kd6, and 2...Kc7 by 3 Ke5.

14. In its time this study was the first example in literature employing the method of corresponding squares to such endings.

It is easy to see that 1 d4 leads immediately to a draw: Black replies 1...Ke4 2 Kc3 Kf5 3 Kd3 Kg4!, and White has no way of strengthening his position. Since it is unfavourable to advance the pawn straight away, White must manoeuvre with his king. Let us try approaching this position from the viewpoint of the theory of key squares and corresponding squares. Suppose that the black king is at f4. Then White can play 1 Ke2 Ke5 2 Ke3 Kd5 3 d4 Kc4 4 Ke4 Kxb4 5 d5 Kc5 (5...Ka3 6 d6 b4 7 b3 8 d8=Q, or 5...Ka5 6 d6 Kb6 7 Ke5 b4 8 Ke6 b3 9 d7, winning) 6 Ke5 b4 7 d6 Kc6 8 Ke6 b3 9 d7 b2 10 d8=Q b1=Q. The queens have appeared simultaneously, but now comes 11 Qc8+ Kb6 12 Qb8+, and the black queen is lost.

This means that e2 will be the first key
square. It is not hard to guess that the second key square is d4; if his king reaches there, White wins very simply.

Now let us try to find the corresponding squares. If the white king moves to c3, Black has only one reply—1 \ldots Ke3. This means that the square corresponding to c3 is e3. If the king goes to c2, Black again has only one reply which enables him to maintain the balance—1 \ldots Kf4, so that the square corresponding to c2 is f4.

Let us now consider the next adjacent squares—suppose that from c2 the king moves to b2. This square is adjacent to c2 and c3, hence the square corresponding to it will be f3.

But what square corresponds to b3? It, like the previous one, borders upon c2 and c3, but Black has no such second square like f3. This means that, if the white king manoeuvres between b2 and b3, Black will inevitably lose the correspondence.

Now White’s plan is clear: 1 Ke2! Kf4 2 Kb2 (or 2 Kb3) 2 \ldots Kf3 3 Kb3! Kf4 4 Ke2! Ke5 (totally bad is 4 \ldots Kf3 5 Kd2 Kf4 6 Ke2 etc.) 5 Kd1!

The play still requires accuracy. 5 Kd2 would be pointless because of 5 \ldots Kd4, when the king does best to retrace its steps, since after 6 Ke2? Ke3 Black breaks through to the critical squares of the b4 pawn, and the game ends in a draw.

5 \ldots Kd5 6 Ke2 Kd4 7 Kd2! Ke5 8 Ke3, and White wins.

Doesn’t it seem to you that the winning method employed in this study is in some way similar to that which we saw in example 13? Yes, it is that same ‘triangulation’! Only, it is turned through 90°, and the squares which come into it are c2, b2 and b3. For all the lack of similarity between positions 13 and 14, the winning methods in them turn out to be identical.

We will meet more complex systems of corresponding squares in chapter 10, but now let us consider certain peculiarities of the geometry of the chess board.

15. The squares e1 and a5 are situated on the same diagonal, and from e1 the king can reach a5 in 4 moves. It is easy to see that the diagonal path will be the shortest of all those possible.

The squares e1 and e8 are situated on the same straight line—the e-file. Moving along this file, the king reaches e8 in 7 moves. It is clear that other routes are also possible, but the reader will undoubtedly be surprised to learn that, apart from the path indicated, there are a further 392(!) routes which enable the king to reach e8 in the same 7 moves. The king can move there, tracing out the most fantastic figures, provided only that they are within the area depicted in the diagram (which encloses the shortest distances along diagonals), and that each time the king moves from one rank onto the next.

Thus the movement of the king in a straight line (along a file or rank) can in case of necessity be replaced by movement in a broken line.

A practical illustration of this rule is provided by the following example.

16. If White begins, he can win the a7 pawn. In order to save the game, Black must reach c7 with his king at the moment when White captures the pawn.

White can approach the opponent’s pawn in one and the same number of moves by various paths. E.g. Kf7-e7-d7-c7-b7, or
Kf7-e6-d6-c6-b7, or Kf7-e6-d5-c6-b7. The following is also possible: Kf7-e8-d7-c8-b7.

A legitimate question arises: can't the king, in heading for the pawn, simultaneously hinder the black king's movements towards the c7 square? It turns out that such a combination of tasks can be carried out. After 1 Ke6 Kc3 2 Kd5! White's king as though pushes away the black king. The latter is forced to move aside, and can no longer reach c7 in time, e.g., 2 ... Kb4 3 Ka6 Ka5 4 Kb7 Kb5 5 Kx a7 Kc6 6 Kbh, and the pawn queens.

This king manoeuvre, which has received the name of 'shoulder-charging', is frequently employed in pawn endings.

It is useful to make the acquaintance of another typical manoeuvre.

17. The first impression is that things are bad for Black: he cannot get to the white pawn, and the fate of his own pawn is settled. But nevertheless: 1 ... Kd2! Black is apparently intending to support the advance of his pawn, and threatens 2 ... a3. Therefore White replies 2 Kxa4, but then comes 2 ... Kc3 f4 Kd4, and Black succeeds in stopping the pawn. By this 'feint'—a deceptive movement of the king to the left, in order then to dart to the right—Black managed to gain the necessary tempo.

And, in conclusion, a rule which enables

one quickly and without calculation to determine whether or not a king can catch a pawn: 'the rule of the square'.

18. The result depends on who it is to move. If White begins, he queens his pawn: 1 b5 Kf4 2 b6 Ke5 3 b7 Kd6 4 b8 = Q+. But if it is Black to move, he succeeds in stopping it: 1 ... Kf4 2 b5 Ke5 3 b6 Kd6 4 b7 Kc7. The dotted line on the diagram shows the 'square' of the b4 pawn (b4–b8–f8–f4). Its sides are equal to the distance of the pawn to its queening square.

The 'rule of the square' is as follows: if the
king is inside the 'square' of the pawn, or on its move can step into it, then it catches the pawn: if this is not so, it cannot catch the pawn.

When calculating at the board it is simpler to draw mentally only one line—the diagonal of the 'square'. After all, in terms of number of individual squares, the diagonal of the 'square' is equal to its side. It should also be borne in mind that, with a pawn in its original position, when it still has the right to advance two squares, the 'square' should be constructed from the square in front of the pawn.

And lastly: the presence on the board of other pawns can prevent the king from stepping into the 'square' of a passed pawn.

19. White fails to win after 1 a4? Ke4 2 a5 Kd5 3 a6 Ke6, etc. Correct is first 1 d5! e x d5 2 a4, when the king is powerless to stop the pawn.
2. King and Pawn against King

With this balance of forces there are more than 80,000 different possible positions (even if symmetric ones, on different flanks, are excluded). Guided by only two considerations—the ‘square’ of the pawn and its system of key squares, we can instantly determine what the result will be.

We met the rule of the ‘square’ in the previous chapter, and it will now be useful for us to establish the full system of key squares of a passed pawn, and its dependence on where the pawn is situated.

Let us first consider a case where the pawn has crossed the demarcation line.

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20

8   7   6   5   4   3   2   1
a   b   c   d   e   f   g   h

Crosses indicate the key squares
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20. We already know that, if White’s king succeeds in occupying one of the three key squares in front of the pawn (c7, d7, e7), it automatically promotes to a queen. But what will be the result with the king at e6? If its opponent is at d8, a by-pass manoeuvre is immediately decisive: 1 Kd6 Kc8 2 Ke7, or 1 ... Ke8 2 Ke7, while if it is at e8, the advance of the pawn is decisive: 1 d6 Kd8 2 d7 Kc7 3 Ke7.

Continuing the analysis, it is easily established that White also wins when his king reaches d6 or c6, irrespective of where the opponent’s king is placed.

This means that, along with e7, d7 and c7, the squares e6, d6 and c6 are also key squares of the d5 pawn: the occupation by the king of any one of these squares leads to the promotion of the pawn. Black can draw the ending if he succeeds in not allowing the white king onto these squares. To do this when the white king is at e5, Black’s king must defend the two squares in front of its opponent—e6 and d6, i.e. it must be at d7 or e7, while when the white king is at c5 it must be at c7 or d7.

Suppose Black’s king is at d7. Then on 1 d6 he must retreat his king so that after 2 Ke6 or 2 Kc6 he can reply 2 ... Ke8 or 2 ... Kc8 respectively. This condition is satisfied by only one move—1 ... Kd8!, whereas 1 ... Ke8 (or 1 ... Ke8) 2 Ke6 (or 2 Ke6) 2 ... Kd8 3 d7 leads to zugzwang and to Black’s defeat.

In the previous chapter, when we made the acquaintance of the concept of key squares, we examined an important final position of mutual zugzwang (cf. example 2).* It serves as a direction finder in the following position.

21. On 1 Kf5 Black loses immediately after 1 ... Ke8 2 Ke6, when White has obtained position 2 with Black to move. But 1 ... Kf7 also does not help: 2 Ke5 Kf8 (Black is stubborn; if now 3 Ke6, then 3 ... Ke8, and it is White to move) 3 Kf6! (the decisive move) 3 ... Ke8 4 Ke6 Kd8 5 d7 etc.

* From now on only the number of the position being referred to will be given in brackets.
King and Pawn against King

21. Other moves lose: 5 d5+ Kd6 6 Kd4 Kd7 7 Ke5 Ke7 8 d6+ Kd7 9 Kd5 Kd8!, as we already know, Black draws the ending.

Thus, depending on which rank the pawn is situated, it has a definite system of key squares. As the pawn advances, the key squares move with it. Thus for a pawn at d3 the key squares will be c5, d5 and e5, and for a pawn at d4 they will now be e6, d6 and c6. Since the struggle is basically for three key squares in a row along a rank, it is natural that the vertical opposition of the kings acquires importance.

22. Another important position of mutual zugzwang. Let us check it.

Suppose it is Black to move. He is forced to play 1 ... Kc5 (or 1 ... Ke5), on which there follows 2 Ke4 (or correspondingly 2 Kc4). Now the pawn promotes automatically, e.g. 2 Kd6 3 Kd4 Kc6 4 Kc5 Kd7 5 Ke5 Ke7 6 Kc6. After several times carrying out a by-passing 'herring-bone' manoeuvre, White has occupied one of the basic key squares of the pawn and can now advance it.

It is another matter if it is White to move. On 1 Ke3 Black replies 1 ... Kc5 (or 1 Ke3 Ke5), not allowing the opponent's king forward. After 2 d4+ Kd5 3 Kd3 Kd6 (3 ... Kc6 or 3 ... Ke6 is also possible) 4 Ke4 Ke6 (all

23. Here White immediately penetrates onto the key squares: 1 Kd2(f2) Kd7 2 Ke3 Ke7? 3 Ke4 Kd6 4 e3!

Here it is, the decisive move. In this way White puts his opponent in zugzwang, and gains the possibility of a by-pass.

4 ... Kd6 5 Kf5 Ke7 6 Ke5 Kf7 7 Kd6 etc. And now let us move the pawn forward one square.

24. The slight change in the position is immediately reflected in the result: White will no longer have a reserve pawn move, and so Black is able to maintain the balance.

1 Ke2 Ke7 2 Kd3(f3). The only correct reply now is 2 ... Kd7(f7)? 3 Ke4 Ke6, retaining control over the key squares.
King and Pawn against King

24. King requires 2 moves, to reach c6 it requires 3, and to reach b6 it requires 4. It is clear that, if the white king immediately advances to b4, Black will be unable to reach b6 in time.

25. What should Black play? If 1 ... Ke7, then 2 Ke5 Kd7 3 Kd5, and the battle for the key squares is lost, while on 1 ... Kd7 White has the immediately decisive 2 Kd5. Black is saved by 1 ... Kd8!, to answer 2 Ke5 with 2 ... Ke7, and 2 Kd5 with 2 ... Kd7.

26. Let us try to find the solution, proceeding from the theory of key squares.

If the white king reaches d4, Black's king must be able to stand at d6, if c4—at c6, and if b4—at b6. Only in this case will the key squares be fully defended against the penetration of the white king. All three squares—d4, c4 and b4—can be reached by the white king in 3 moves, whereas to reach d6 the black

27. From the previous example it is clear that the white king must advance to a5.

1 Kc2! Ke7 2 Kb3 Kd6 3 Ka4 Kc6 4 Ka5 Kb7 5 Kb5! Ka7 6 Kc6 Ka6 7 b5 + Ka7 (28).
28. White has penetrated onto the basic key squares, but the struggle is not yet over, since due to the proximity of the edge of the board he has to be careful—there is the possibility of stalemate. Thus 8 b6 + ?? leads to a draw after 8 ... Ka8!: on 9 Kc7 Black has no move.

The correct continuation is 8 Kc7! Ka8 9 Kb6 (driving the king out of the corner) 9 ... Kb8 10 Ka6 Ka8 11 h6 Kb8 12 b7, and White wins.

Mandler, 1969

29. The battle for the key squares begins at the very distant approaches: 1 Kb2!

Bad is 1 Ka3 Ka5 or 1 Kb3 Kb5, when Black seizes the key squares. If now 1 ... Kb5 2 Kb3!, and White maintains the balance.

1 ... Kb6 2 Kc2! Ke6 3 Kd2! Kd6 4 Ke2! Ke6 5 Kf2! Kf6 6 Kg2! Draw. By moving along a 'narrow road', White's king has gained control of the key squares.

If the pawn is a rook's pawn, the number of key squares is sharply reduced, and with it the chances of winning.

30.

Crosses indicate the key squares

30. Here the pawn has only two key squares—g7 and g8; h7 will no longer be a key square.

31. Black to move plays 1 ... Kg8, when his king cannot be evicted from the corner, e.g. 2 h6 Kh8 3 h7—stalemate.
It is White to move, he can play 1 Kh7 Kf7 2 h6, but after 2 ... Kf8 (2 ... Kf6? 3 Kg8) he can make no progress, e.g. 3 Kh8 Kf7 4 h7 Kf8, or 3 Kg6 Kg8 4 h7+ Kh8.

An exception to the rule is provided by the following position.

In conclusion we should mention those very rare cases, where to avoid stalemate the pawn must be promoted to a rook.

32. White can penetrate to g8, but in doing so he leaves his pawn undefended, and Black is saved by 1 ... Kg5. Therefore in this case g8 will not be a key square.

How the struggle develops for the key squares when there is a rook’s pawn is shown in example 33.

33. 1 Kg4 Kc5 2 Kg5 Kd6 3 Kg6 Ke7 4 Kg7, and the struggle is over; the king cannot prevent the advance of the pawn.

It is a different picture with Black to move: 1 ... Kc5 2 Kg4 Kd6 3 Kf5 Ke7 4 Kg6 Kf8 5 Kh7 Kf7, and by controlling g7 and g8, Black draws (31).

34. 1 c8=Q? gives stalemate, but 1 c8 = R! wins.
3. King and Two Pawns against King

An advantage of two pawns normally ensures an easy win. A draw is possible only in exceptional cases, when both pawns are lost, or one is lost and the second proves insufficient for a win, or finally, with rook's pawns or far-advanced pawns, when there is a possibility of stalemate.

Connected pawns, if they are defending each other, always win.

When there is a possible attack from the side on one of a pair of doubled pawns, the second pawn may be able to defend the critical squares of the first against invasion by the opponent's king.

Grigoriev, 1935

35. 1... Ka8 2 Kd6 Kb7 3 a8 = Q + ! K x a8
4 Kc6 Kb8 5 b7 and wins.

With doubled pawns, the second can always be used to gain a tempo.

36. 1 Kc5 Kb7 2 Kb5.
A bad blunder, making the second pawn worthless, would be 2 b5??, when White can no longer win.

2... Kb8 3 Ka6!
This is stronger than 3 Kc6 Kc8 4 b7 + Kb8 5 b5 Ka7, when the only way to win is by 6 b8 = Q + ! K x b8 7 Kb6 etc.

3... Ka8 4 b5 Kb8 5 b7 Kc7 6 Ka7, and White wins.

37. There is only one way to win—1 g4!, covering the f5 square against the opponent's
King and Two Pawns against King

But if it is Black to move, he loses after both 1 ... Kxf6 2 Kg8, and 1 ... Kf8 2 Kg6 etc.

Isolated pawns can usually defend each other indirectly: in capturing one, the king moves out of the 'square' of the other.

40. The king can attack either of the pawns, but this does not bring Black any relief. On 1 ... Ka5 comes 2 c5, and on 1 ... Kc5—2 a5. By advancing one step, the critical square of the first pawn is covered against attack from the side by the second pawn. This allows White to gain time for the approach of his king.

The following position is truly unique.

38. After 1 ... Kc7 2 Ke6 Kd8 White must play 3 Kd5! Kxd7 4 b6, reaching the winning example 21. But if the position is moved one file to the left, this plan no longer succeeds due to the rook's pawn.

39. Here the white king cannot get out from in front of the pawn, e.g. 1 Kh8 Kf8 etc.
41. White achieves his aim by the splendid move 1 Ke4!!, and, depending on where the opponent's king moves, White captures the pawn on that side. Pawn advances also do not help, e.g. 1 ... d3 2 K×d3 Kf5 3 Ke2 Kg4 4 Kf2. Draw.

But if the position is moved one rank down the board, after 1 Ke3 the sacrifice of either pawn (1 ... d2 or 1 ... f2) wins, since a position of type 21 arises.

\[42\]

\[8\]
\[7\]
\[6\]
\[5\]
\[4\]
\[3\]
\[2\]
\[1\]
\[a\]
\[b\]
\[c\]
\[d\]
\[e\]
\[f\]
\[g\]
\[h\]

42. 1 ... Ka5 2 d5.

The sacrifice of the pawn is pointless: 2 Kb2 K×a4 3 Kc3 Kb5 4 Kd3 Ke6 5 Ke4 Kd6 with a draw.

2 ... Kb6 3 Kb2 Kc5, with a draw.

From the solution it becomes clear why the defensive possibilities of the pawns are poorer here than in example 40, where the critical square of one pawn was defended by the other.

Up till now we have been examining examples, where isolated pawns have been unable to advance independently to the queening square. A. Studenetsky has established a simple rule, called by him the rule of the 'wandering square', which allows one to determine quickly whether or not pawns are capable of queening without the aid of their king.

If the common 'square' of two isolated pawns extends to the edge of the board or beyond it, the pawns can advance to the queening square independently, without the support of their king.

If the common ‘square’ of the pawns does not extend to the edge of the board, and the pawns are left to their own devices (i.e. their own king cannot support them), the decisive factor is the distance between the pawns. If the pawns are separated by one square, they can successfully oppose a frontal attack by the enemy king. If there is a distance of two squares, the pawns are lost, if they have not reached the 5th rank. If the distance between the pawns is 3 squares, they are not lost only if they have reached the 4th rank.

Studenetsky, 1939

\[43\]

The dotted line indicates the 'wandering square'

43. If the pawns are on the same rank, one can construct a square, the length of one side of which will be the distance between the pawns (including the files on which the pawns themselves stand—translator’s note). As the pawns advance, this 'square' changes its position (hence the name 'wandering'). When it reaches the edge of the board, as in our example, this is a sign that the pawns can queen without the support of their king.

Suppose, for example, that in the diagram
position the black king is at b7. With White to move there follows 1 d6 Ke6 2 a6 K×d6 3 a7, while with Black to move 1 ... Ka6 is met by 2 d6 Kb7 3 a6+ etc.

Of course, the rule of the 'wandering square' is applicable only when the opponent's king has not yet invaded the critical squares of either of the pawns.
4. Pawn against Pawn

Material equality gives reason to suppose that the ending of pawn against pawn should normally end in a draw, and in many cases this is indeed so. But in fact the decisive role is played not by the material equality, but by the presence or absence of a positional advantage to one of the sides. The forms of this advantage are varied: if the pawns are not passed—activity of the kings; if they are passed—the greater degree of advancement of one of them, and the ability of the kings to battle against the passed pawns.

It will be expedient to divide the endings under consideration into three main groups:

4.1 Pawns on the same file.
4.2 Pawns on adjacent files.
4.3 Both pawns passed.

4.1 PAWNS ON THE SAME FILE

In the first chapter we have already considered several positions with blocked pawns (4, 5, 7, 8). It was established that alongside a blocked pawn there are several critical squares, which must be defended against invasion by the enemy king. At the time we did not try to determine the entire system of critical squares, and it will be useful to do this now.

44. We will consider various relative placings of the kings. First, a case where one king attacks, and the other defends, the critical squares of the pawn:

g8/e8. In this position White’s king inevitably enters the zone of critical squares of the black pawn, and therefore he wins it both with and without the move. E.g., with White to move: 1 Kg7 Ke7 2 Kg6 Ke6 3 Kg5! As we see, in order to achieve success White had to take his king onto one of the main critical squares of the pawn, which for White are key squares. The remainder is simple: 3 ... Kd7 4 Kf6 Kd6 5 Kf5 Kc7 6 Ke6 Kc6 7 Ke5.

from Durand, 1860

Crosses denote the critical squares of the d5 pawn
Dots denote the critical squares of the d4 pawn

You will no doubt have noticed how the white king gradually, one after another, won back the key squares, which in the end led to the loss of the pawn, although, as we know, in giving it up but defending the key squares of the d4 pawn against invasion by the white king, Black can still gain a draw: 7 ... Kc7 8 Kxd5 Kd7 etc.
a8/c8. Events develop similarly in this position: 1 Ka7 Kc7 2 Ka6 Kc6 3 Ka5! Kd7 4 Kb6 Kd6 5 Kb5 etc.
b4/f6. Here the pawn cannot be won: 1 Kg4 Kg6 2 Kf4 Kf6. By contrast, it would be fatal to play 1 Kh5? Kf5, since Black’s king
not only keeps the opponent’s king out of the critical squares of the d5 pawn, but itself aims for the critical squares of the opponent’s pawn.

\( \text{d8/d6}. \) This is a position by Leick (1948). White’s king is too far cut off from his pawn, and if it is his move he even loses: 1 \( \text{Kc8 Kc6} \), or 1 \( \text{Ke8 Ke6} \), and the black king picks up the d4 pawn.

\( \text{f3/e7}. \) Here Black keeps the opponent’s king off the critical squares of his pawn only by 1 ... Kf7!

\( \text{g5/a4}. \) Here we have a double-edged position—both kings are already on critical squares of the pawns. The question is—who is it to move? The side that begins, wins a pawn. E.g., with White to move: 1 \( \text{Kf5 Kb5} \).

Before it is too late, Black must switch to defence—the attempt to counter-attack leads to defeat: 1 ... Kb4 2 Ke6 Kc4 3 Ke5 etc. With Black to move, it is the other way round: 1 ... Kb4 2 Kf4 Kc4 3 Ke3 Kc3 etc.

2 Ke6 Kc6 3 Ke5 Kc7 4 K × d5 Kd7, with a draw.

Thus ideas about the critical squares of a blocked pawn are an important guide, determining the manoeuvring of the kings, and they allow one to establish in advance whether or not the pawn can be won.

\( \text{46.} \) In this position White is unable to win, e.g. 1 e4 Kf8 2 Kd7 e5!, or 1 e3 Kd8 2 e4 Ke8 3 e5 Kd8 4 Kf7 Kd7 5 Kf8 (5 e6 + even loses) 5 ... Ke6 6 Ke8. Draw.

\( \text{47.} \) Black thought that he would win here, but in reply to 1 Kb2 Kc4 2 Ka3! b2 came the unforeseen 3 Ka2!, when White won the battle for the key squares of the b5 pawn.

\( \text{Horwitz & Kling, 1851} \)

\( \text{Yates-Tartakover} \)

\( \text{Bad Homburg, 1927} \)
48. 1 e5! Kf7 (1 ... Kd7 2 Kb5) 2 Ke5 Kg6 3 Kc6!

In breaking through with his king onto the critical squares of the pawn, White takes account of the imminent position of mutual zugzwang.

3 ... Kg5 4 Kd7 Kf5 5 Kd6, and White wins.

50. The attempt to break through to c7—a critical square of the b7 pawn—does not prove successful: in the same 5 moves the black king reaches a6, and White loses his pawn.

But if this is so, the route of the white king must change: in reply to the capture of the b6 pawn White must be able to reach b4—in this case Black will be unable to penetrate to the key squares of his pawn, and White will draw.

But moving directly towards the b4 square leads to defeat: 1 Kg4 Kc2 2 Kf4 Kd3!, and Black's king pushes aside its opponent.

White saves the game by a subtle manoeuvre, enabling him to avoid an unpleasant encounter with the opponent's king; 1 Kg3! Kc2 2 Kf2! Kd3 3 Ke1! Kc4 4 Kd2 Kb5 5 Kc3 K×b6 6 Kb4. Draw.

To study the 'shoulder-charging' mechanism and ways of avoiding it, it is useful to consider also other placings of the kings in example 50.

Thus, for example, with the black king at c1 the same saving manoeuvre is still possible: 1 Kg3 Kd2 2 Kf2 Kd3 3 Ke1 etc.
But if the king is at d1, e1 or f1, White is unable to avoid defeat: 1 Kg3 Ke2!, and the path to e1 is closed.

With Black's king at g1 or h1, he is unable to reach the opponent's pawn in time, and so here it is White who wins.

Thus in position 50, if the black king is at a1, b1 or c1 White draws, if it is at d1, e1 or f1 he loses, and if it is at g1 or h1 he wins.

Let us now consider various positions of the white king on the rook's file. With his king at h1, h2 or h3 it is even easier for White to avoid the 'shoulder-charge', but with his king at h5 (as well as h6, h7 or h8) he loses: 1 Kg4 Ke2 2 Kf3 Kd3!, and the king reaches b4 too late.

But if we move the black king from b1 to a1, White is again able to avoid being pushed away: 1 Kg4 Kb2 2 Kf3 Kc3 3 Ke2! etc.

The following is perhaps an even more expressive example, showing how to avoid being pushed away.

Averbakh, 1982
from Grigoriev

52. This position is to a certain extent a critical one. Black will draw if, after the capture of his pawn, his king can reach c7, controlling the key squares b7 and b8. To be able to do this when it is the opponent to move, it is essential for Black's king to be within the marked zone. Note that, in the lower half of the board, there is only one square (f4) from which his king, stepping behind its opponent and thus avoiding being pushed away, can reach c7 in time.

The white king can approach d5 via two diagonals—from a2 or g8, and from g2. We will consider all three routes.

53. However far away Black's king is, he gains a draw in this position, since the white king is not able to push aside his king and prevent it moving to the cherished f4 square.
Pawn against Pawn

53

E.g., with the black king at h1: 1 Kb3 Kg2(h2) 2 Kc4 Kf3(g3) 3 Kd5 Kf4 etc.
Or, finally, with the king at c1: 1 Kb3 Kd2 2 Kc4 Ke3 3 Kd5 Kf4 etc.
The situation is slightly different with the white king at g8.

55

White wins if the black king is inside the zone

54

White wins if the black king is at a1 or b1

54. Black gains a draw if his king is on the 1st rank between h1 and c1: in this case his king is able to reach f4 at the moment when the opponent’s king is at d5. But if Black’s king is at a1 or b1, he loses, since he is not able to avoid being pushed away. E.g. with the king at b1: 1 Kf7 Kc2 2 Ke6 Kd3 3 Kd5!, and wins.
The most favourable position for the white king is at g2, which is quite natural, since in moving to d5 it can take control both of f4, and of the squares adjoining it.

55. Here the losing zone includes the entire 1st rank. E.g., with the king at c1: 1 Kf3 Kd2 2 Ke4 Ke3 3 Kd5 etc.
It is useful to consider also the position with the white king at f3.

56

White wins if the black king is inside the zone

56. The losing zone now includes all the back two ranks, with the exception of the h2 square. Indeed, if his king is at h2, Black saves the draw: 1 Ke4 Kg3 2 Kd5 Kf4 etc.
57. Since we are familiar with the previous example, the solution does not cause any difficulty: 1 Kh8! Kf6 2 Kh7! etc.

If the pawns are not yet blocked, in calculations one has to take into account the reserve tempi, which may be used to achieve a position of zugzwang.

58. White cannot defend his pawn, therefore he must bring his king behind the opponent's king, in order to avoid being pushed away. But the straightforward 1 Kf1 loses after 1 ... c5! 2 Kf2 Kd2. Correct is 1 Kf2

Kd2 2 c5! Kd3 3 Ke1! with a draw (or 1 ... c5 2 Ke3 Kc2 3 Ke2! Kc3 4 Kd1! etc.).

After our very thorough analysis of examples with blocked rook's pawns, the solution of the following two studies will not cause any difficulty. One must merely count the tempi correctly.

59. When the black king arrives at a3, the white king must reach e4. Therefore: 1 Kb7! a5 2 Ke6! a4 3 Kd5 a3 (or 3 ... Kb2 4 Kc4 Kx a2 5 Kc3 etc.) - 32) 4 Ke4. Draw.
60. 1 Kb7 a5 2 Kb6 a4 3 Kb5 a3 4 Kb4 Ke3 5 K×a3 Kd4 6 Kb4, and White wins. 1 ... Ke3 does not help after 2 K×a7 Kd4 3 Kb6!

If the black king were at g4, the natural 1 Kb7 would be a mistake, leading to a draw: 1 ... a5 2 Kb6 a4 3 Kb5 Kf5 4 K×a4 Ke6 5 K b5 Kd7. Correct is 1 Kb5!, not allowing the pawn to advance, e.g. 1 ... Kf5 2 Ka6 Ke6 3 K×a7 Kd7 4 Kb7, and White wins.

If the pawns are at a significant distance from each other, and the kings are fairly remote from the pawns, the assessment of such positions becomes a difficult matter, demanding precise calculation.

Grigoriev, 1938

Here the attempt at a by-pass leads to a fiasco: 4 Kd7? b5 5 Kc8 b4 6 Kb8 b3 7 Ka7 Kb5!, and White's king is too late in reaching a3 to defend his pawn.

4 ... b6.

If 4 ... b5, then 5 Ke5 Kc5 6 Ke4 Kc4 7 Ke3 Kb3 (7 ... b4 8 Kd2 Kb3 9 Kc1 Ka2 10 Kc2 with a draw) 8 Kd4! with a draw. Therefore Black tries to preserve the tempo.

5 Ke5! Kc5 6 Ke4 Kc4 7 Ke3 b5 (7 ... Kb3 8 Kd4) 8 Kd2 Kb3 9 Kc1 Ka2 10 b4! Draw.

Grigoriev also examined a similar situation with certain other king positions. If, for example, in the previous example the kings are replaced at h3 and f2 respectively, the draw is achieved even more quickly—in analogous fashion to the variation given in the note to Black's 3rd move.

But if the kings are placed at h5 and f4, White is no longer able to save the draw.

Grigoriev, 1938

61. Black is threatening to attack the b2 pawn, so that extreme measures must be taken.

1 Kg5 Ke4 2 Kf6 Kd5 (if 2 ... Kd3, then 3 Ke5 Kc2 4 Kd4 K×b2 5 Kc5) 3 Ke7 Kc6.

In reply to 3 ... b5 White embarks on a distant by-pass: 4 Kd7 b4 5 Kc7 Kc5 6 Kb7 b3 7 Ka6 Kb4 8 Kb6 Kc4 9 Ka5, and it is now Black who must force a draw by giving up his pawn. If after 6 Kb7 Black plays 6 ... Kb5, then 7 Kc7? loses after 7 ... b3 8 Kd6 Kc4. Correct is 7 b3!, when it is Black who has to think in terms of drawing.

4 Ke6!

62. 1 Kg6 Ke5 2 Kf7 Kd6 3 Ke8 b5 4 Kd8 b4 5 Kc8 Kc6! 6 Kb8 b3 7 Ka7 Kb5!, and the white king is too late.

63. The immediate advance of the king towards the opponent's pawn leads only to a draw: 1 Ka2 Kg2 2 Kb3 Kf3 3 Kc4 Ke4! 4 b4 Ke5 5 Kc5 Ke6 6 Kb6 Kd5 etc.
irrespective of which square on the a-file the enemy king occupies. E.g., with the king at a4: 1 Kg4! Kb5 2 Kf5! (2 Kg5? Ke5 3 Kg6 Kd5 4 K ×g7 Ke5 with a draw) 2 ... Ke6 3 Ke6! Ke5(c7) 4 g4, or 2 ... Ke5 3 Ke5! Ke6 4 Ke6! In all cases White succeeds in pushing aside the opponent's king.

Moreover, White also wins when the black king is at b1 or b2. E.g., with the king at b2: 1 Kg4 Ke3 2 Kf5 Kd4 3 g4 Ke3 4 g5 Kf3 5 g6, and white wins.

Having his king on any of the remaining squares of the b-file guarantees Black a draw. E.g., with the king at b3: 1 Kg4 Ke4 2 Kf5 Kd5 3 g4 Kd6 4 g5 (4 Kg6 Ke5) 4 Ke7 etc. The position of the king at c1 also leads to a draw: 1 Kg4 Kd2 2 Kf5 (2 Kf3 Kd3 etc. — 61) 2 ... Ke3 3 g4 Kf3 4 g5 Kg3 5 g6 Kh4 etc.

4.2 PAWNS ON ADJACENT FILES

With pawns on adjacent files, the only positions of interest to us are those where one side has a certain positional advantage. For the given balance of forces this advantage consists of a superior (more active) king position.

We will first consider cases where the pawns are as close as possible, and there are no reserve moves. Here a typical defensive procedure is the sacrifice of the pawn with the aim of transposing into a drawn ending.
65. Black is threatening to win the white pawn, while retaining control over the key squares of his own pawn. But by sacrificing his pawn, White forces a favourable displacement of the key squares: 1 d6! c×d6 2 Kb3 Kc5 3 Ke3. Draw.

66. Here the goal is achieved by 1 ... f4! 2 g×f4 Ke8! 3 Ke4 Kf8! Draw.

67. It would be a mistake to play 1 f6? g×f6 2 Kc2 Ke6 3 Kd2 Kf5 4 Ke3 Kg4, when Black wins. Correct is 1 Kc2 Kd6 2 f6! g×f6 3 Kd2! Draw.

68. Here Black gains a draw, only because his pawn is a rook’s pawn. After 1 Kc7 Ka8 2 Kb6 he has the reply 2 ... a5 with an immediate draw. E.g., 3 K×a5 Ka7, or 3 b5 Kb8! With the pawns on other files, this sacrifice would not have worked, due to the capture with the pawn.

Let us now move this position one rank up the board.

Berger, 1890

69. The composer shows that after 1 Kc8 a5 2 b6 White wins, or, more precisely, gives mate in 4 moves. However, as was
pointed out by Zinar (1974), in the diagram position it must be Black to play, since he has no previous move. Therefore the solution becomes: 1 ... a5 2 b6 etc. This is thus an example in retro-analysis, although it is unlikely that its composer suspected this.

from Tattersall’s *A Thousand End-Games*, 1910

70. In order to win, White must capture the black pawn and at the same time seize the key squares of his own pawn (a4, b4 and c4). But the plausible 1 Kc3 is wrong because of 1 ... a3! 2 b3 Ke5, when the draw is obvious. 2 b4 is also useless.

The correct continuation is 1 Kb1! a3 2 b3! Ke5 3 Ka2 Kd5 4 Kx a3 Kc5 5 Ka4 Kb6 6 Kc4, or 1 ... Ke5 2 Ka2 Kd4 3 Ka3 Kc5 4 Kx a4, and White wins.

71. 1 Kf2! (threatening 2 Kg3) 1 ... h4 2 Kg1!, and the rest is already familiar.

72. The black pawn is doomed, but, in moving towards it, White has to reckon with the threat of ... h3: 1 Kd3 Kb4 2 Ke4 Kc5 3 Kf5 (of course, not 3 Kf4 h3 with a draw, but 3 Ke5 is possible) 3 ... Kd6 4 Kg4, or 3 ... h3 4 g x h3 Kd6 5 Kf6, and wins.

If the pawns have not converged, the play is considerably complicated by the existence
of reserve tempi. It is essential to take them into account, since positions of mutual zugzwang often arise.

*Moravec, 1940*

73. The direct 1 Kc3 Kg5 2 Kd4 does not succeed, since after 2 ... Kf4 it is White who ends up in zugzwang. He wins by attacking the pawn from the side: 1 Kb4! Kg5 2 Kc5 Kf4 3 Kd4!, and it is Black who is in zugzwang.

As in other situations, an important role here is played by 'shoulder-charging'.

*Dobias, 1926*

74. By 1 Kd4! White not only suppresses the attack on his pawn from the rear, but also occupies the long diagonal, along which his king will then approach the opponent's pawn. Black has no satisfactory reply: if 1 ... Kb4 2 f4, or 1 ... Kc6 2 Ke5 Kc5 3 f4, and White wins.

In the following example White does not let the opponent's king off the rook's file, until he has improved the position of his own king and prepared a zugzwang position.

*Adamson, 1915*

75. The immediate approach of the king to the pawn does not succeed: when the king captures on h6, Black will have the reply ... Kf4. And 1 g4 is even more premature. White first aims to eliminate the attack on his pawn from the rear.

1 Kc7 Ka6 (1 ... Ka8 is decisively met by 2 g4 Ka7 3 Kd6, when Black is too late) 2 Kc6 Ka5 3 Kc5 Ka4 4 Kc4 Ka3 5 Kc3 Ka2 6 Kc2 Ka3.

6 ... Ka1 is bad because of 7 g4, while 6 ... h5 is met by 7 Kd3.

7 g3!

Only this modest advance of the pawn ensures a win. If 7 g4 Kb4 8 Kd3 Kc5 9 Ke4 Kd6 10 Kf5, then 10 ... h5! 11 g × h5 Ke7,
and the king reaches the saving f8 square just in time.

7 ... Ka4.

The most tenacious defence. If 7 ... Ka2 8 g4!, while after 7 ... h5 8 Kd3 Kb4 9 Ke4 Kc5 10 Kf5! White wins as in example 71.

8 Kc3!

A necessary finesse. The opponent’s king must still be kept at the side of the board. A draw results from 8 Kd3 Kb4 9 Ke4 Kc5 10 Kf5 Kd4!

8 ... Kb5 9 Kd4 Kc6 10 Ke5! (not 10 Ke4 Kc5! 11 g4 Kd6 12 Kf5 h5! with a draw) 10 ... Kc5 11 g4, and White wins.

Grigoriev, who published this splendid study in the chess column of Izvestiya (1925), pointed out that, with the white pawn at g3, it would not have been possible to win.

Let us change the position of the white king.

Moravec, 1952

77. This position, without the white king, was put forward by the composer with the question: where should White’s king be placed, so that he can win? The extent of the white king’s winning zone is shown in the diagram. It is clear that from d8, d7, d6, d5, d4, d3, e3, e2, f2, g2, or h2 the king can eliminate the black pawn in 4 moves, while Black is not able to capture the white pawn. With his king at c3, b1, c1, d1, or d2 White wins by 1 Kc2!, when Adamson’s study is reached. However, as will be seen, the c2 square does not come into the zone, since in this case it is White who is in zugzwang.

78. Here approaching the pawn with the king does not succeed: Black’s king aims for f8, and by a timely ... h3 he saves the game. E.g. 1 Kc5 Kb6 2 Kd6 Kc8 3 Kc6 Kc7 4 Kf6 Kd6(d8) 5 Kg5 h3 etc.

Correct is 1 Kc6! Kb8 2 Kd7! (in this way
White wins an extremely important tempo) 2
... Kb7 3 Ke6 Ke8 4 Kf5 h3 5 g×h3 Kd8 6 Kf6
Ke8 7 Kg7, and White wins.

80. To save the game, White must manage
to reach d5 at the moment when the black
king is at d3. The only way to do this is by 1
Kh8!, e.g. 1 ... Kf6 2 Kg8 Ke5 3 Kf7 Kd4 4
Ke6 Kd3 5 Kd5. Draw.

4.3 BOTH PAWNS PASSED

If the pawns of both sides are passed, the
result depends on who is the first to queen a
pawn. Cases are also possible where the
queens appear simultaneously, but either the
new-born queen can be won, or mate given. If
neither of the pawns can be Queened, the
ending, naturally, ends in a draw.

Apart from the advance of the pawns, a big
role is played here by the manoeuvring of the
kings. The aims of this manoeuvring can be
very varied: the gaining of a tempo, pushing
away the opponent's king, or luring it onto
an unfavourable square, so that, for example,
one's pawn can then be promoted with check
or the enemy queen can be won by a double
attack, and so on. All these features will be
explained in more detail in the analysis of
eamples.

81. The pawns are at an identical distance
from the promotion squares, but White's
king is more active. It can simultaneously
stop the opponent’s pawn and support its own, and this proves decisive.

Duras, 1905

81

1 \text{Ke5!} \hspace{1cm} 82. \text{Where should the king make for, in order to clear the way for its pawn? 1 Kb5 suggests itself, but after 1 \ldots Ke4 2 c4 Kd4 3 c5 Kd5 4 c6 Kd6 Black avoids danger. Correct is 1 Kd4!, maintaining control over the black pawn and ‘shoulder-charging’ the opponent's king. If now 1 \ldots a5 2 c4 a4 3 c5 a3 4 Kc3 and the game is over, so Black tries to bring his king into play. 1 \ldots Kf4 2 c4 Kf5 3 Kd5 (3 \ldots a5 is not dangerous, since after 4 c5 the pawn queens with check) 3 \ldots Kf6 4 Kd6 Kf7. Here too 4 \ldots a5 does not help, since after a check at h8 the new-born queen is immediately lost. 5 c5 Ke8 6 Kc7! a5 7 c6 a4 8 Kb7 a3 9 c7 a2 10 c8 = Q+, and White wins. Had White played 1 Kb4, after 1 \ldots Ke4! 2 c4 Ke5! 3 c5 Ke6 the draw would have been obvious. But White has interesting possibilities after 1 Kb4 Ke3, when the following position is reached (83).}

Grigoriev, 1931

83

3 b5 Ke7 4 Ke6! Kd8 5 Kb7 g5 6 b6 g4 7 Ka7(a8), and White wins by queening with check.

Grigoriev, 1929

82

83. \text{White succeeds in putting the opponent in zugzwang: 1 c4 Kd4 2 c5 Ke5! The strongest. If 2 \ldots a5 +, then 3 Kb5! a4 4 c6 a3 5 c7 a2 6 c8 = Q a1 = Q 7 Qh8+, winning the queen, while after 2 \ldots Kd5 3}}
Kb5 Black is in zugzwang. But now 3 Kb5 Kd5 leaves White in zugzwang.

3 Ka5! Ke6 4 Ka6!, and White wins, e.g. 4 ... Kd5 5 Kb5 Ke6 6 Kc6 a5 7 Kb7, and the pawn queens with check.

This same idea—queening with check—is the basis of the play in position 84.

Moravec, 1952

84. 1 b5 Ke5 2 b6 Kd6 3 Kb5 h4 4 Ka6 h3 5 b7, and White wins.

Grigoriev, 1928

86. 1 f4 is met by 1 ... Kb5, when Black's king is able either to stop the opponent's pawn, or to queen his own.

1 Kd4!

Now Black has two possibilities:

(a) 1 ... Kb5 2 Kd5! Ka6 3 f4 Kb7 4 f5 Kc7 5 Ke6 Kd8 6 Kf7! b5 7 f6 b4 8 Kg7, and White wins.

(b) 1 ... b5 2 f4 b4 3 f5 b3 4 Kc3 Ka3 5 f6 b2 6 f7 b1=Q 7 f8=Q+, and White either wins the queen or gives mate.

1 g4 b5 2 g5 b4 3 g6 b3 + 4 Kc3 b2 5 g7 b1=Q 6 g8=Q+ Ka1.

By skilful manoeuvring White overcomes this difficulty: 1 Kc3! Ka3 2 Kc4! Ka4 3 g4 b5 + 4 Kd3!

This subtle move is the whole point: in this way White regains his lost tempo, since on 4 ... b4 he has the immediately decisive 5 Kc2.

4 ... Ka3 5 g5 b4 6 g6 b3 7 g7 b2 8 Kc2!

Ka2.

The kings have returned, but the situation has changed significantly. After 9 g8=Q+ White gives mate.

The following example shows the winning of the new-born queen by a vertical check.

Grigoriev, 1928

85. The immediate advance of the pawn does not succeed, since Black queens a move earlier.
Both Pawns Passed

Mandler, 1938

87. After 1 K×b7 Kb3 Black succeeds in eliminating the white pawn.

Correct is 1 Kd6! Ka3! (if 1 ... b5, then 2 Kc5! Kb3 3 K×b5 Kc3 4 Kc5 Kd3 5 Kd5, and the king manages to defend the pawn) 2 Kc5! Ka4 3 f4 b5 4 f5 b4 5 Kc4! (now White lures the king to a3) 5 ... b3 6 Kc3 Ka3 7 f6 b2 8 f7, and White wins (86).

Some of the unexpected finesses which are possible in this ending are shown by the following study.

Iriarte, 1968

89. 1 Kf5! Kc5 2 Kf4!

Only in this way can the king safely leave the dangerous diagonal. After 2 Ke4? Kc4 3 Ke3 Kc3 4 Kf2 b4 5 e4 Kd4 6 Kf3 b3 Black wins.

2 ... b4 3 Ke3 Kc4 4 Kd2 Kb3 5 e4 Ka2 6 e5 b3, and the queens appear simultaneously. Draw.

Had the white pawn been at d2, the draw would have been achieved differently: 1 Kf5 Kc5 2 Ke5! (as before, 2 Ke4 is bad because of 2 ... Kc4 3 Ke3 b4 4 d4 b3 5 Kd2 K×d4) 2 ... b4 3 d4+ Kc4 4 d5 b3 etc.
Iriarte, 1968

90. 1 Kf5! Kc5 2 Ke4! Kc4 3 Ke3 Kc3, and now the pawn can be advanced: 4 g4 b4 etc. Draw.

In the following examples too, White manages to avoid stepping on a 'mined' square.

Prokop, 1943

91. The natural 1 Kc5 h5 2 d4 h4 3 d5 h3 4 d6 loses after 4 ... Ke6, since the king is forced onto the fatal c6 square. The draw is achieved by subtle manoeuvring, with the aim of diverting the black king from c6.

Moravec, 1941

92. If White immediately leads his pawn forward, this ends catastrophically for him: 1 Ka5? f5 2 b4 f4 3 b5 Kc5! 4 b6 Kc6 5 Ka6 f3 6 b7 f2 7 b8 = Q f1 = Q+ 8 Ka5 Qa1 = , and Black wins the queen.

An original king manoeuvre saves the draw: 1 Kb3! Kd3 2 Ka2!! f5 3 b4 Kc4 4 b5! (4 Kb2? Kxb4 5 Kc2 Kc4 leads to a loss) 4 ... Kxb5 5 Kb3. Draw.

Moravec, 1952

1 Kd4! Kf4 (otherwise 2 Ke3) 2 Kc5 h5 3 d4 h4 4 d5 Ke5 5 d6 Ke6 6 Kc6 (White has managed to gain a tempo, and now this move has become possible) 6 ... h3 7 d7. Draw.
93. After 1 b4 d4 2 Kc4 Ke4 3 b5 d3 White loses his queen.

The draw is achieved by 1 Kb4! Ke4 2 Kc3 Ke3 3 Kc2 Ke2 4 Kc3. If instead Black replies 1 ... Kd4, then 2 Ka5! followed by the advance of the pawn becomes possible.

In conclusion we give a group of positions, in which the theme of the play is king manoeuvring with a double aim. We have already met a number of such instances in the preceding examples, but here such manoeuvring comprises the entire strategy of the play. We will begin with a famous study, in which this idea was first expressed in striking form.

Reti, 1921

94. White's king is hopelessly behind the opponent's pawn, whereas its black opponent is ready to eliminate the white pawn. At first sight the task seems impracticable, but nevertheless...

1 Kg7.

For the moment the king does not appear to be threatening anything, so Black has a choice:

(a) 1 ... Kb6 2 Kf6 h4 3 Ke5 (threatening 4 Kf4) 3 ... h3 4 Kd6 (the king has unexpectedly ended up beside the pawn) 4 ... h2 5 c7 Kb7 6 Kd7, and the draw is obvious.

(b) 1 ... h4 2 Kf6 h3 (the pawn has escaped, but ...) 3 Ke7(e6) h2 4 c7 Kb7 5 Kd7 with a draw.

Reti's original idea, which later the chess composer Gurvich picturesquely called 'chasing two birds', made a strong impression in its time, stimulated a search for similar positions, and considerably enriched chess theory. Reti himself returned several times to this theme.

Reti, 1922

95. 1 c6 h5 (if 1 ... Kb6, then 2 Kb4 h5 3 Kc4 K×c6 4 Kd4) 2 Kb4 Kb6 (or 2 ... h4 3 Ke5! h3 4 Kd6) 3 Kc4! h4 4 Kd5! Kc7 5 Ke4. Draw. (1 Kb4 h5 2 c6 is also possible).

Reti, 1928

96. Here the draw seems even more improbable. After 1 Kg6 Black has three possibilities, but they all lead to a draw:
Pawn against Pawn

(a) 1 ... Kb6 2 K × g7 h5 (2 ... f5 3 Kf6 f4 4 Ke5 f3 5 Kd6) 3 K × f6, and then as in example 94.
(b) 1 ... h5 2 K × g7 h4 3 K × f6.
(c) 1 ... f5 2 K × g7 f4 3 Kf6 f3 4 Ke6(e7).

Prokes, 1946

Rinck, 1922

97. Thanks to the fortunate position of his king, White is able to refute Black’s plan: 1 a4 Kb3 2 a5 Ke3.
   2 ... Ke4 3 a6 Kd3 4 a7 f2 5 a8 = Q f1 = Q 6 Qa6 + leads to the loss of the queen.
   3 Kg1!
   The only move to win. If 3 a6? Kd2!, or 3 Kg3 Kd4! 4 a6 Ke3 with a draw.
   3 ... Kd4 4 a6 Ke3 5 Kf1, and White wins.

Adamson, 1922

98

99

99. 1 f4? is wrong: 1 ... Kg4 2 Kf7 Kf5.
   Correct is 1 Kf7 a5 2 f4! (but not 2 Ke6? a4 3 f4 a3 4 f5 a2 5 f6 a1 = Q 6 f7 Qa3, when
   Black wins) 2 ... a4 (or 2 ... Kg4 3 Ke6 a4 4 f5 with a draw) 3 f5 a3 4 f6 a2 5 Kg8 a1 = Q 6
   f7. Draw. Had White played 5 Ke7?, then after 5 ... a1 = Q 6 f7 Qe5 + 7 Kf8 Kg5(h5)
   Black would have won.

Moravec, 1952

100
100. Here the play is more complicated, since it involves pawn advances: 1 Kg4 b5.

There is nothing better. If 1 ... Kb3, then 2 Kf5 Kc4 3 Ke5 Kd3 (3 ... b5 4 d4, and the queens appear simultaneously) 4 Kd5 with a draw.

2 d4 b4 3 d5! Kb5 4 d6! Kc6 5 Kf5 b3 (5 ... Kx d6 6 Ke4) 6 Ke6. Draw.

Leick, 1948

101. The composer analyzed this position for the most varied positions of the white king. He established that, wherever his king stands, White cannot win, but he loses only if his king is on one of the following 6 squares: a8, b8, c8, g3, h1 or h8.

In all other cases the game ends in a draw. E.g., with the king at h2: 1 g4 Kf4 2 g5! (the standard pawn sacrifice) 2 ... Kxg5 3 Kg3. And with the king at d8 the draw is achieved by 'chasing two birds': 1 g4 Kf4 2 Ke7 b5 3 Kf6! Kxg4 4 Ke5, and the king enters the 'square' of the pawn.

A study with similar king manoeuvres, combined with a pawn sacrifice, was earlier created by the Dutch study composer Feijter.

102. 1 Kb7 a5 2 Ke7 Kc5 3 Kd7 Kd5 4 Ke7 Ke4 (White's resources appear to be exhausted, but ...) 5 Ke6! Kx f4 6 Kd5. Draw.

Prokes, 1947

103. 1 Kc8 Kc6 2 Kb8 Kb5 3 Kb7! Kx a5 4 Ke6, and the king has entered the 'square' of the pawn.

* In the Lasker-Tarrasch game the following position arose: (Kg5, pawns b2, b3, h2/Kf5, pawns a5, b5, c5). The continuation was 1 h4! Kg4 2 Kg6! Kx h4 3 Kf5, and it was now Black who had to fight for a draw.

Feijter, 1939

Prokes, 1947

from Grigoriev
5. Two Pawns against One

Practice has shown that, the more pawns there are on the board, the greater the importance of one side being a pawn up. Therefore with two pawns against one the material advantage must be considered minimal. As a rule, it is nevertheless sufficient for a win, although there are numerous exceptions. The most favourable positions for the stronger side are those where his pawns stand at some greater or lesser distance from the enemy pawn, and his king is in front of the pawns. Therefore one should as far as possible avoid the premature advance of the pawns, and in particular their maximum convergence (blocking).

As the basis for arranging the material we have taken the pawn structure in combination with the presence or absence of a passed pawn for the stronger side. It has therefore been divided into the following three main groups:

5.1 Connected pawns.
5.2 Isolated pawns.
5.3 Doubled pawns.

In each of these groups the task is to differentiate between those cases where the possibility of a draw is caused by the position itself, and those where a draw is gained (or eliminated) by the employment of certain methods.

5.1 CONNECTED PAWNS

5.11 All pawns passed

If all the pawns are passed, the question is whether or not the stronger side’s king can stop the enemy passed pawn. If it can, play reduces to a straightforward ending of king and two connected pawns against king. But if the pawn cannot be stopped, the material advantage of course loses its significance, and everything depends on who is the first to obtain a queen.

The following examples illustrate both the normal course of play, and also certain exceptions, caused in the main by the proximity of the edge of the board.

104. Black is faced with an impossible task—that of restraining the enemy pawns and defending his own. If it is Black to play, he loses immediately: 1 ... Ke5 2 g6(h5) Kf6 3 h5.

If White begins, he must give his opponent the move. He achieves this by a familiar procedure—‘triangulation’: 1 Kf2(e2) Kg6 2 Ke2 Kf5 3 Ke3 and wins. However, 2 Kg2 Kf5 3 Kf3 is also possible.

105. Here White carries out a mating attack. For example, with Black to move: 1 ... e2 2 h7+ Kh8 3 Kf7 e1=Q 4 g7+ K×h7 5 g8=Q+ and 6 Qg6 mate.
If White begins, he can also give mate in a different way: $1 \text{g7 e2 2 Kg6 e1 = Q 3 h7 mate.}$

Had the black pawn been at h3, with Black to move White would have been unable to win: $1 \ldots \text{b2 2 h7 + Kh8 3 Kf7 b1 = Q 4 g7 + K x h7 5 g8 = Q + Kh6,}$ and there is no mate, since g6 is controlled by the queen.

Now we will consider a case where, due to the unfortunate position of his king, it is the stronger side that has to aim for a draw. The procedure used here is that of sacrificing a pawn with the aim of entering the ‘square’ of the opponent’s passed pawn.

Gorgiev, 1928

$1 \text{g4 +! K x g4.}$
Declining the pawn sacrifice does not help:

1 ... Kg5 2 Kg7! (but not 2 h4 +! K x h4 3 Kg6 K x g4 4 Kf6 Kf4 and wins) 2 ... c5 3 h4 +! and now either 3 ... K x h4 4 Kf6 e4 5 g5, or 3 ... K x g4 4 Kg6 K x h4 5 Kf5 with a draw.

2 Kg6 c5 (otherwise 3 Kf6) 3 h4! Draw.

Prokes, 1948

107. 1 Ke7 (threatening 2 Ke6) 1 ... Kd5 2 Kd7! b5 3 e4 + K x d4 (3 ... K x e4 4 Kc6 b4 5 d5) 4 Kd6 K x e4 5 Ke5. Draw.

If only one of the two connected pawns is passed, two types of pawn formation are possible:

(a) an immobile formation, where the weaker side’s pawn is blocked;
(b) a mobile formation, where all the pawns retain the possibility of moving.

5.12 One passed pawn, pawns blocked

In this group of endings the stronger side has a protected passed pawn, restricting the mobility of the enemy king, as a result of which a positional advantage is added to the material one. Nevertheless, here too draw positions are possible, in particular with pawns at the side of the board.

We will first consider the pawn formation where the protected passed pawn is closer to
the centre, and the immobile pawn supporting it is closer to the side of the board. In these positions the stronger side's king supports the passed pawn, which normally ensures an easy win. Exceptions occur mainly with wing pawns which are too far advanced, or, on the contrary, with pawns which are not advanced at all.

It is expedient to begin the analysis with the most often-occurring, and therefore the most important practical positions with wing pawns, remembering that the features caused by the proximity of the pawns to the side of the board will disappear as the pawns approach the central files.

From the finish it will be apparent that a win is not possible in the position moved one rank up the board, since the concluding moves lead to stalemate.

The king manoeuvring in such situations may also prove to be rather more complicated.

Grigoriev, 1930

109. 1 Kd6! Kf7 2 Kd7 Kf8 (2 ... Kg7 3 Ke7 Kg6 4 Ke6) 3 Ke6 Kg7 4 Kf5, and wins.

The stalemate shown in the note to White's 9th move in example 108 can be achieved by force in certain positions.

Selezniev, 1930
from an idea by Troitsky

108. White wins by occupying c5 with his king. Black can create only a temporary hindrance.

1 ... Kc5 2 Kd3 Kd5 3 Kc3 Ke5 4 Kf3 Kd5 (the king must of course remain within the 'square' of the b5 pawn) 5 Kf4 Kd6 6 Ke4 Ke6 7 Kd4 Kd6 8 Kc4 Kc7 9 Kd5!

Not immediately 9 Kc5, in view of 9 ... Kb7 10 b6? Ka6 with a draw. The c5 square must be occupied when the black king is at b7. Of course, in an analogous position moved to the right there will be no stalemate, and the necessity for such manoeuvring will not arise.

9 ... Kb6 (9 ... Kd7 10 b6 Kc8 11 Kc6) 10 Kd6 Kb7 11 Kc5 Kc7 12 b6 + Kb7 13 Kc5, and White wins.
110. 1 a5! b5 2 a6! Kd5 3 Kb4 Kc6 4 Ka5 Kc5. Stalemate.

111. White wins easily by sacrificing his passed pawn at the appropriate moment, and penetrating into the critical zone of the black pawn: 1 Kd5 Kc8 2 Kd6 (or 2 Ke6 Kc7 3 Ke7 Kc8 4 c7) 2... Kd8 3 c7+ Ke8 4 Ke6 Kx c7 5 Ke7 etc.

If the position is moved a further file to the right, another solution becomes possible: White transfers his king to a5, and in reply to ... Kb7 continues d6-d7. With a protected passed pawn on the 6th rank, such a sacrifice is a typical procedure.

An obvious draw results in positions of type 111, moved as far up the board as possible.

112. But if this position is moved one file to the right, it will no longer be a draw, since the white king acquires the possibility of a by-pass to the left.

If the position in question is moved as far down the board as possible, White to play can no longer win, since if his king moves away Black has time to win the rear pawn and to queen his own.

113. But if this position is moved one file to the right, White acquires the additional possibility of a by-pass to the left and can win. For example: 1 ... Ke3 2 Kd1, and if 2 ... Kd4 3 Ke2, or 2 ... Kf3 3 Kc1 etc (but cf. position 12 — translator's note).

Some peculiar features are displayed by the following position—with the same pawn formation, but moved one rank up the board.
114. In Rabinovich's Endspiel (1938) this position is incorrectly evaluated as drawn in view of the following variation: 1 Kd2 Ke4 2 Ke2 Kf4 3 Kf2 Ke4 4 Kg3 Kd3 5 d5 K×c3 6 d6 Kb2 7 d7 c3 8 d8=Q e2 with a draw (Black has a bishop's pawn, and the white king is far away).

Euwe (1940) showed that 1 Kc1 leads to a win. Indeed, in this case White remains at the same distance from the d2 and a3 squares, whereas Black is forced to move away from one of the corresponding squares—e4 or b5. E.g.: 1 ... Ke4 (otherwise 2 Kd2 and 3 Ke3) 2 Kb2 etc. Chéron (1952) repeated Euwe’s indication, and other authors declared 1 Kc1 to be the only move to win.

But in fact any move by the white king wins. 1 Kc1!! is simplest, but 1 Kd1 or 1 Kd2 is also possible, e.g. 1 Kd1 Ke4 2 Kd2 Kd5 3 Ke3, or 2 ... Kf4 3 Kc2; 1 Kd2 Ke4 2 Kd1 Kd5 3 Ke2 Ke4 4 Kd2 etc.

That which has been stated in examples 108–114 shows that, the further the pawns are from the side of the board, the fewer the drawing chances for the weaker side; and for central pawns (on the d- and e-files) there are altogether none.

If the protected passed pawn is closer to the side of the board, and the immobile pawn is closer to the centre, the stronger side’s king can no longer give its passed pawn direct support, and the manoeuvring of the kings takes on a quite different character.

The essence of this manoeuvring is disclosed in the following examples.

115. The manoeuvrability of the black king is restricted by the a4 pawn: it must remain within its 'square', and at the same time defend the critical squares of the b4 pawn. The squares along the lines c4–e4 and c4–e8 are the boundary of Black’s critical zone, the main line of his defence.

For the play in such positions, old books give the following recipe: ‘Assume the opposition and then retain it on each move’. But in the given position this rule does not work. If it is White to move, assuming the opposition (1 Kg3) does in fact win, but it is much simpler to play 1 Kf3 and 2 Ke4, occupying one of the critical squares. The situation is even more striking with Black to move: 1 ... Kc6? (taking the opposition) loses, whereas 1 ... Kd6! (rejecting the opposition!) gives a draw. A vivid example of how ruinous a mechanical use of the opposition can be!

Black merely has to defend the invasion squares, and if it is him to move he is perfectly capable of doing this.

In 1922 Grigoriev showed that, as long as the white king is moving along the 2nd rank, it is sufficient for Black to move his king between d6 and e6, but as soon as the white king steps onto the 3rd rank a precise choice of corresponding square becomes essential. To Kc3 or Kg3 Black must answer ... Ke5!, while to Kd3, Kf3 or Kh3 his only reply is ... Kd5! Therefore Black must aim to occupy d6 or e6 as soon as possible, regardless of opposition considerations.

Now it becomes clear why, if it is Black to move, he must play 1 ... Kd6! The white king is ready to step onto the 3rd rank, so that Black’s replies must be extremely precise. E.g. 2 Kg3 Ke5! 3 Kf3 Kd5! 4 Kf4 Kd4! 5 Kf5 Kd5! (of course, not 5 ... Kc3? 6 a5) etc. The fact that, at the main line of defence, the corresponding squares fully coincide here with the
Connected Pawns

concept of opposition is merely a feature of the given position.

After the incorrect 1 ... Kc6? White continues so as to prevent Black from taking the close opposition: 2 Kg3! (2 Kf3? Kd5) 2 ... Kc5 (if 2 ... Kd6, then 3 Kf4! Kd5 4 Kf5 invading the critical zone, but now 3 Kf4? is met by 3 ... Kd4) 3 Kg4! (exploiting the fact that c4 is inaccessible to Black) 3 ... Ke6 4 Kf4! (threatening 5 Ke4) 4 ... Kd5 5 Kf5 and wins. White has gained the opposition on one of the main ranks (here there are three: the 5th, 6th and 7th).

We now give four examples by Dzindzio (1925) with the same pawn formation, but with different king positions, illustrating the possibility or otherwise of taking the opposition on the main ranks:

(a) Kg2–Ke4. Draw, whoever it is to move: 1 ... Kd4! 2 Kf2 Ke4! etc.
(b) Kg3–Ke5. White wins only if it is Black to move.
(c) Kg4–Ke6. White wins whoever it is to move (in this example the critical square is e4).
(d) Kg6–Ke6. White wins whoever it is to move: 1 Kg5 Kc5 2 Kg4! The close opposition (Kg6–Ke6) would assure Black of a draw, but the distant opposition loses due to the presence of the ‘crucial’ c4 square.

This last position shows that, if in example 115 after 1 ... Kd6! 2 Kh3 Kd5! White moves his king up the h-file, Black should keep his king on the d-file, retaining the possibility of taking the close opposition. E.g. 3 Kh4 Kd4! 4 Kg4 Ke4! 5 Kh5 Kd5! 6 Kh6 Kd6! 7 Kg6 Ke6! etc.

116. If the protected passed pawn has reached the 5th rank, i.e. it has crossed the middle of the board, one of the critical squares of the enemy pawn (in this case e5) ends up outside the 'square' within which Black must keep his king, and this immediately shows that his position is indefensible, wherever his king stands and whoever it is to move. E.g. 1 ... Kd5 2 Kg2 Kc6 3 Kf3 Kd5 4 Ke3, and wins.

**Crosses indicate the key squares of the b5 pawn**

In positions 115 and 116 it is easily established that neither the character of the play nor the result changes, if they are moved one file to the right.

In position 115, where the protected passed pawn has not reached the 5th rank, the result, as we have seen, depends on the placing of the kings. If an invasion of the critical zone is not possible, a draw is inevitable.

117. After 1 Kd2 Kd4 2 Ke2 Ke4 3 Kf2 Black can continue either 3 ... Kf4, or 3 ... Kd3 followed by capturing on b2.
118. Here the pawns stand one rank further up, and the latter possibility no longer works (Black defends as in example 115).
The attack on the pawn also does not work when the position is moved one file to the right.

119.  1 ... Ke4 2 Ke2 Kf4 3 Kf2 Ke4 4 Kg3 Kd3? (only 4 ... Ke5! gives a draw) 5 b5 K × c 6 b6 Kd2 7 b7 c3 8 b8 = Q c2 9 Qb2 Kd1 10 Kf2, and if 10 ... c1 = Q then 11 Qe2 mate. This finish proved possible, because the white king was sufficiently close.
If the kings are replaced at g3 and e5 respectively, Black does not lose even if it is him to move.

120. Exploiting the fact that he has a bishop's pawn, Black can play 1 ... Ke4! 2 Kg4 Ke5! 3 Kg5 Ke4! 4 Kf6 Kd3!, with a draw since the white king has moved away from f3.

5.13 One passed pawn, pawns mobile

In this group of endings the stronger side has a passed pawn, but the pawns are not blocked. This mobile structure, in which all the pawns retain the possibility of advancing, possesses certain features which were discovered by Grigoriev.

121. White to move wins simply by 1 Kd2 and 2 Kd3 etc.
But if it is Black to move he can draw by 1 ... Kc3! 2 Kd1 (or 2 Kb1) 2 ... d3 3 c × d3
Connected Pawns

K × d3! (or 3 ... K × b3! with the white king at b1).

The attempt to undermine the pawn chain immediately by 1 ... d3? fails to 2 c4!

But if the position is moved one rank up the board, the c-pawn no longer has the right to advance two squares, and along with 1 ... Kc4 Black can play 1 ... d4. Now 2 c × d4 is met by 2 ... Kc4!, when White is in zugzwang; if 3 Kd2 K × d4!, or 3 Kb2 K × b4! But if in this position it were Black to move, he would be in zugzwang, and he would lose after 3 ... K × d4 (3 ... K × b4 4 Kd3!) 4 Kb3! Kd5 5 Ka4 etc.

Grigoriev, 1933

122. From the notes to example 121 it is clear that after 1 Ke6! Ke8 White must advance his d-pawn in such a way as to be able to play d5–d6 only when the black king is at d8 or f8 (but not at e8). Therefore: 2 d3! Kd8 3 d4 Ke8 4 d5 Kd8 5 d6 e × d6 6 K × d6—draw.

Such calculations of advances by a pawn (or pawns) are typical, and to facilitate them it is useful to be guided by the following idea.

If the black king is at e8, it will end up at d8 or f8 in an odd number of moves, therefore the white pawn should complete its path to d6 in an even number. But if Black had started, then before the advance of the pawn his king would have already been at d8 or f8, and would have ended up on these squares within an even number of moves, and therefore White would have had to play 1 d4!, in order to reach d6 in an odd number.

This is a general rule, also providing for cases where White has to advance two pawns. But in the given example, where only one has to be moved, the calculation is simpler: if it is White to move, the black king is not yet on the necessary square, and so the pawn must be placed on a square of the same colour as that occupied at the given moment by the enemy king.

Having explained the basic idea of these endings, it will be easy to understand the manoeuvres of the two sides in the following position.

Grigoriev, 1933

123. 1 Ka1!

An unusual way to win, but both 1 Kb2? d4 2 Kc1 Kc3 and 1 Kb1? Kc3! 2 Kc1 (2 Ka2 K × c2) 2 ... d4 lead only to a draw.

1 ... Kc3 (1 ... d4 2 Kb2) 2 Kb1 Kb4 (2 ... d4 3 Kc1) 3 Kc1 Kc3 4 Kd1 d4 5 Kc1, and wins.

124. Here after 1 ... Kc4 roughly the same situation is reached as in the previous example after White's 4th move, but the advance of the d-pawn by one or two squares has to be reckoned with.

2 Kd1 (or 2 Kc1, but not 2 Kc2? d5!—
**Two Pawns against One**

Grigoriev, 1933

125. 1 Ke1! (the only move) and the threat of ... d4 is parried.

As an addendum to example 124, it remains to consider the case where the white pawns are one rank further up the board.

Moravec

126. 1 Ke2 Kd6 2 Kf3 Ke5 3 Kg4! f6 4 Kh5! f5 5 d6 K × d6 6 e × f5 and wins.

Grigoriev, 1936

127. 1 Kd2! (not 1 g4? h5) 1 ... Kf5 2 Ke3.

Not 2 Ke2? in view of 2 ... Ke4 and 3 ... h5, or 2 Kd3? h5 3 Ke3 Kg4 4 Kc4 K × g3.

2 ... h6.

Black loses immediately after 2 ... h5 3 Kf3, or 2 ... Kg4 3 Ke4, but now 3 Kf3 is not possible due to 3 ... h5.

Alatortsev

(from a correspondence game, 1934/35)
3 Kf2! Kg4.

If 3 ... Ke4, then 4 Kg2 h5 5 Kh3 Kf5 6 Kh4 Kg6 7 f5+, and for the f-pawn White obtains the h-pawn, at the same time controlling the critical squares of the g3 pawn.

But how is White to play now? The situation is quite different from that in examples 123 and 124. It obviously doesn't work to obtain the position with the white king at g2 and black pawn at h5, even with Black to move, since he can play ... h4, exploiting the fact that after g x h4 the pawn becomes a rook's pawn and ... K x f4 is possible.

It is evident that White must play to realize his f-pawn, especially since his king is manoeuvring on the other side of the passed pawn.

Since after 4 Ke3 h5 White is in zugzwang, he can only play 4 Ke2! h5.

Or 4 ... Kf5 5 Ke3 Kg4 6 Ke4. Now it is Black who ends up in zugzwang.

5 Ke3! K x g3, and after 6 f5 h4 7 f6 h3 8 f7 h2 9 f8 = Q h1 = Q White wins.

Leick, 1939

Although the idea of giving up the passed pawn for the enemy pawn has already been shown in position 127 (the note to Black's 3rd move), for greater clarity we will give some separate examples.

Herberg, 1936

129.

1 Kg3! (not 1 g3 in view of 1 ... Kg4 and ... f5-f4) 1 ... f5 (if 1 ... f6, then 2 Kh3 f5 3 g3) 2 Kf3! K x h4 3 Kf4 and wins.

Originally the composer had the position moved 2 files to the left. It is clear that in this case, apart from 1 Ke3, White also wins by 1 e3, e.g. 1 ... Ke4 2 Ke1 (or 2 Kd1) 2 ... Kf5 3 Ke2 Ke4 4 Kd2 etc.

Ebersz, 1942

128. With the pawns on the central files, the win involves fewer subtleties.

1 Ke3 f6 (1 ... Ke4? 2 Kc4) 2 Kd2 (in contrast to example 127, here 2 Kc2 is also possible) 2 ... Ke4 3 Ke1 (3 Kd1 is also possible) 3 ... Kd5 4 Ke2 Ke4 5 Kf2 f5 6 Ke2, and wins.
Two Pawns against One

130. 1 a4 Kb4 2 Kd3 Ka5! 3 Kc3! K × a4 4 Kc4 Ka5 5 K × c5 etc.

Up till now we have been examining positions with two connected pawns against one, with the stronger side having a passed pawn. We will now cover cases where there is no passed pawn. Here two types of pawn formation are again possible:
(a) an immobile structure with a pair of blocked pawns;
(b) a mobile structure.

5.14 No passed pawn, pawns blocked

Endings with blocked pawns and no passed pawn are of course less favourable for the stronger side. He normally wins, irrespective of who it is to move, if one of the connected pawns has reached the 6th rank—in other words, if the enemy pawn is still on its initial square. In remaining cases the result is determined by the placing of the kings and on whose turn it is to move. In positions with wing pawns the manoeuvring is often of the same character as that in positions of type 115. But if the pawns are situated between the c- and f-files, the character of the manoeuvring becomes different, in view of the appearance of invasion points on both wings.

We will first consider positions where the stronger side's backward pawn is closer to the central files.

131. The black pawn is still on its initial square, and White wins irrespective of who it is to move. E.g. 1 ... Kc8 (if 1 ... Ke7 2 c6) 2 Ke6! (but not 2 c6? Kb8!—draw) 2 ... Kd8 3 Kd6 Kc8 4 Ke7 Kb8 5 Kd7 Ka8 6 c6! b × c6 7 Kc7, and mate in 3 moves.

If it is White to play, he uses 'triangulation' to give his opponent the move: 1 Ke5 Kc6 2 Kd4 Kd7 3 Kd5 and wins (but not 1 c6+? Kc8! 2 Kd6 Kb8! with a draw).

132. Rook's pawns cause the usual exceptions. After 1 ... Kb8 (1 ... Kd7? 2 b6!) 2 Kd6 Ka8 the draw is obvious.

Let us move both positions one rank down the board.

133
133. If it is White to move, he cannot win: 1 Ke4 Ke6! (of course, not 1 ... Ke5?, which would allow White to use 'triangulation'; here Black does not have to fear the 2 c5 break) 2 Kf4 Kd6! 3 Ke4 Kc6 4 Ke3 Ke7 (or 4 ... Kd7), with a draw.

But if it is Black to move, he again loses: 1 ... Ke6 2 c5, or 1 ... Ke7 2 Ke5 (2 c5 is also possible) 2 ... Kd7 3 Kd5 (seizing the critical squares of the b6 pawn), or 1 ... Ke7 2 Ke5!

It is clear that the result will not change if the position is moved further down the board.

Black to move. But here it is important that in the position in question the kings have already taken up front-line positions. If this has not yet occurred, the result is determined by the position of the kings and their respective manoeuvring, as is shown by the following examples.

134. Here (or in positions moved further down the board) a draw is inevitable: on 1 ... Kd6 it is pointless for White to play 2 b5, while if it is his move, 1 Kd4 can be met by 1 ... Kd6 or even 1 ... Kb5 (2 Kc3 Kc6), since the loss of the opposition is not dangerous for Black.

135. If it is Black to move, to defend the critical squares b6, c6 and d6 he must play 1 ... Kd8! (2 Kc5 Ke7 or 2 Kd5 Kd7).

If position 131 is moved to the right, the result does not change: White wins, whoever it is to move (if it is White's move, apart from 1 Kf5 the simpler 1 d6+! also becomes possible).

The result is also unchanged if position 133 is moved to the right: White wins only if it is

136. While manoeuvring in the rear, Black need not worry about the opposition: only, on Ke4 or Kf4 he must reply ... Ke6 or ... Kf6 respectively. The attempt to by-pass on the right is hopeless: on Kg4 Black replies ... Ke6 with the threat of ... Kd5. If he tries a by-pass on the left, White requires 4 moves to reach a5; it follows that, for the black king to reach b7 in time it can even be on the f-file. In this lies the whole idea of Black's defence.
1 ... Kf7!

Not 1 ... Kd7? in view of 2 Kf4! Ke8 (there is nothing else) 3 Ke4! Kd8 4 Kf5, or 3 ... Kf8 4 d5 and wins.

2 Kf3 Ke7! and the draw is obvious: 3 Kg4 Ke6! 4 Kf4 Kf6! 5 Ke3 Kf7 (or 5 ... Ke7) 6 Kd3 Ke7 (or 6 ... Kd7).

Just how great the defensive resources are here is shown by the following position, in which the stronger side even has a tempo in reserve.

Ebersz, 1942

137. White cannot immediately bring his king into the front line: 1 Kf4? d5 2 Kf3 (2 Kg4 d4) 2 ... Kf5 3 Ke3 Ke5, or 1 Kg4? d5 2 Kf4 Ke6 3 Kf5 and Black wins. 1 Kf3? is met by 1 ... Ke5, and 1 Kg2? by 1 ... Kf5 2 Kf3 d5, winning.

Correct is 1 Kf2! (in the given instance this defence only accidentally takes the form of the opposition) 1 ... Ke6 2 Ke2 (it is of crucial and practical importance that, apart from the composer’s solution, 2 Ke3 is also possible) 2 ... Kf5 3 Ke3 Ke5 4 Kf3 Kd5 5 Kf4 (5 Ke3 can also be played) 5 ... Ke6 (if 5 ... Ke6, then 6 Ke3!) 6 Ke4 Kf5 7 Ke3 d5 (7 ... Kb5 8 Kd4) 8 Kd2 Kb5 9 Kc2 Ka4 10 Kb2, with a draw.

138. How can the change in result be explained here in comparison with position 136?

Since the weaker side’s pawn is a central one, on the Q-side Black has to defend a wider front: no longer 2, but 3 squares. With the white king at b5 it is now insufficient to reply ... Ke7 in view of Ka6; on Kb5 it is essential to play ... Kd7. Hence the black king cannot move onto the g-file, since from there to b7 it requires 5 moves, whereas to reach b5 the white king as before requires 4 moves.

If it is Black to move, he loses: 1 ... Ke7 2 Kg4!, or 1 ... Kg7 2 Ke3! Kf7 3 Kd3! (so as to meet 3 ... Kf6 with 4 Kd4 Kg5 5 e5) 3 ... Ke7 4 Ke4.

With White to move there is no win. Black is saved by maintaining the opposition (on the main f- and b-files); 1 Kg3 Kg7! 2 Kh3 Kf6! 3 Kg3 Kg7! (but not 3 ... Kg5?, since after 4 Kf3 the f5 square is inaccessible to Black) 4 Kf2 Kf6(f8) 5 Ke3 Ke7 6 Kd3 Kd7(d8) 7 Kd4 Kd8(d7) etc.

Let us now turn to positions in which the rear pawn is closer to the side of the board. This pawn structure is to a certain extent even less favourable for the stronger side. This is mainly explained by the fact that the king is not alongside its extra (rear) pawn, and therefore the advance of the latter (a breakthrough), which is sometimes possible, loses its effectiveness.
139. The black king cannot be driven out of the corner: 1 ... Ke8 2 Kd6 Kb8 3 Kd7 Ka8 4 a6 Kb8! Draw.
If the position is moved one or more ranks down the board, the result remains the same.

140. 1 ... Ke6! 2 Ke4 Kd6! with a draw (after 3 Kf5 Kc5 it is now White who has to worry about forcing a draw).

141. Even the existence of an extra tempo for White does not change the result: 1 Ke4 Kd6 2 Kb4. Now Black can play 2 ... Kd7, to answer 3 a4 with 3 ... Ke7, but the immediate 2 ... Ke7 3 a4 Kb7 is also possible, since after 4 Ke4 Ke7 5 Kd5 Kd7 Black defends the critical squares in time, and White's reserve tempo has already been used up.

142. 1 Ke3 Kd5 2 Kb3 Ke6.
Black's moves are forced: he cannot allow either the position Kd4/Kd6 because of a2-a3, nor the position Kb3/Kd6 in view of a2-a4. The squares d3, c3 and b3 strictly correspond to e5, d5 and c6.
In the resulting position White wins easily if he is able to give his opponent the move.
The usual method for this is 'triangulation', which in the given instance means using the rear squares b2 and c2. To maintain the balance, Black must use an analogous 'triangle'. But it transpires that, in reply to 3 Ke2 or 3 Kb2, Black cannot use the rear squares c7 and d7 because of Ke3 and Kd4, winning, while the triangle c6–c5–d6 is altogether ruled out in view of the fact that c5 is inaccessible. It follows that the only triangle remaining for Black is d6–d5–c6, but, in placing his king at d5 or c6, he allows the white king to occupy the corresponding square e3 or b3, and thus is bound to lose.

3 Ke2 or 3 Kb2, and Black is helpless: 3 Ke2 Kd6 (3 ... Kd5 4 Ke3, and if 4 ... Ke5, then 5 Kb3 and 6 a4, or 4 ... Kd6 5 Kd4, while 4 ... Kc6 can be met by 5 Kd4 or 5 Kb3) 4 Kb2 Kc6 5 Kb3 Kc5 6 Kd6 Ke6 7 Kd4 Kd6 8 a3, or 3 Kb2 Kd6 4 Ke2 Ke6 (4 ... Kd5 5 Ke3) 5 Kb3, and wins.

The solution offered establishes, for methodological aims, the correspondence of the three main squares (the 'front line'). Knowing this, one can shorten the solution by one move (by immediately using the 'triangle'): 1 Kc2! Kd6 (1 ... Kd4 2 Kb3; 1 ... Kd5 2 Ke3) 2 Kb2! Kd5 3 Ke3 Ke6 4 Kb3 etc.

If position 139 is moved one or more files to the right, White wins irrespective of who it is to move.

143. For example, with White to move: 1 Kd5! (1 Kf5? Kd6) 1 ... Kd8 2 Ke6 Ke8 3 b6 and wins.

But if this position is moved one or more ranks down the board (so that the black pawn is no longer on its initial square!), there is no win, irrespective of who it is to move.

144. In the analogous position with blocked bishop's pawns, but with the pawn at d4 instead of b4, White won if it was Black to move, 1 ... Kf6 being decisively met by 2 d5. But here the 2 b5 break is ineffective: 2 ... cxb5 3 c6 Ke6, or 3 Kd5 b4. White also achieves nothing by 2 Kf4 Ke6 3 Ke4 Kf6 4 Kd4 Ke6 5 Ke4 (with the threat of 6 b5) 5 ... Kd7 etc.

If it is White to move, he succeeds in driving the enemy king back one rank due to the fact that the d6 square is inaccessible to it, but the result is nevertheless a draw: 1 Kd4 Kd7! (1 ... Ke7? 2 Ke5, or 1 ... Kf7? 2 Kc4 Ke7 3 b5) 2 Ke5 Ke7 3 Kf5 Kf7 (4 Kg5 Ke6).

If position 142 is moved one file to the right (after which blocked bishop's pawns result), neither the result nor the character of the play change. By slightly varying this position, Grigoriev enriched theory with an interesting and original idea.

145. White has two tempi in reserve, but how is he to make use of them? At any rate, it
is clear that a direct attack on the critical squares d6, e6 and f6 does not hold any promise: the black king will move between e6 and f6, and the play will revert to the previous position.

White wins by a clever plan. Manoeuvring with the stated aim of seizing the centre, White transfers his king to a5, provoking the reply ... Kb7. After this he uses one of his tempi on b2–b3, forcing the diversion of the black king to a7. But this allows him to occupy e5, with the tempo needed for the win still in reserve. In carrying out this plan White has to overcome certain technical difficulties.

1 Ke2 Kg7.
1 ... Kb7 2 Kd3 Ka6 is unsatisfactory in view of 3 b4 Kb5 4 Kc3 Ka6 5 Kc4! Kb7 6 Kd4, winning.

2 Kd3 Ke7!
A critical point. If now 3 Kc4, then 3 ... Ke6!, and White is in zugzwang: 4 Kd4 (4 Kb4 Kd5) 4 ... Kf6! 5 Kc3 Ke5! 6 Kc4 Ke6! — draw. But there is another road open to White.

3 Kc3! Ke6.
Or 3 ... Kd7 4 Kb4. But now Black ends up in zugzwang. The whole point of these tactical manoeuvres lies in the decisive correspondence of the c4 and e6 squares.

4 Kc4! Kd7.
4 ... Ke5 is decisively met by 5 b4 and 6 b5.

Now the rest is simple: 5 Kb4 Kc7 6 Ka5 Kb7 7 b3! Ka7 8 Kb4 Kb7 (or 8 ... Ka6 9 Kc3, and if 9 ... Kb5 10 b4, or 9 ... Ka5 10 Kc4 Ka6 11 b4) 9 Kc4 Kc7 10 Kd4 Kd7 11 Ke5 Ke7 12 b4, and wins.

If one of the connected pawns is a central one, then in positions of type 143 a win can always be achieved, irrespective of who it is to move. But if such a position is moved down the board, the stronger side wins only if it is the opponent to move.

146. After 1 ... Kg6 (or 1 ... Ke7 2 Kg5 Kf7 3 Kf5) the black king is too far from a7 (6 moves), and White reaches a5 more quickly (5 moves).

Dedrle, 1921
147. This position is a classic example on the theme of the by-pass. White has to obtain the previous position with Black to move: 1 Kg2! (1 Kg1 is also possible) 1 ... Kf6.

If Black passively keeps his king at e7 and e8, the white king goes to g5.

2 Kf2!

On 1 ... Kf7 there would have followed 2 Kf3! Now White chooses his move, depending on where the black king goes to. For example, on 2 ... Ke5 he decides matters by taking his king to the Q-side: 3 Ke6! Kf6 4 Kd4 Ke7 5 Kc3 Kd7 6 Kb4 Kc7 7 Ka5 Kb7 8 Kb5, and White wins.

2 ... Ke7 3 Kg3! Kf7 4 Kf3! (4 Kf4 Kf8!) 4 ... Ke7 5 Kg4! Kf6 6 Kf4, and White has achieved his aim.

It should be mentioned that the routine taking of the opposition—1 Ke2(e1) would have been an irreparable mistake. By replying 1 ... Ke8! Black gains a draw.

If it is Black to move, he maintains the balance after 1 ... Kf7!, e.g. 2 Kg2 Kg8(g6) 3 Kh3 Kf7 (of course, not 3 ... Kh7 4 c5) 4 Kg4 Kg6 5 Kf4 Kf6 6 Ke4 Ke7 7 Ke3 Ke8! 8 Kd4 Kd7(d8) 9 Kc3 Kc7 10 Kb4 Kb6 11 Ka4 Ka6. Draw.

We will now turn to an examination of mobile pawn structures with no passed pawn, i.e. cases where the pawns are not blocked.

148. The simplest is 1 Kd3 (supporting the more mobile pawn!) 1 ... Kd6 2 Kc4 Ke6 3 e5, and White wins.

But he also wins by 1 Kf4 Kf6 2 Kg4 (2 e5 +? Kf7!—draw) 2 ... Kg7 (on 2 ... Kg6? White has the decisive 3 e5, since after 3 ... Kh6 Black is too late to defend his Q-side) 3 Kg5 Kf7 4 Kh6 Kf6 5 e5+.

If Black begins and plays 1 ... Kf6, White must not play 2 Kf4? because of 2 ... e5 + 3 dxe5 + Ke6 with a draw. 2 Kf3 or 2 Kd3 is correct.

It is clear that White also wins if the position is moved up or down the board. With the black pawn at e7 (white pawns at d5

5.15 No Passed Pawn, Pawns Mobile

If there is a direct convergence of the pawns, these can transpose into positions with blocked pawns.

A basic feature of this pawn structure is that one of the connected pawns has opposite it an enemy pawn on the same file, while the other, having no direct barrier, has a greater ability to advance. It is clear that, if the stronger side's king is on the side of the more mobile pawn, this increases the pawn's significance, and this is more favourable than having the king on the opposite side. Also of considerable significance is the fact that the stronger side has a reserve tempo when transposing into a blocked position.

Earlier we first examined positions with wing pawns, and then with central pawns. Here it will be expedient to do things in reverse order, since central positions are comparatively simple, while the flank positions, which are more important in practice, are not only more complicated, but also reveal a greater number of special features.

If the weaker side's pawn is on one of the central files (d- or e-), the game can be saved only in exceptional cases, e.g. when the king is so far away that it is unable to give the connected pawns the necessary support. The stronger side normally wins.
and e5) after 1 Kg5 Kf7 he also wins by 2 e6+.

The stronger side should not, without necessity, bring the pawns so close together that he has only one tempo in reserve. It is easier to win if the tempi are preserved.

150. 1 Kg3 Kf6 2 Kf4 e3!
Now 3 d × e3? loses to 3 ... d3 4 Kf3 Ke5! 5 Kf2 (or 5 e4 Kd4 6 e5 Ke3!, winning) 5 ... Ke4 6 Kf1 Kf3! 7 Ke1 K × e3.
Correct is 3 Kf3! Ke5 4 Ke2! — draw.

Fine, 1941

149. White’s immediate task is to occupy e6 with his king: 1 e3 Kd6 2 Kf5 Kd5 3 d3 Kd6 4 e4 Kd7 5 Ke5 Kd8 6 Ke6 Ke8 (now the king must go to d7 or f7) 7 e5 (or 7 d4) 7 ... Kf8 8 Kd7 Kf7 (it only remains to win the e7 pawn) 9 d4 Kf8 10 e6, and White wins.

In certain positions the manoeuvring of the kings is more complicated.

151. 1 Kc2! (1 Ke2? Kc3 2 Ke3 e5!) 1 ... Kc5 2 Kd1! (not 2 Kc3 or 2 Kc1, since after 2 ... e5 Black has the opposition on the main file, e.g. 2 Kc1 e5 3 Kd1 Kd6! 4 Kd2 Kd7! 5 Kc3 — White has won back the rank, but not the opposition — 5 ... Ke7!, with a draw) 2 ... Kd4 (2 ... e5 3 Ke2!; 2 ... Kb4 3 Kd2! e5 4 Kc2! Kc3 5 Kc3) 3 Kd2 e6 (3 ... Kc5 4 Kc3) 4 Kc2 Kc5 5 Kd1 Kd4 6 Kc2 Kc5 7 Ke3 e5 8 Ke2 Kd4 (8 ... Kd6 9 Kf3; 8 ... Kb4 9 Kf3 Kc3 10 Ke3 Kdb 11 d4) 9 Kd2 Kc5 10 Ke3, and wins.

The following position has a solution of typical ‘Grigoriev’ difficulty.

152. Nothing is given by the plausible 1 e4 in view of 1 ... Ka2! (1 ... Kb2? 2 e5! e6 3 Kb4 with a draw) 2 e5 e6 3 Kc3 (the attempt to win the d-pawn is hopeless, since the black king occupies d5 in time) 3 ... Ka3 4 Kc4 Ka4 5 Kc5 Kd3, and wins.

1 Ke5 also fails to save the game after 1 ... Kb2 2 Kd6 Kc3 3 Kc7 e5 (or 3 ... d5).
The only defensive possibility lies in exploiting the poor position of the black king. To this end 1 Kc3? does not work, since it leaves Black the reply 1 . . . Ka2.

1 Kb3! e6.

If 1 . . . Kb1, then 2 e4 Kc1 (after 2 . . . e6 3 e5 the king cannot move off the 1st rank) 3 Kc3 Kd1 4 Kd3 Ke1 5 e5! (5 Ke3? e5!) 5 . . . Kf2 (or 5 . . . e6 6 Ke3) 6 e6 d6 7 Ke4 Kg3 8 Kf5, with a draw.

2 e4! d6 (or 2 . . . Kb1 3 e5—draw) 3 e5!

This forces the move. After 3 . . . d5 4 Ka3 the black king remains shut in, and the advance of the d-pawn is not any use.

If the weaker side has a bishop's pawn, a factor of considerable importance is whether or not one of the connected pawns is a central one, and also, in certain cases, on which side of the pawns is the king supporting them, and who it is to move.

153. Due to the lack of space, a breakthrough by the king on the right is not possible. But as before White wins by 1 Kg4 Kg6 2 Kf3 (2 f5+? Kf7 or 2 . . . Kg7 with a draw) 2 . . . Kf7 3 Ke3 Ke7 4 Kd4 Kd6 5 f5 etc.

With Black to move: 1 . . . Kg6 2 Kg3! Kf7 3 Kf3 Ke7 4 Ke3 Kd7 5 Kd4 Ke6 (5 . . . Kd6 6 f5) 6 Ke4 Kd6 7 Kb5, and wins.

154. Here White does not have a central pawn, and therefore it is a draw irrespective of the turn to move. On reaching the position Kd4/Kd6 White can agree a draw after 1 f5 Kc6 or 1 Kc4 Kc6; he can win only if it is Black to move (1 . . . Kc6 2 Ke4, or 1 . . . Ke6 2 Kc5), but it is not possible to achieve such a situation.

If the position is moved up the board, White wins, while if it is moved down the result is a draw. But if the kings are moved to the d-file, as before White wins only if it is his opponent to move.

If position 153 is moved up the board, the win becomes even simpler. But if it is moved one or two ranks down the board, Black
acquires an amazing drawing possibility, but only if it is his move.

155. The secret of the defence is that Black must always be able to answer Kh4 with ... Kg6, Kg2 with ... Kf6, and Kg3 with ... Kf7. His king must be one file closer to the centre than White's. Why this is necessary will now become clear.

1 ... Kg5!

If 1 ... Kg6?, then 2 Kh4, and White wins; since 3 Kh5 cannot be allowed, Black is forced to play 2 ... Kh6, but then White transfers his king to d3, attaining the position Kd3/Kd5, and wins by f3–f4, as in example 153. For a successful defence Black must answer Kd3 with ... Ke5!

2 Kg2 (2 Kg3 f4+; 2 Kh2 Kg6) 2 ... Kf6! 3 Kg3, and we reach Dedrle's study (156).

156. 1 ... Kf7! (retaining the possibility of answering 2 Kh4 with 2 ... Kg6, and 2 Kg2 or 2 Kf4 with 2 ... Kf6) 2 Kf2 Ke6 3 Ke2 Kd5(d6) 4 Kd3 (4 Kd2 Kd6!, or 4 ... Ke6!) 4 ... Ke5! 5 Kc3 (5 c4 Kd6!) 5 ... Kd5 6 Kb4 f4! (this is the whole point; if 7 e×f4, then 7 ... Ke6, and if the pawn is not captured, it is sufficiently close to the queening square; this was not the case in example 153) 7 e4 + Kd4 8

Kb3 Ke3 9 e5 K×f3 10 e6 Kg2 11 e7 f3 12 e8 = Q f2. Draw.

If it is White to move in position 155, he wins by Kg3–f2 etc., and in Dedrle's study by 1 Kf4.

157. 1 ... Kg4! 2 Kg1 f3!, and after 3 e3 Kf5 4 Kf1 (4 Kh2 Kg4; 4 Kh1 Kg5) 4 ... Ke4 we reach Grigoriev's study (12): 5 Ke1 Kd3 6 Kd1 Ke3 7 Kc1 Kd3. Draw.

White to move wins by 1 Kg2(g1) etc.

If the kings are replaced at d2 and d4
respectively. Black is no longer saved by the turn to move: \(1 \ldots \text{Kc}4 \text{ 2 e}4\!), and wins.

Keres, 1943

\[158\]

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158. 1 \text{Kh}2! \text{Kf}5 2 \text{Kg}2! \text{Kf}6 3 \text{Kf}1 \text{Ke}5 4 \text{Ke}1 \text{Kd}4 5 \text{Kd}2 \text{Ke}4 6 \text{e}4! \text{Kd}4 7 \text{f}3 \text{Kc}4 8 \text{Ke}2 \text{Kd}4 9 \text{Kf}2 \text{Ke}5 10 \text{Kf}1, \text{and White wins as in example 114.}

The existence of reserve tempi naturally makes the win more probable, although sometimes there may be certain difficulties.

Berger, 1890

\[160\]

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a & b & c & d & e & f & g & h & \\
\hline
\end{tabular}
\end{center}

160. Fine (1941) suggested the following solution: \(1 \text{Kd}4 \text{Kf}5 (1 \ldots \text{Kd}6 2 \text{g}4) 2 \text{Ke}3 \text{Kg}4 (2 \ldots \text{Ke}6 3 \text{Ke}4) 3 \text{Ke}4 \text{Kh}4! (3 \ldots \text{Kg}3? 4 \text{Kf}5) 4 \text{Kf}3! (4 \text{Kf}5? \text{Kg}3) 4 \ldots \text{f}5 (4 \ldots \text{Kh}5 5 \text{g}3 \text{f}5 6 \text{Kf}2! \text{Kg}6 7 \text{Ke}3, \text{or 6} \ldots \text{Kg}4 7 \text{Kg}2 \text{Kh}5 8 \text{Kf}3 \text{Kg}6 9 \text{Ke}3) 5 \text{Kf}2! (5 \text{g}3+) \text{Kh}3 6 \text{Kf}2 \text{Kg}2! 5 \ldots \text{Kg}4 6 \text{g}3 \text{Kh}5 (6 \ldots \text{Kh}3 7 \text{Kf}3 \text{Kh}2 8 \text{g}4) 7 \text{Kf}3! \text{Kg}6 8 \text{Ke}3, \text{and wins.}

This winning method retains its significance, if, for example, the position \text{Ke}3;\text{Kf}5 or \text{Ke}4,\text{Kg}4 arises with Black to move. But in the initial position the win is achieved much more simply ‘à la Grigoriev’ by \(1 \text{f}5 +! \text{Kd}6 2 \text{Kf}4! \text{etc. (145).}

161. This position arose in a game \text{Ed.-Lasker-Ward (1913)}, in which \text{g}4+ led to a win as a result of a mistake by the opponent, but the position is of course drawn (154).

The opinion was expressed that there was nevertheless a win, but that it was achieved by 1 \text{Kh}2, and if 1 \ldots \text{Kh}6, then 2 \text{Kg}2 \text{Kg}6 3 \text{Kf}2 \text{etc.}, transposing to position 159.
Leick showed that after 1 Kh2 Black can immediately force a draw by 1 ... f5. A stronger try is 1 Kg2, but against accurate defence it too is ineffective: 1 ... Kg4! (all other moves lose: 1 ... f5 2 Kf3; 1 ... Kg6 2 Kf2 Kg5 or 2 ... Kf7 3 Kf3 Kc6 4 Kg4) 2 Kh2 (if 2 Kf2, then 2 ... Kh3 3 Kf3 f5) 2 ... Kf5! (the only move: 2 ... Kf3 3 Kh3; 2 ... Kh5 3 Kg1) 3 Kg1 (3 Kg2 Kg4; 3 Kh3 Kg6 4 K∞ f5) 3 ... Ke6! Again the only move, but now Black threatens 4 ... f5, and after 4 g4 the draw is also obvious.

Mattison, 1918*

163. White’s pawns are under threat, and sooner or later he will have to play c2–c3, so as to parry the attack ... Ke2 with b2–b4. The reply ... c4 is not to be feared, since after Kg4 the king reaches d4 in time. But even so the immediate 1 c3 does not work in view of 1 ... Kf3! 2 Kh4 (2 Kh2? even loses after 2 ... c4) 2 ... Kf4 with a draw. It is clear that the white king must avoid ‘imprisonment’ and come into play.

1 Kg4! Ke3.

Should White now continue 2 Kf5? But Black is already threatening 2 ... Kd2, on which there follows 3 c3 Kc2 4 b4, and here a new defensive resource appears: 4 ... c4!, i.e. the threat of drawing thanks to a bishop’s pawn against a queen. To prevent this, the white king must be sufficiently close (at f3—119). Therefore, for the moment it cannot be moved off the 4th rank. The pawn must nevertheless be advanced.

2 c3! Ke4 (an attempt to save the game in a different way) 3 Kg5 Ke5.
If 3 ... Kd3, then of course 4 Kf4! The following king 'walk' is to White's advantage: the danger to his pawns is reduced, and it only remains for him to attack the black pawn from the rear. To do this he must go round the enemy king (standing a knight's move away from it), and in this he is helped by the existence of reserve tempi (b2–b3 or c3–c4).

The subsequent play is now understandable, although in each variation a precise concrete calculation is involved.

4 Kg6 Ke6 5 Kg7!

It is early as yet for 5 b3 in view of 5 ... Ke5 6 Kf7 c4! Now there are three possibilities:

(a) 5 ... Ke7 6 c4! Ke6 7 Kf8 Ke5 (7 ... Kf6 8 b3) 8 Ke7! Kd4 9 b3 Ke3 10 Kd6 and wins.

(b) 5 ... c4 6 Kg6 (6 Kf8? Kf6—draw) 6 ... Ke5 7 Kg5 Ke4 8 Kf6! Kd3 9 Ke5 Kc2 10 Kd4 and wins.

(c) 5 ... Kf5 6 Kf8!! (the only move: 6 Kf7 c4; 6 c4 Ke4; 6 Kg8 Kf6!) 6 ... c4 (or 6 ... Ke6 7 c4) 7 Kf7! Ke5 8 Ke7, and White wins.

When the defender has a knight's pawn, it is also of great importance whether or not the opponent's 'semi-passed' pawn is closer to the centre.

165. An interesting position. It is of no importance who begins, since both on his own move, and in reply to 1 Kd5, Black plays the identical 1 ... Kf7 (1 ... Kh7 2 h6). After 2 Kd6 Kf8 3 Ke6 the draw is attained by 3 ... Ke8! 4 Kf5 Kf7!

But it would be wrong, in apparent analogy with example 139, to hide the king in the corner before g5–g6 is played. E.g. 3 ... Kg8? 4 Ke7 Kh7 (4 ... Kh8 is no better: 5 Kf7 Kh7 6 h6! or 6 g6 + ) 5 Kf8.

Also possible is 5 Kf7 Kh8, and now not 6 h6? Kh7!, but 6 Kg6! Kg8 7 h6 Kh8 (7 ... g × h6 8 K × h6) 8 h × g7 + ! Kg8 9 Kh6, and wins.

5 ... Kh8 6 h6, and White wins.

If White's king is replaced at g4 and Black's at h7, then Black to play can draw by 1 ... Kg8!, but if it is the opponent to move he loses: 1 Kf5 Kg8 (here 1 ... g6 + does not force a draw, since there follows 2 Kf6 g × h5 3 Kf7) 2 Kg6 Kh8 3 Kf7! (3 h6? Kg8!—draw) 3 ... Kh7 4 h6 g6 5 Kf6 etc.
black king on the 7th rank are won.) Black must maintain the opposition on the e- and d-files: 1 \textit{Kd4 Kd6} (2 \textit{Kc4 Ke5}).

If 1 \textit{Kf4}, then 1 \ldots \textit{Kf6}, guarding the g5 and e5 squares against invasion. But how should Black play in reply to 2 \textit{Kf3}? He cannot play 2 \ldots \textit{Kc6} 3 \textit{Ke4}, or 2 \ldots \textit{Ke7} 3 \textit{Kd5}, when White has the opposition. The only move is 2 \ldots \textit{Kf7}! Now 3 \textit{Kg3} can be met by 3 \ldots \textit{Kf6}, since 4 \textit{Kf4} achieves nothing due to 4 \ldots g5+. When Black is threatened with the loss of the opposition, his king finds a shelter on the f-file. E.g. 2 \textit{Ke3 Ke5} (or 2 \ldots \textit{Kc7}) 3 \textit{Kf3 Kf6}!

Now this has been explained, it will be easy to understand the play in the following example. Incidentally, here a new defensive resource for the weaker side is revealed, when the white pawns are on the 3rd, and also the 2nd rank.

\ \[\text{Grigoriev, 1936}\]

\[\text{168. In view of the threat of h3-h4, Black must step into the 'square' of the h-pawn, i.e. it must move onto the c-file, but in any case the opposition remains with White. Salvation must be sought on the f-file!}\]

1 \ldots \textit{Kc7} 2 \textit{Kc1} (after 2 \textit{Kc2 Kc6}! the opposition passes to Black) 2 \ldots \textit{Kd7} 3 \textit{Kd1 Ke7} 4 \textit{Ke1} (White still cannot begin a by-pass:}

166. White cannot win: 1 \textit{Kh4 Ke6}! (not 1 \ldots \textit{Kf5}? 2 \textit{g6 Kf6 3 Kg4 etc.}) 2 \textit{g6 Kf5}!, or 1 \textit{Kg4 Ke6} 2 \textit{g6 Kf6}! 3 \textit{Kf4 Ke7} 4 \textit{Kg5 Kf8} 5 \textit{h6 Kg8}—draw. The point is that, after g5–g6, the squares g4/f6 and h4/f5 are mutually corresponding (in view of the threat of h5–h6).

Maintaining the opposition also saves Black in positions moved one or more ranks down the board.

167. Due to the existence of a reserve tempo for White (g4–g5), the squares d5 and e5 are critical. (Therefore analogous positions with the white king at d5 or e5 and the
Two Pawns against One

4 Ke2 Ke6! 5 Kc3 Ke5 6 Kd3 Kd5—draw) 4 ... Kf6.

Now 5 Kf2 Kf5 would lead to an obvious draw, so it is here that White begins a dangerous by-pass.

5 Kd2 (Black seems to have reached an impasse: all king moves lose) 5 ... g4! Here it is, the saving move! After 6 b4 Ke6! Black attains position 115 (the h-pawn has not crossed the middle of the board!), and for a draw it is sufficient for his king to move between d6 and e6, as long as White’s king remains on the 2nd rank.

The stronger side has every justification in hoping for a win, if the pawns have not yet converged as far as possible, and especially if his king is in front of his pawns.

169. Here the only move not to win is 1 g5? in view of 1 ... g6. Curiously, in the 1st edition of his Chess Fundamentals, Capablanca asserted (he later corrected this) that 1 f5 also did not win in view of 1 ... g6 (he left the analysis of the variation to the reader). Capablanca gave the solution 1 Ke4 Ke6 2 f5+ Kf6 3 Kf4 etc.

But it is precisely by 1 f5! that White wins more quickly. If 1 ... g6, then 2 f×g6 Ke7.

Or 2 ... Ke6 3 g5! Ke7 4 Ke5 Ke8 5 Ke6 Kf8 6 Kf6 Kg8 7 g7 Kh7 8 g8 = Q+! By 2 ... Ke7 Black provokes his opponent into a premature advance of his rear pawn: 3 g5? Kf8 4 Ke5 Kg7 5 Kf5 Kg8 6 Kf6 Kf8, or 6 ... Kh8. Draw.

3 Ke5!, and White wins.

In the following two examples the pawns are weakened by the lack of support from their own king.

Novikov, 1930

170. 1 Kf2 Kg7 2 Kg3 h5 3 Kh4 Kg6 4 g3!, and now 4 ... Kh6 leads to stalemate.

Position 171 is of great theoretical significance.

Gorgiev, 1936

171. 1 Kf2 Kh6 (1 ... Kg7 2 g4 h4 3 g5; 1 ... g5 2 g3) 2 Kg3 Kg5 3 Kh3! Kf5 (3 ... Kf4 4 Kh4 Kf5 5 g3! Kf6 6 g4 with a draw) 4 Kh4! Kf4 5 g4! h×g4. Stalemate.
172. This position—the object of numerous discussions in the past—received a correct evaluation only 80 years after its publication!

Contrary to the opinions of von der Lasa (who suggested this position), Berger and Fine, White cannot win if it is the opponent to move. The demonstration of a win with White to move has also contained mistakes, and requires correcting.

Berger incorrectly gave a win by 1 Kg3 Kg5 2 h3 Kh5 3 Kf3 Kh4 (or first 3 ... Kg5 4 Ke4 Kh4) 4 Kf4, continuing now 4 ... g5 + ? or 4 ... Kh5?, which does indeed lose. But, as shown by Sacconi, 4 ... g6! ensures a draw.

Chéron gave a more precise solution: 1 Kg3 Kg5 2 h4+! Kh5 (2 ... Kf5 3 Kf3 g6 4 g3) 3 Kh3 g6 4 g3! g5 5 g4+ and wins.

An instructive, although more lengthy solution, is that given by Sacconi: 1 Kf3 Kg5 (1 ... g6 2 h4; 1 ... g5 2 g4+, transposing to example 142, but 2 Kg3 and 3 h4 is of course simpler) 2 Ke4 Kg4 3 Ke5 Kg5 4 Ke6 Kg6 5 Ke7 Kh6 6 Kf7 Kh6 7 g4 Kh7 (7 ... Kg5 8 h3 Kh6 9 h4) 8 g5 Kh8 9 Kg6 etc. (cf. the notes to position 122; in the given case after 9 ... Kg8 the king stands on the required square for h5–h6, therefore the pawn must move to a square of the opposite colour: 10 h4).

With Black to move the play is more complicated. The main variation of the solution given below is based on an unpublished analysis by Grigoriev.*

1 ... Kf4! (now the white king is unable to stand in front of its pawns) 2 Ke2.

If 2 g3+, any move by the king onto a white square is sufficient to draw: 2 ... Kf5 3 Kf3 Kg5, as in the main variation; 3 h4 g4 4 h5 g4; 3 h3 g5 4 Ke3 Ke5 (or 4 ... g4 5 h4 Ke5—115). Draw. Simplest of all is 2 ... Kg4:

2 ... Ke4! 3 g3 Kf5!

Berger and Fine continue 3 ... g5?, which loses immediately to 4 h3. Black also loses after 3 ... g6? 4 h4 Ke5 (4 ... Kf5 5 Kg3 g5 6 g4+ or 5 ... Ke5 6 Kg4 and 7 h5) 5 Ke3 g5 6 h5 Kf5 (6 ... g4 7 h6) 7 g4+ Ke5 8 Kd3 (this is simpler than 8 Kf3 Kf5).

4 Kg3.

4 h4 Kg4 5 Kf2 is obviously weak in view of 5 ... g5, while 4 h3 transposes into the main variation after 4 ... Ke4 or 4 ... Ke6 (4 ... g5? 5 Kd3!; 4 ... g6? 5 Ke3!) 5 Kf2 (or 5 h4 Kf5 6 Kf3 g6! 7 g4+ Kf6!—167) 5 ... Kf5 6 Ke3 (6 Kf3 g5!) 6 ... Ke5 7 Kf3 Kf5 8 g4+ Kg5!

4 ... Kg5! (but not 4 ... g6? 5 h4, or 4 ... g5? 5 g4+—142) 5 h3 (if 5 h4+, then 5 ... Kh5 6 Kf4 g6—171, while on 5 g4 there follows 5 ... Kh4!) 5 ... Kf5!

Chéron continues here 6 h4 g6! (Sacconi considers only 6 ... Kg6? and 6 ... g5? 7 g4+ Kf6!—draw (167). The same finish results from 6 g4+ Kg5 7 Kg3 g6! 8 h4+ Kf6!)

Grigoriev's analysis sets Black one further small test:

6 Ke3 Ke5 7 Kd3 Kd5!

* This variation, without any comments, was written by Grigoriev in 1925 in the margin of a copy of Berger's book (refuting the latter's conclusions). The correct solution was also shown by Chéron (1952), quoting Sacconi; however, the analysis of the latter, beginning with the 6th move, is incorrect, and leads to the erroneous conclusion that White wins.
Two Pawns against One

How should one interpret this exclamation mark, attached without any explanation?

It is natural that on 7...g5 White wins by 8 Ke3; but if Black keeps a tempo in reserve by playing 7...g6, White achieves the same by the subtle 8 Ke2!! with the following continuations: 8...Ke4 9 h4; 8...Kf5 9 Ke3; 8...Ke6 or 8...Kf6—9 Kf3, and wins.

8 h4 Ke5 9 Ke3 Kf5 10 Kf3 g6! 11 g4+ Kf6!, with a draw as in example 167.

White’s active king position enables him to win easily in the following example.

Sarkozy-Zinner
Budapest, 1925

\[\text{Diagram 174 (piece configuration)}\]

5 Kf6! Kh4 6 Kf5!

A characteristic position for the kings: the side that achieves it by his move places the opponent in zugzwang.

6...Kh5 7 Ke5!

Again the only move. If 7 Ke4?, then 7...Kg4 and 8...f3. But now on 7...Kg4 there follows 8 Kf6.

7...Kg6 8 Ke4! Kh5 (9 g3 was threatened) 9 Ke5! Draw.

Nothing changes if after 1 Kd5 Black plays more reservedly, and tries to dislodge the white king from the corresponding squares:

(a) 1...Kg5 (1...Kh6 2 Ke6 f5 3 g3!, or 2...Kg7 3 g4!; 1...f5 2 Ke6 f4 3 Kf6 g5 4 Ke5!) 2 Ke6 f5 3 Kf7 f4 (3...Kh5 4 Kf6 Kh6 5 Ke6! Kg5 6 Ke5 Kg4 7 Kf6 Kh5—7...g5 8 Ke5—8 g3! with a draw) 4 Ke6! Kg4 5 Kf6 Kf5 6 Ke6! (6 Ke5? g5!) 6...Kg4 7 Kf6 g5 8 Kg6! Kh4 9 Kf5!

(b) 1...Kg4 2 Ke6! Kg5 3 g3! (3 Kf7? Kf5!) 3...f5 4 Kf7! Kh5 5 Kf6! Kh6 6 Ke6!

It should nevertheless be clearly understood that, although Black cannot win here, this is only because for free manoeuvring he does not have at his disposal another file (to the right of the h-file). This is explained in the following examples.

66
175. In this position White resigned. Rabinovich (1938) gave the following possible variation: 1 Kh6! f5 2 Kh5! Kd4!! (here is the move, the equivalent of which Black did not have in example 174) 3 Kh4 (or 3 Kg5 Ke4!) 3 ... Kd3! 4 Kg3 (or 4 Kg5) 4 ... Ke4, and wins.

177. Here 1 f4 or 1 f3 cannot achieve anything after 1 ... Kf2 (this is clear from example 174). To win, White must exploit the distant position of the black king.

1 Kg2! Ke2.

Now the king must not be granted the e3 square, e.g. 2 f4? Ke3 3 Kh3 g6!, or 2 f3? Ke3 3 g4 g6! 4 Kg3 g5—draw.

2 g4! Kd3 3 Kf3!

Other moves lead to a draw: 3 Kh3 Ke4 4 Kg3 g5; 3 f4 Ke4 4 Kg3 g6!; 3 Kg3 Ke4! 4 f3 + Ke5 5 f4 + Ke4 6 g5 (if 6 f3, then 6 ... Ke5 7 Kh4 Ke4?) 6 ... Kf5 7 Kf3 Ke6 etc.

3 ... g5 4 Kg2! (4 Kf3? Ke5 5 f3 + Ke3) 4 ... Kd4.

If 4 ... Ke4, then 5 Kg3 Ke5 (otherwise 6 f4) 6 Kf3 Kd4 7 Ke2 Ke4 8 f3 + Kd4 9 Kd2 Kd5 10 Kd3, and wins.

But now nothing is achieved by either 5 Kf3 Kd3, or 5 Kg3 Ke4, or 5 Kh3 Ke5 6 Kg3 Ke4.

5 Kf1! Ke5 (on 5 ... Kd3 there follows 6 f4)

6 Ke2 Ke4 (if 6 ... Kf4 or 6 ... Kd4, then 7 f3) 7 f3 +, and White wins.

An analytical gem!
178. 1 b3! (the threat of ... b4 must be averted, but 1 b4? does not work because of 1 ... Ka6! with a draw) 1 ... Ka5 (1 ... Ka6 2 b4! Kb6 3 Kb8, and wins) 2 Kb8! (2 b4+? Ka6; 2 Kb7? b4 3 c4 — stalemate) 2 ... b4 (2 ... Kb6 3 b4) 3 c4 Kb6 4 Kc8 Ke6 5 Kb6 6 Ke8 Ke6 7 Kf8 Kf6 8 Kg8 Kg6 9 Kh8 Kf6 (9 ... Kh6 10 c5) 10 Kh7 Kf7 11 Kb6 Kf6 12 Kg5 Kf5 13 Kb4 Kf4 14 Kh3 Kf3 (14 ... Kf3 15 c5) 15 Kg3 Kg5 16 Kf3 Kf5 17 Ke3 Ke5 18 Kd3, and White wins.

This amusing king march provoked a number of imitative studies.

If the weaker side has a rook’s pawn, the number of drawn positions increases considerably. These endings occur very often in practice, and therefore deserve special attention.

179. It is of no importance whether the king is at f7 or g7. If it is Black to move he plays 1 ... h6 2 g6+ Kg7, but he can also continue 1 ... Kg7 2 Ke6 Kh8 3 Kf7 h6, or 2 ... Kg8 3 Kf6 Kh8 4 g6 Kg8+, with a draw.

If it is White to move, 1 h6 leads to position 132.

180. Black is no longer saved by 1 ... h5 2 g5+ (108), or by 1 ... Ke6 2 Ke4 Kf6 3 Kd5 Kg6 4 Ke5 Kg7 5 Kf5 Kf7 6 h5.

With White to move, 1 h5 leads to a draw (134).

If the kings have not approached as near as possible, the manœuvreing takes place on the basis of the normal opposition.

181. After a move by the white king onto
the f-file, Black draws by taking the opposition on it. If it is Black’s move, he draws by 1... Kg6!

182. The win is achieved by 1 Kf1! Black to move draws by 1... Kg5(g6) or 1... Ke5(e6).

The result clearly does not change if position 180 is moved one rank down, but if it is moved two ranks down the result will again be a draw, irrespective of the turn to move.

183. 1... Ke4 (1... h3? 2 g3+ Kg4 3 Ke3 Kg5 4 Kd4!) 2 Ke2 Kf4 3 Kd3 h3! 4 g3+ Kf3 etc.

Thus in the examples considered (179–183) a draw is inevitable if the black pawn or the white pawns are on their initial squares; in intermediate positions the possession of the opposition is the decisive factor.

If the kings are differently placed, the result may change. For example, if in example 183 the black king stands at g4, White to play can win.

An interesting situation is illustrated by the following example.

184. 1 g6 leads to a draw, but with Black to move (after 1... Kh8) 2 g6 wins. We have here a zugzwang position, in which the turn to move is unfavourable for either side. There is a similar zugzwang in the position with the black king at h8 and the pawn at g4.

It follows that, when the connected pawns stand on squares of different colour, the kings too should occupy squares of different colour; while if the connected pawns are on squares of the same colour, the kings too should stand on squares of the same colour.

This observation (Bähr, 1936) simplifies preliminary calculations.
185. The connected pawns stand on squares of the same colour; therefore a draw is given by 1... Kg8! 2 Kh6 Kh8!. If 1 h4, Black must play 1... Kh8(!!) 2 Kh6 Kg8!

Having free control over g8 and h8, Black cannot lose, whatever move order White resorts to.

186. But here, where the g-pawn has not yet moved, White can always tip the scales in his favour. If he begins, he plays 1 g4!, while if his opponent begins he answers 1... Kg8 with 2 g3!

This ‘colour rule’ provides a quick evaluation of a position.

Exceptions are provided by positions 187 and 188, in which White cannot win, in spite of the fact that one of his pawns has not yet moved.

187. White cannot place his king at h6, without making a preliminary move with his g-pawn, but in advancing it he loses all his advantage, and the result is a draw. After 1 Kf5 Black draws by any king move, except 1... Kh6? in view of 2 Kg4 Kg7 3 Kg5, when it is Black to move.

188. One of the pawns has advanced too far, and White does not have a tempo in reserve. If 1 Kh5, then 1... h6! with a draw.

Marshall-Schlechter
1911

189. It is easy to see that after 1... Kg4 2 Kf2 Kh3 3 Kg1 White is unable to occupy the corresponding (by colour) h1 square and is bound to lose.
190. As we have seen, Black to move cannot win, but even with White to move there is no win: 1 Kg2 Kg4 2 Kh1 (2 Kf2 or 2 Kf1 is also possible) 2 ... Kf3 (2 ... Kh3 3 Kg1—185) 3 Kg1 g4 (or 3 ... h4) 4 Kf1! (4 Kh1? Kf2 5 h3 g3 or 5 h4 Kg3) 4 ... h4 5 Kg1 (or 5 h3) 5 ... g3 6 h x g3 or 6 h3. Draw.

Thus here the stronger side is not even helped by the fact that his king is in front of his pawns.

The following examples show the manoeuvring of the kings in positions with various features.

Grigoriev, 1930

191. In this position, which is a very important one for theory, White wins only if it is his move.

It should first be mentioned that the black king should not be allowed to go to g6 earlier than necessary: 1 g4? Kg6 2 Kg3 h5; 1 Kg4? Kg6 2 Kf4 Kg6 3 g4 Kg6 4 h4 Kg6—draw. White's preparations are complete 3 Kg4! Kg6.

Now 4 Kf4 Kg6 5 g4 appears to win, but in fact on 4 Kf4? there follows 4 ... Kh5!, and 5 Kf5 is not possible due to stalemate, while if 5 Kf3, then 5 ... Kg6 6 Kc4 Kg6 7 g4 Ke6—draw. This zugzwang position arising after 4 ... Kh5! is the reason for the subsequent play.

4 Kf3!! Kg5.

Other moves do not help: 4 ... Kh5 5 Kf4 (now it is Black who is in zugzwang!) 5 ... Kg6 6 Ke5! Kh5 7 Kf6!! 4 ... h5 or 4 ... Kf6—5 Ke4; 4 ... Kg7 5 Kf4! (5 g4? Kf7!!; 5 Ke4? Kf6) 5 ... Kf6 6 g4 Ke6 7 Kf4, and wins.

5 g4 + Ke5 6 Ke3, and White wins, e.g. 6 ... Kd5 7 Kf4 Ke6 8 Ke4 Kf6 9 Kd5 Kf7 10 Ke5 Kf7 11 Kf5 Kg7 12 Ke6.

Here Grigoriev, wishing to take play into a theoretically interesting position with the maximum difficulties for White, committed an inaccuracy: 12 ... Kg6 (with the intention of 13 Ke7 h5), overlooking 13 h5+ (13 ... Kg5 14 Kf7). But the position for which he was aiming can be achieved by force by a simple transposition of moves: 12 ... h5! 13 g5 Kg6. Now 14 Ke7 Kg7 and 14 Ke5 Kf7!! 15 Kf5 Kg7 16 Ke6! Kg6 lead only to a loss of time. Correct is 14 Kd6!!, since the moves necessary for retaining the opposition, 14 ... Kf6 or 14 ... Kh6, are not possible (109).

A comment should be added to Grigoriev's analysis. Black does not lose because his pawn has been weakened by its advance to h6, as might seem to be the case at first sight. The reason for his defeat lies in the unfavourable position of his king at h7. With his king at g7 it would be a draw, e.g. 1 Kh5 Kh7 2 h4 Kg7 3 Kc4 Kf6! (this is the whole point; 3 ...
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Kg6? loses to 4 Kf3!). Now on 4 Kf3 there follows 4... Kg6!, while 4 Kf4 can be met by 4... Ke6 or 4... h5.

Thus if it is Black to play in our example, he draws by 1... Kg7!

The same idea, but in a different situation, provides the main interest in example 192.

Grigoriev, 1930

192. 1 Kh5!

Of course, not 1 h3? or 1 Kh3? because of 1... Kf4, nor 1 Kg3? in view of 1... Kg5 2 Kf3 Kh4, transposing into position 186. It is important for the theory of these endings that on 1 Kh3 Black also wins by 1... Kg5 2 Kg3 h5!, e.g. 3 Kf3 Kh4; 3 Kh3 h4 4 Kg2 Kg4; 3 h3 Kf5 4 Kh4 (4 Kf3 g5) 4... g6 (the same zugzwang as in example 191) 5 Kg3 Ke4 etc.

1... Kf6.

1... Kf4 2 Kg6 Kf3 3 Kxg7 h5 does not give a win in view of the by-passing manoeuvre Kf6-e5-d4-e3 (59).

2 h4! Kf7.

In contrast to example 191, here it is the weaker side that uses zugzwang to his advantage. A draw results both from 2... Ke7 3 Kg6 Kf8 4 h5, and from 2... Ke5 3 Kg6 Kf4 4 Kxg7 h5 5 Kf6!

3 Kg4 Ke6 (3... g6 4 Kf3!) 4 Kf4 (4 Kh5? Kf6! 5 Kg4 Ke5, and wins) 4... g6 5 Ke4.

Draw.

Loll, 1763

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194. This position initially provoked conflicting assessments. With Black to move the draw is simple: 1... Kf4 2 g3+ (2 h3 h6! 3 g3+ Kf5 4 Kf3 h5) 2... Kg4 3 Kg2 h5 4 h3+ Kg5! 5 Kf3 Kf5.

It wasn’t immediately that the correct defence was found with White to play: 1 Kf3 (1 Kg3 Kg5 2 h4+ Kf5 or 2 h3 h6!) 1... h5! (Reichhelm, 1873) 2 Ke3 (2 Kg3 Kg5; 2 h3 h4; 2 h4 Ke5 3 g3 Kf5) 2... Ke5 3 g3 Kf5 4 h3 Ke5 etc.

White can obtain winning chances only if he is able to occupy h6 with his king (184), or to attack the h7 pawn from the rear.

194. This position arose from the previous one at the time when (under the influence of
Connected Pawns

Lolli and Walker) it was thought that Black did best not to move his pawn from h7 (1 Kf3 Kg5 2 Ke4 Kg4 3 Ke5 Kg5). Analysis by White in 1873 showed a win here.

1 Ke6 Kg6.

Chéron (1952) observes that if 1 ... h5 (if the king moves anywhere other than g6, then 2 Kf6, while if 1 ... h6 2 Kf7 h5 3 h4 4 Kg7), then 2 Ke5! (2 Kf7? h4!—with the threat of 3 ... h3—3 h3 Kf5! with a draw) 2 ... Kg4(h4) 3 Kf6 and 4 h3+.

2 h3 Kg7.

Other moves also fail to draw: 2 ... h6 (2 ... h5 3 h4) 3 g3; 2 ... Kg5 3 Kf7 Kh6 (3 ... h5 4 g3 Kf5 5 h4 Kg4 6 Kg6) 4 Kf6 Kh5 5 Kg7 h6 6 g3 Kg7 7 h4+ Kh5 8 Kh7.

3 Kf5, and White wins, e.g. 3 ... Kf7 4 Kg5 Kg7 5 h4 Kf7 6 Kh6 etc. (186).

195. The winning path here is not the only one, but accurate play is required, taking account of the positions examined earlier. With reserve pawn moves at his disposal, White should leave at least one pawn in its place, and move it only in definitely won positions.

With White to play: 1 Kg4 Kg6 2 h3 h6 3 g3 Kf6 4 Kf4 Kg6 (4 ... Ke6 5 h4 Kf6 6 g4) 5 Ke5 (5 g4 also wins, but not 5 h4? Kh5!—192) 5 ... Kh5 6 Ke6 Kg5 7 Kf7 h5 8 h4+ Kf5 9 Kg7 Kg4 10 Kg6 etc.

With Black to play: 1 ... Kg6 2 Kg4 (2 Ke5 is also possible, transposing into example 194) 2 ... Kh6 3 Kf5 Kh5 4 Kf6 Kh6 5 h4 (5 Kf7 Kg5 6 Kg7? h5!—draw) 5 ... Kh5 6 g3 Kg4 (6 ... Kh6 7 Kf7) 7 Kg7 K × g3 8 h5, and White wins.

In the event of 1 ... h6, any pawn move wins. E.g. 2 h3 Kg6 3 h4! Kh5 4 g3 (192); 2 h4 Kg6 (2 ... Ke6 3 g3! Kf6 4 g4) 3 Kg4!! (192, cf. the note to White’s 1st move) 3 ... Kf6 4 g3 Kg6 5 Kf3 (191).

196. This example combines many of the ideas presented earlier, with the black king having rather greater freedom of manoeuvre (in comparison with previous positions).

1 Kd6 (1 Kd4? Kf4; 1 h3? Kf4 2 Ke6 Kg3 3 Kf6 K × g2 4 h4 Kf3!, with a by-pass) 1 ... h6.

1 ... Kf6 2 Kd7 Kf7 is futile in view of 3 h3 Kf8 (3 ... Kf6 4 Ke6 h5 5 g3) 4 Ke6, while if 1 ... h5, then not 2 Ke7? h4 (with the threat of 3 ... h3) 3 h3 Ke5!, when the white king is shut out, but 2 Kd5! h4 3 h3 Kf4 4 Ke6, and wins.

2 h3! Kf6!

On 2 ... h5 there follows 3 Kc7 h4 4 Kf7; if instead 2 ... Kf4, then 3 Ke6 Kg3 4 Kf5! K × g2 5 h4 etc. (the 5 ... Kf3 by-pass does not succeed due to the king at f5).

3 Kd7!
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3 g3? fails to win after 3... Kf5 4 Kd5 Kf6 5 Ke4 Ke6 (189) 6 g4 Kf6 7 Kf4 Kg6 8 h4 Kf6, or 4 Ke7 Ke4 5 Kf6 Kf3 6 h4 K×g3 7 h5 Kf4! (the white king has not managed to occupy f5).

3... Kf7 4 g3! Kf6.

Or 4... h5 5 h4 Kf6 6 Kd6! Kf5 7 Ke7! (195). Now White only has to avoid 5 Kd6? Kf5 with a draw (cf. the note to White's 3rd move).

5 Ke8! Kf5 6 Kf7 Ke4 7 Kg6, and wins.

It will be expedient in conclusion to return to the Grigoriev positions of type 191 and 192 (some of the most complicated in this kind of ending), and make the acquaintance of certain earlier positions, and also of some later analyses.

Reti and Mandler, 1921

Bähr, 1934

197. Black has a tempo in reserve (... g6). Therefore White must avoid taking the vertical opposition, since after ... g6 he will be in zugzwang (and the opposition will pass to Black). This reveals the following correspondence of the front-line squares (the critical squares being e5 and f5, onto which the black king must not be allowed): ... Ke6 or ... Kg6 corresponds to Kf4, and ... Kf6 to Ke4 (at the distance of a knight's move!).

But on ... Kf6 White cannot reply Kg4 (because of ... Ke5, winning). This disturbance of the correspondence harmony is explained by that tactical feature, characteristic of the given pawn structure, which was shown in position 192 (on ... Kf6 White can play Kh5, since ... Kf5, which would ensure a win in the position moved to the left, does not work here because of stalemate). Hence the following corresponding squares are additionally defined: ... Kf6/Kh5 and ... Kf7/Kg4.

The establishment of these features is sufficient for the playing procedure to be perfectly understandable: 1 Kg3! (to answer 1... Ke7 with 2 Kg4) 1... Kc7 2 Kf3! Kf6 3 Ke4!, or 1... Kf7 2 Kg4! Kf6 (2... Ke6 3 Kf4; 2... Kg6 3 Kf4 Kh5 4 Kg3—192) 3 Kh5. Draw.

198. Here too White has a reserve tempo, and it is clear in advance that Black will place his king at a knight's move from White's. One position of mutual zugzwang is Kf4/Kg6 (in the event of h3-h4 there follows ... Kf6; if instead of h3-h4 the king moves, then ... h5, and vice versa; if ... h5, then g4-g5, while on ... Kf6 there follows h3-h4). But Kf4/Kc6 is also a zugzwang position, and thus the front-line corresponding squares are determined.

Black is ruined by the fact that, until h3-h4
is played, he cannot take the opposition. The
win is achieved by 1 Kd2! If now 1... Ke7,
then 2 h4!, and a winning position is reached
where White begins a by-pass with the aim of
transforming the distant opposition into
close opposition. Therefore the play develops
as follows: 1... Ke7 (2 Kd7 2 Ke3!) 2 Kd3!
Ke6 3 Kd4!, and White wins, since on 3...
Kd6 or 3... Kf6 there follows 4 h4! (180).

Bähr, 1936

199. White has one reserve tempo at his
disposal, and Black has two. The advance of
any one of the pawns leads to situations
already considered—each with its own
features, i.e. its own system of critical and
corresponding squares.

Since the black king cannot be prevented
from occupying e5 or f5, the most important
thing for White is to defend the critical
squares e4 and f4. With the king at f5 this can
apparently be achieved by Kf3 or Ke3. But
Ke3? in reply to ... Kf5 does not work, since
by ... h5 Black can make a favourable trans-
position into positions of type 197, or by ...
g5 can transpose into position 198, in both
cases occupying the necessary correspon-
dence with his king.

Therefore the only correct defence to ...
Kf5 is Kf3 (the opposition!): if ... g5, then
Kg3, while if ... h5, then h3–h4 or Ke3. The
other front-line corresponding squares will be
e3/e5 and g3/g5, and hence also the adjacent
rear squares f2/f6 and e2/e6 (for the remain-
ing rear squares further away, the opposition
is no longer demanded).

It is clear that the draw is given by 1 Ke1!
Kg7 2 Kh1! (retaining the possibility of taking
the opposition at e2 or f2 when Black moves
to e6 or f6) 2... Kf6 3 Kf2 Kg5 4 Kg3 Kh5.
Now the simplest is 5 h4!, but 5 Kg2 also does
not lose (but not 5 Kh2?—191).

Obviously a draw is also inevitable when it
is Black to move.

Bähr, 1936

200. Here White’s possibilities are wider:
he can bring about either the necessity for
knight’s opposition by g2–g4, or the necessity
for opposition play by g2–g3. But this latter
move can be neutralized by ... h5.

Therefore Black should play for the opoposition, so that in reply to g2–g4 he can switch
to knight’s correspondence.

E.g., with Black to move: 1... Kf7 2 Kf1
Ke7 (manoeuvring deep in the rear, Black
waits) 3 Ke2 Ke6! 4 Kf3 Kf5 5 g3 h5, or 5 g4+
Kg5. Draw.

But if White begins, he himself seizes the
opposition, forcing Black either to be the first
to move a pawn, or to make way for the white
Two Pawns against One

king, or to allow White to switch to a favourable system of corresponding squares.

1 Ke2! (with the critical squares being e4, f4 and g4, and the main file the f-file, White begins a by-pass) 1... Kf6 2 Kf2! Kg5 3 Ke3! Kf5 4 Kf3 Ke5 (4... h5 5 g3; 4... Kg5 5 Ke4) 5 Kg4 Kf6 6 Kf4! (6 Kh5 is pointless—191, but now White controls the critical squares) 6... Kg6 (or 6... Ke6) 7 g4, and White wins.

5.2 ISOLATED PAWNS

If there is no threat of the weaker side's pawn queening, two pawns usually win easily against one, when both pawns are passed or they stand sufficiently far apart. In the latter case the weaker side's king will be unable simultaneously to defend its own pawn and to prevent the advance of the enemy pawn. But if the exchange of one of the pawns is inevitable, the outcome will naturally be decided only by the possibilities afforded in the ending of king and pawn against king. The material will be grouped according to whether or not the stronger side has a passed pawn.

5.21 All pawns passed

These cases are on the whole straightforward, provided they do not transpose to queen endings, or that the material advantage is not countered by significant positional defects. The latter may complicate the play, and at times lead to an unexpected result.

201. Black's defeat results from the close placing of his pawns and the unfavourable position of his king. At the basis of the study is the ending of pawn against pawn.

1 Kd4! (a draw follows after 1 K×c6? Kf3 2 Kd5 Kf4 etc.) 1... e5+.

But now 1... Kf3 is inadequate in view of 2 a4! Kf4 3 a5 e5+ 4 Kc3, while if 1... e5+, then 2 K×e5 (2 Kd4? Ke2 3 a4 e5 4 K×e5 Kd3—draw) 2... Ke3 3 a4 (3 Kd6? Kd4) 3... Kd3! 4 a5! e5 5 a6 c4 6 a7 c3 7 a8=Q c2 8 Qd5+! and wins.

Grigoriev, 1936

2 K×e5 Kg3!

2... e5 is hopeless: 3 Kd5 Ke3 4 K×e5 Kd3 5 Kd5 Ke3 6 Ke5. But now 3 Kd4 is not possible, due to 3... Kf4 4 a4 e5 + 5 Kc3 e4 (with the threat of 6... Ke5) 6 a5 e3 7 a6 Kg3!, with a draw.

3 a4! e5 4 a5 e4 5 Kd4!, and White wins: 5... Kf4 6 a6 e3 7 Kd3 Kg3 8 a7 etc.

Grigoriev, 1928

202. 1 Kd4! e5+ (or 1... Kb3 2 g4 e5 + 3 Ke3!) 2 Kc3! and wins.

203. 1 Kb4 Kg7 2 Kg5 K×h7 3 K×h5, and White wins.
5.22 One passed pawn

When the stronger side has only one passed pawn, his second pawn is usually on the same file as the enemy pawn: in this case they are frequently blocking each other.

Of course, formations also occur where the pawns are not on the same, but on adjacent files. In this case the result is quickly determined by the possibility and expediency of an exchange. Theory has devoted hardly any attention to these endings, and here are virtually the only examples to be found in theoretical literature.
207. The exchanging attempt 1 ... d5 is countered by 2 c5.

Alekhine-Reti
Vienna, 1922

208

208. 1 ... Kg7, and if White tries to win the b-pawn, Black picks up the h-pawn and then reaches c8 in time.

With pawns at a3/b5 (instead of a2/b4) White wins, while with pawns at a4/b6, apart from approaching the b-pawn, White can also win by stalematign the black king in the h8 corner and answering ... b5 with a4-a5 etc.

This last example brings us by analogy to the practically important cases with blocked rook's pawns.

209. The plan here is extremely simple: using his passed pawn to divert the enemy king, White heads for the rook's pawn with

210

210. 1 Ke4 Kg5 2 Kd5 K × g4 3 Ke5 Kf5 4 Kb5 Ke6 5 K × a5 Kd7 etc.
211. 1 Kd4 Kf5 2 Kc4 K × f4 3 Kb5 Ke5 4 K × a5 Kd6 5 Kb6 Kd7 6 Kb7 etc.

Bähr, 1935

212. Black can eliminate the a4 pawn, but White nevertheless wins by transposing into Lolli’s ancient position.

1 e3+ Kc4 2 Kc2(e2) Kb4 3 Kd3! K × a4 4 Kc4! Ka3 5 e4 Kb2 6 e5! (6 Kbs? Kb3! 7 K × a5 Kc4 with a draw) 6... a4 7 e6 a3 8 e7 a2 e8 = Q a1 = Q, and White easily sets up a mating net: 10 Qe2 + Ka3 11 Qd3 + Kb2 12 Qd2 + Kb1 13 Kb3, and wins.

An exception to Bähr’s rule was found by Grigoriev.

Grigoriev, 1936

213. In manoeuvring with his king, White must avoid the opposition of the kings at f2/ f4 when it is him to move: 1 Kg3! Ke4 2 Kg2! Ke3.

If 2... Kf4, then 3 Kf2! Kg4 4 Ke3 K × h4 5 Kf4! Kh3 6 e4 h4 7 e5 Kg2 8 e6, and White wins as in example 212.

3 Kf1 Ke4 4 Ke1! Ke3 5 Kd1 Kf4 6 Kd2 Ke4 7 e3 Kf3 8 Kd3 Kg3 9 Ke4! Kg4 10 Ke5 K × h4 11 Kf4! etc.

In the following example White uses Bähr’s rule to draw the ending.

Bähr, 214

214. It is clear that 1 Ka3? Kc3 2 a5 Kc4 3 Ka4 Kc5 4 a6 Kb6 5 Kb4 K × a6 6 Kc5 Kb7 7 Kb5 loses, since the black pawn is inside the zone.

Correct is 1 Kc2! Kb4 2 Kd3! K × a4 3 Kc4, pressing the opponent’s king to the edge of the board and creating a zugzwang position. After 3... a6 4 Ke5! Ka5 5 Kc4 Kb6 6 Kb4 the draw becomes obvious.

The triangles shown in diagram 209 are an important guide, and a skilful use of them enables the correct plan to be found in the most varied positions.

215. Here White is saved by the active position of his king. His problem is to lure the pawn out of the winning zone (the triangle
Two Pawns against One

Zinar, 1982

The straightforward 2 Kc4? f5! 3 Kd4 Kg5 4 Ke3 Kg4 5 Kf2 a6! 6 Kg2 Kf4 leads to a loss.

2 ... Kf6!
This is stronger than 2 ... Kg5 3 a6! f5 + 4 Kf3 with a draw, while 2 ... a6 would have allowed a false attack: 3 Kd4! f5 4 Ke3! etc.

3 Kd5! Ke7 4 Kc6! (4 a6? Kd7) 4 ... f5 5 Kd5!
This return of the king is the whole point! White loses after 5 Kb7 f4 6 K×a7 f3 7 a6 f2 8 Kb7 f1 = Q 9 a7 Kd7 10 a8 = Q Qb5 + 11 Ka7 Kc7 etc.

5 ... Kf6 6 a6, and Black is in an unusual form of zugzwang: if 6 ... Kg5, then 7 Ke6, since 7 ... f4 now leads only to a draw, while if 6 ... f4, then 7 Ke4 Kg5 8 Kf3, and the black pawn is outside the winning zone.
The manoeuvring of the kings in such situations is of a fairly complex nature, but the problem is normally always the same—to create a zugzwang position.
Here are two further typical examples.

Zinar, 1982

216. The primitive 1 a6 does not succeed: the f7 pawn is inside the winning triangle, and the problem is again how to lure it out of there.

1 Kd3! Kg6! 2 Ke4!

217. If 1 Ke2 1 ... f3! 2 f4, then, as in the previous position, Black is saved by 2 ... Kd4! 3 Kf3 a3!, when White is in zugzwang. White is also unsuccessful with 1 Kd1 Kd3 2 Kc1 a3.
Correct is 1 Kd2 Kd4.
Isolated Pawns

After 1 ... Kb4 2 Kc2 Ka3 3 Kb1 Kb4 4 Kb2 a3+ 5 Kc2 Kc4 6 f4 Kd4 7 Kb3 Ke4 8 Kx a3 K x f4 9 Kb4 White wins.

2 Ke1! Ke5.

There is nothing better. If 2 ... Ke3, then 3 Kb2 K x f3 4 Ka3 Kc4 5 K x a4 Kb5 6 Kb5, while if 2 ... Ke3, then 3 f4 Kd4 4 Kb2 Ke4 5 Ka3 K x f4 6 K x a4 Ke5 7 Kb5.

3 Kd1!

3 Kb2 Kd4 4 f4? is correct due to 4 ... a3+! 5 Ke1 Ke5 6 Kc2 Kc4 7 f5 Kd5 8 Kb3 Ke5 9 K x a3 K x f5 with a draw.

3 ... Kd5 4 Ke2! Kc4 (4 ... Kd4 5 a3) 5 Ke3! a3 6 Kd2! Kd4 7 Kc2 Kc4 8 f4, and White wins.

Zinar, 1982

218. On 1 Kg2 there follows 1 ... Ke4 2 Kg3 Kd5!, attacking the a3 pawn in order to lure the second pawn out of the winning zone: 3 f4 (forced) 3 ... Ke6! 4 Kg4 Kf6 with a draw.

The correct move is 1 Kf1!, which sets Black a difficult choice.

If, for example, 1 ... Kf3 2 Ke1 Kf4, then 3 Kd2, and White wins by picking up the a4 pawn. If instead 2 ... Ke4, then 3 Ke2 Kd4 4 f3! Kc4 5 f4! (the pawn has moved out of the zone, but White has the more active king, and this is decisive) 5 ... Kd5 (5 ... Kd4 6 Kf3 Kd5 7 Kg4 Ke6 8 Kg5 Kf7 9 Kf5 Ke7 10 Kg6, and wins) 6 Kd3! Ke6 7 Kc4 Kf5 8 Kb4 K x f4 9 K x a4 Ke5 10 K b5 Kd6 11 K b6, and wins.

1 ... Ke5 2 Ke1! Kd5 3 Kd1!

The positions of mutual zugzwang are: c2/c4, d2/d4 and e2/e4. Black is no longer able to maintain the distant opposition.

3 ... Kd4 4 Kd2! Kc4 5 f3! Kd4 6 Ke2 Kc4 7 f4, and White wins.

If the stronger side’s king is in front of the pawn, the chances of success are naturally improved.

Zinar, 1982

219. Here the drawing zone includes only 4 squares: h3, h4, h5 and g4.

We have already considered the position with the pawn at f4 in the analysis of the previous example. But here, with the pawn at g4, it is a draw: 1 Kf5 Kf7 2 Ke5 Kg6 3 Kd5 Kg5 4 Kc4 K x g4 5 Kb5 Kf5 6 K x a4 Ke6 7 Kb5 Kd7 8 Kb6 Kc8 etc.

220. Here a further 3 squares enter the drawing zone: f4, g3 and h2. E.g., with the pawn at f4: 1 Ke5 Ke7 2 Kd4 Kf6 3 Ke4 Kf5 4 Kb4 K x f4 5 K x a3 Ke5 6 K b4 Kd6. Draw.

If the stronger side’s rook’s pawn has crossed the demarcation line, Bähr established that the material advantage can usually be realized.
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Zinar, 1982

With a blocked pawn on the 6th rank, positions with a d-pawn turn out to be drawn, but here too positions of mutual zugzwang are possible.

Zinar, 1982

These are the only known exceptions to Bähr's rule:
(a) (Kc2, pawns a5, d2/Kc4, pawn a6).
Draw, whoever it is to move.
(b) (Kc3, pawns a5, d3/Kc5, pawn a6).
Draw only with White to move.
(c) (Kd2, pawns a5, e2/Kd4, pawn a6).
Draw only with White to move.
(d) (Kd2, pawns a6, e2/Kd4, pawn a7).
Draw, whoever it is to move.

On the basis of this last position the following study was composed.

Crum, 1913

222. 1 Kd4? would be a mistake because of 1...

Kb4 2 Kd3 Kc5 3 Kc3 d6 4 Kb3 Kd4 5
K × a3 Kc3!, when Black wins.

Situations of mutual zugzwang develop on the squares d4/b4 and d5/b5. Correct is 1
Ke4! Ka4 2 Ke5! Kb5 3 Ke4! Kb6 4 Kd4! Kd5 5
Kd5 Kb4 6 Kd4 d6 7 Kd3 Kc5 8 Kc3 Kd5 9
Kd3. Draw.

Let us now turn to an examination of examples where the blocked pawns are rook's
pawns, and the passed pawn in on the bishop's file.

223. White wins easily, by the typical man-oeuvre of 'shoulder-charging' the opponent's
king: 1 Ke3 Kg4 2 f4 K × h4 3 Kf3! Kh3 4 f5 h4
5 f6 Kh2 6 f7 h3 7 f8 = Q etc.

But White can also win by 1 Kg3 Ke4! 2
f3+.

Not 2 f4 Kd5! 3 Kf3 Kd4 with a draw, but
2 Kg2 is perfectly possible: 2...

Kf4 3 f3 Ke5
4 Kf1 Kf4 5 Kf2 Ke5 6 Ke3 Kf5 7 f4 Kg4 8
Ke4 K × h4 9 Kf3 etc. Incidentally, 9 f5 also
wins after 9...

Kg5 10 Ke5 h4 11 f6 h3 12 f7
h2 13 f8=Q h1=Q 14 Qg8+ and 15 Qh7+, winning the queen.

2 ... Ke3!

The strongest reply. After 2 ... Ke5 3 Kg2 play reduces to the variation just considered.

3 Kg2 Kd4 4 Kh3! (4 Kf2 would be pointless: 4 ... Kd3!) 4 ... Kd3! 5 Kh2!

The decisive move! The corresponding squares are g3/e3, g2/d4, h3 and f2/d3, but there is no square corresponding to h2!

5 ... Kd4 6 Kg2! Kd3 7 Kh2! Kd4 8 Ke2, and White wins.

225. This example is taken from a practical game (1884). After 1 Kd2 c3 + 2 Kc2 Kc4 White, taking account of the pawn's position at a5, saved the draw by 3 Kc1! Had Black played 2 ... a4, White would have been saved by 3 Kd1(b1).

Evidently unaware of this game, a few years later Horwitz and Kling suggested the following study.

Horwitz & Kling, 1889

226. Here is the composers' solution: 1 a5 b x a5 2 Kd1 a4 3 Kc1 Kd4 4 Kc2 Kc4 5 a3(?), with a draw (!?). But 5 a3 in fact loses (224). Correct here (Neustadtl, 1899) is 5 Kd1 (if 5 Kc1?, then 5 ... Kd3 6 Kd1 c2 + 7 Kc1 Kc3 8 a3 Kb3, winning) 5 ... Kd3 (it is also an
obvious draw after 5 ... a3) 6 Kc1 c2 7 a3.

Draw.

If after 1 a5 b × a5 2 Kd1 Black plays 2 ... c2+, all the same it is a draw: 3 Kc1 Kc3 4 a3! Kb3 5 a4.

Neustadt also showed that White is by no means obliged to begin with 1 a5 (possible is 1 Kd1 c2 + 2 Kc1 Kc3 3 a5 with a draw), and that if the a4 pawn is removed and the b6 pawn replaced at b7, then Black wins, whoever it is to move.

Grigoriev, 1933

227. In the previous position it was of no significance whether the king stood at c1 or d1, since the a2 pawn, depending on circumstances, could move either to a3 or a4. But in Grigoriev’s position the pawn stands not at a7, but at a6, and no longer has the right to move two squares. Therefore the black king has only one good retreat square: 1 ... Kd8!, and if now 2 Kd6 Kc8 3 c7? then 3 ... a4 4 b × a4 a5 with a draw.

But the position of the pawn at a6 makes the b6 square available to the white king, and White wins by giving his opponent the move: on 1 ... Kd8 comes 2 Kg4(a4) Kc8 3 Kg4(e4)! Kd8 4 Kg5 etc.

The concluding remark to position 227 is illustrated in the next example.

Grigoriev, 1920

229. 1 Kf2 Kg4 2 Ke3! (this king man-
oeuvre is the idea of the defence—White forces the advance of the h7 pawn) 2... h6 3 Kf2 Kf4 4 Ke1.

If the pawn were at h5, then 4 Kf1! would be necessary (225). The king must stand on a square of the same colour as the enemy rook's pawn.

4... Ke3 5 Kf1 h5 (if 5... f2, then 6 h3! Kf3 7 h4 Kg3 8 h5) 6 Ke1 f2 + 7 Kf1 Kg3 8 h3 Kg3 9 h4. Draw.

If the passed pawn is on the 4th rank, a win is possible under certain circumstances, and the procedure is extremely instructive.

Maizelis, 1956

230. 1 Kd5 Kd7 2 a5! Kc7 3 Ke6! (White begins a lengthy by-passing manoeuvre, the aim of which is to attack the a7 pawn!) 3... Kc6 4 a6! Kc7.

In the event of 4... Kb6 5 Kb6 K×a6 6 c5 Kd7 7 Kd7 White wins, and things are not improved by chasing the other pawn: 4... Kc5 5 Kd7 K×c4 6 Kc6! (6 Ke7? Kc5 7 Kb7 Kd6 with a draw) etc.

5 Ke7 Kc6 (5... Kc8 6 Kd6 Kd8 7 Kc6 Ke8 8 c5 Kb8 9 Kd7 etc.) 6 Kd8 Kd6 7 Kc8 Kc6 8 Kb8 Kc6 9 c5 + Kc6 (9... K×a6 10 c6 etc.) 10 K×a7 Kc7 11 c6, and White wins.

If Black begins, the by-pass proves unsuccessful: 1... Kd7 2 Kd5 Kc7 3 a5 Kd7 4 a6 Kc7 5 Ke6 Kc6 6 Ke7 Kc7! 7 Ke8 Ke8! Draw.

This same idea was expressed somewhat earlier in the following study.

Wallace, 1949

231

230

231. 1 a5 Kd7 2 Kb5! (2 Kc5? Kc7 3 Kd5 Kd7 with a draw) 2... Ke7 3 Kc5! (3 Ka6? Kb8 4 c5 Ka8 5 c6 Kb8 leads only to a draw) 3... Kd7 4 Kd5 Ke7 5 Ke6!, and then as in example 230.

To demonstrate the conclusion that, with a pawn at a3 and the second pawn on the 4th rank a win is also possible, Maizelis suggested the next position.

Maizelis, 1955

232
232. With Black to move there are no
difficulties: 1 ... Kc5 2 Kb3 Kb6! 3 Kb4 Kc6 4
a4 Kb6 5 a5+ Kc6. Draw.

The composer thought that, if it were
White to move first, he could hope to win: 1
Kb4 Kb6 2 a4 Kc6 3 a5, and so on as in the
study just examined.

But, as was shown by a number of readers
of the magazine Shakhmaty v SSSR, after 1...
Kd7!! all White's winning attempts are in
vain, e.g.:

(a) 2 Kc5 Kc7! 3 Kd5 Kd7 4 a4 Kc7 5 Ke6
(there appears to be nothing better; if 5 a5,
then 5 ... Kd7 with a draw) 5 ... Kc6 6 a5
Kc5 7 Kd7 K × c4 8 Kc6 a6 9 Kb6 Kd5 with a
draw.

(b) 2 Kb5 Kd6! (2 ... Kc7 3 Kc5!, winning)
3 a4 Kc7 4 Kc5 Kd7 5 Kd5 Kc7, leading to a
continuation already examined.

(c) 2 a4 Kd6! 3 Kb5 Kc7 4 Kc5 (4 a5 Kb7 5
Kc5 Kc7) 4 ... Kd7 5 Kd5 Kc7 etc.

We thus conclude that in position 232, with
his pawn at a3 or a4, White is unable to win
against correct defence.

And now two further interesting drawn
endings with similar pawns.

Chéron, 1952

233. Black is saved by the active position
of his king: 1 Ke1(g1) h5 2 Kf1 h4 3 Kg1 (no
better is 3 Ke1 Kg2, or 3 h3 Ke4! 4 Ke2 Kf4
f3 Kg3 etc.) 3 ... Kg4! (not 3 ... h3? 4 Kf1,
when White wins) 4 Kg2 h3+ 5 Kg1 Kf4! 6
Kf1 Kf3 7 Ke1 Kg2. Draw.

Goldenov, 1967

234. Here, in spite of his extra pawn, it is
Black who has to think in terms of saving the
position. Bad, for example, is 1 ... Kh5 2
Kg7 f5 3 K × h7 f4 4 Kg7 f3 5 h7 f2 6
h8=Q+, or 1 ... Kf4 2 Kg7 f5 3 K × h7 Kg3
4 Kg6 f4 5 h7 f3 6 h8=Q f2 7 Qh1, and White
wins.

The way to draw is as follows: 1 ... Kf4
nevertheless) 2 Kg7 f5 3 K × h7 Ke3! (but not
3 ... Kg3?) 4 Kg6 f4 5 h7 f3 6 h8=Q f2 7 Qh1
Ke2 etc.
Isolated Pawns

Also possible is 1 Ke5 Kc6 2 d4 Kd7 3 Kf5 Kd6 4 Kg5 Ke6 5 d5 + Kd5 6 Kc6 Ke5 (Salvioli continued 6 ... Kd6 7 Kh7 Kd7 with a draw, but on 7 Kh7? Black wins by 7 ... Ke5!) 7 Kg5! (7 Kg7? Ke6). Draw.

Salvioli showed that with his pawn at d2 White would win by 1 Ke5 Kc6 2 Kf5 Kd5 3 Kg5 Ke5 4 d3! Kc6 5 d4 Kd5 6 Kh6 etc.

If the position is moved to the right or left, it is still a draw. Only the position obtained by moving it three files to the left is won for White.

A development of this analysis is provided by the following Grigoriev study, which shows, in particular, that in the previous example White cannot win even if it is Black to move.

Grigoriev, 1936

Also possible is 1 Ke1 Ke5 2 Kd1 Kd4 3 Ke2 Ke4.

Nothing is achieved by 3 ... Kc4 4 Ke3 Kb4 5 Kd4, or 4 ... Kb5 5 Kd3 (or 5 Ke4, but not 5 Kd4? Kb4 6 Kd3 e5 and wins).

4 Kf2 Kf4 5 Ke2 Kg3! 6 Kd3!

But not 6 Ke3? e5! Now on 6 ... Kf3 there follows 7 Kd4! e5 8 Kd2 Kf2 9 Kd3!, while after 6 ... e5, 7 Kd3 is not possible because of 7 ... e4.

6 ... e5 7 Kf3! Kg2 8 Ke2! e4 9 Ke1! Kf3 10 Kf1 e3 11 Ke1 e2. Stalemate.
Guided by the study just examined, White gained a draw in the following position.

Westerinen-Smyslov
Szolnok, 1975

239. 1 Kc3 Ke4 2 Kd2 Kd4 3 Ke2 Ke4 4 Kd2!

Of course, not 4 Ke3 Kc3 5 Ke4 Kd2 6 K × e5 Ke3, or 5 Ke2 Kc2 6 Ke1 Kd3, when the black king breaks through to the enemy pawn.

4 ... Kd4 5 Ke2 Kc3 6 Ke3 Ke2 7 Ke2! e4 8 Ke1! (White loses after 8 Ke3 Kd1 9 K × e4 Ke2!) 8 ... Kd3 9 Kd1 e3 10 Ke1 e2. Stalemate.

Berger, 1890

240. In 1890 Berger mentioned only 1 Ke4 Kc5 2 d4+ Kd6, with the following manoeuvre of Horwitz and Kling: 3 Ke3 Kc6 4 Kd2 Kd6 5 Ke3 Kd5 6 Kd3.

In 1922 he added the variation 2 ... Ke4 3 Ke3 Kb5 4 Kd3 Kb4 5 d5 (this wins here because the f-pawn has crossed the middle of the board) 5 ... Kc5 6 Ke4 Kb6 7 d6 etc., and mentioned two other solutions: 1 Kc4 Ke5 2 d4+ K × f5 3 d5, and (the equally simple) 1 d3 Kc6 2 Kc4 Kd6 3 d4, which, however, no longer work in the following position, where the black pawn is closer to the queening square.

241. In his analysis of this position (in the 8th edition of Bilguer’s Handbuch. 1916) Berger committed several instructive mistakes. After 1 Kd3 his continuation ‘1 ... Kb5!’ (mechanically playing for the opposition!) is simply not the best defence, since White wins easily by 2 c3 Kc5 3 Kd2 etc. (240). More tenacious, of course, is 1 ... Kd4! with the sequel 2 e3+! Kb3 3 Kd2 Ka4!, and now ‘à la Grigoriev’ 4 Ke2(e3)!, winning as in example 14.

But this is not the main point. After 1 Kd3 Kb5 Berger considers 2 Ke3?, and again recommends the opposition ‘2 ... Kc5!’. But in fact 2 ... Kc4! is the only way to draw, and 2 ... Kc5? does not lose in Berger’s analysis, only because he continues 3 Ke2? Kc4 4 Kf3
Kd4. As was shown by Dedrle in 1921, White should answer 2 ... Ke5? with 3 Kf2! Kc4 4 Kg3! Kd4 5 Kf3 Kc3 6 Kg4, winning.

In example 241, where 1 Kd3 followed by the advance of the c-pawn wins, there is of course no necessity for such a by-pass, but it becomes necessary if the black pawn is a bishop's pawn. (Note, incidentally, that if position 241 is moved one file to the right, White wins only if it is Black to move)*.

242. According to analysis by Dedrle (1921), White fails to win here by 1 Kd3? Kf5! (the variations are clear from preceding analysis), or 1 e3? Ke4, or 1 Kb3? Kd4 (in the event of 2 e3 + White loses both pawns).

Correct is 1 Kb2! Ke4 2 Ka3! Kd4 3 Kb3 Ke3 4 Ka4, and wins. The corresponding squares are determined here by the decisive positions Kb3/Kd4 and Ka3/Ke4.

243. The critical position is Kd3/Kb4, and the solution is in every way analogous to example 242. But before the winning manoeuvre is begun, the black king must be forced to retreat to the 6th rank (otherwise it is too close to the a2 pawn). To give Black the move, use is made of the triangle a3-b2-b3.

1 Ka3 Kb6 2 Kb2! Ka5 (2 ... Ka6 3 Ke2! Ka5 4 Kd2!) 3 Kb3! Kb6 4 Kc3 Ka5 5 Kd2! (pointed out by Kling; Walker continued 5

* Cf. position 236 (translator's note).

244. For promoting his b-pawn (1 Kd5? etc.) White is short of one tempo, while 1 Kf6? also loses in view of 1 ... Kg4 2 Kx f7 Kf5. Therefore 1 Kf5! is correct, creating a
double-edged position, where in the event of inaccurate play Black even risks losing.

1... Kh4 2 Kf4 Kh3 3 Kf3 Kh2 4 Kf2.

It is readily apparent that at no point can White go for the win of the f-pawn, since by a by-pass to the rear Black would reach the b5 pawn. But now Black attempts to free his king.

4... f6.

Black reckons as follows: while White is picking up the pawn, he will succeed (in the same 4 moves) in transferring his king to d4, winning.

5 Kf3 Kg1 6 Ke4!!

With a dual aim: if 6... Kg2?, then 7 Kf5 winning, while after 6... Kf2 the path of the f-pawn is blocked, which grants White the tempo that he has been lacking from the very start.

6... Kf2! 7 Kd5! f5 8 Ke6. Draw.

while if 5 f4, then 5... Kd4 6 f5 Ke5 7 Kc7 K×f5 8 Kd6 Kf6! 9 Kd5 Ke7 10 Kd4 Kd6 11 K×c3 Kc5 (the second possibility).

A draw also follows from 1 f3? Ke5 2 Kf7 Kf4 3 Ke6 Ke3! 4 f4 Kd2 etc.

From these variations it is apparent that White should not: (a) prematurely advance his f-pawn (the c3 square becomes accessible to the black king!), or (b) approach with his king along the 8th rank, since in this case the distance to the c3 pawn remains too great.

From this the solution is clear: 1 Kg7! Ke5 2 Kf7! (it is sufficient to approach the c3 pawn along the 7th, rather than the 8th rank) 2... Kf5 (2... Kf4 3 Ke6 Kf3 4 Kd5 Ke2 5 Kc4 and wins) 3 Ke7 Ke5 4 Kd7 Kd5 5 Kc7! (5 f3? Kd4; 5 f4? Ke4 6 Ke6 Ke3! or 6 f5 K×f5 7 Kd6 Kf6!) 5... Ke5 (threatening 6... Kb4) 6 f4! Kd4 f5! (7 Kd6? Ke3) 7... Ke5 8 Kc6! (8 f6? K×f6 9 Kd6 Kf7?) 8... K×f5 9 Kd5!, and wins.

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Grigoriev, 1936

Moravec, 1925

245. Black's saving chances are based on two possibilities: (1) on the capture of the c2 pawn, so as to draw against the queen thanks to the bishop's pawn; (2) on the capture of the f-pawn, so as then to answer K×c3 with... Kc5.

For example: 1 Kf8? Kf6! 2 Ke8 Ke6 3 Kd8 Kd6 4 Kc8 Kc5!, and if 5 Kc7, then 5... Kb4 6 f4 Ka3 7 f5 Kb2 (the first possibility),

246. White parries the threat of the capture on h2 by the counter-threat of an attack on f3.

The only way to draw is by 1 Kd5! Kg2! 2 h4 K×f2 3 h5 Kg2 4 h6 f2 5 h7. It is clear that,
had White played 1 Kf5?, 5 ... f1 = Q+ would now have been decisive, while in the event of 1 Ke5? Black would have been able to meet 6 h8 = Q with 6 ... Qa1+.

247

White cannot win

247. 1 Kf6! Kd3 (1 ... Kc3, 1 ... Kb3 and 1 ... h5 are also possible) 2 K × e5 Kc4! 3 Kf4 Kc5!, or 3 Kf5 h5!—draw.

5.23 No passed pawn

Here there is only one possible pawn structure: the stronger side’s pawns are separated by one file, and it is on this file that the opponent’s pawn is situated. A win is normally possible when the king can occupy a blocking position in front of the enemy pawn, and then attack it from the side or else carry out a favourable exchange of pawns.

The existence of reserve tempi is often of decisive importance, and therefore much depends both on the distance between the pawns, and on the placing of the kings.

We will first analyze several simple final positions, with the kings in vertical, and then in horizontal opposition.

248. On 1 ... Kd7 there follows 2 d5! (1 ... Kb7 2 b5!), winning. With White to move the exchange is pointless, and there is no win.

Glazier, 1939

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249. Here there is the same idea: 1 Ke6! Kd8 (or 1 ... f4) 2 K × d5, or 1 ... Kf8 (or 1 ... d4) 2 K × f5.

At the side of the board, where one of the pawns is a rook’s pawn, the result is different.

250. Black draws by 1 ... Kc7! (since White does not achieve anything by 2 c5, or by 2 Ka6 Kc6), but, of course, 1 ... Ka7? loses to 2 a5.
If positions 248 and 250 are moved up or down the board, the result remains the same, but if the black pawn is on its initial square, certain special features come to light.

251. After 1 ... Kd8 the immediate 2 d6? leads only to a draw in view of 2 ... Kc8! 3 d7 + Kd8, but White wins by 2 Kb7! Kd7 3 Kb8! (following Chéron, 1927, certain textbooks continue here with the excessively subtle 3 Ka8!?, starting as though to win 'for the second time', but such manoeuvring is appropriate in a completely different group of endings, considered later—268) 3 ... Kd8 (3 ... Kd6 4 Kc8) 4 d6! c × d6 5 b6 etc.

252. A draw is given by 1 ... Kc8!, but not 1 ... Ka8? 2 Kc7! (2 a6? Kb8!) 2 ... Ka7 3 Kc8 Ka8 4 a6! etc.

This corner position was known by Carrera as long ago as 1617.

253. 1 ... Kd7, or with White to move 1 Ke4 Kc6 2 Kb4 Kc7 3 Kb5 Kc8 (3 ... Kb8 4 Kb6 Kc8! 5 Ka7 Kc7 is also possible) 4 Kb6 Kb8.

Thus the position with the white king at b5 is also drawn, and it should be mentioned that this result holds on all ranks and files.

254. After 1 Kd7 (1 Ke6 Ka6) 1 ... Kb7 2 Kd8 (2 Ke8 Ka8!, but not 2 ... Kc8? since the c6 pawn prevents Black from retaining the
opposition: 3 Ke7 Kc7 4 Ke6 and wins; study by Neustadt (1890) 2... Kb8, 3 b5 no longer works as in example 251 (the black pawn is closer to the queening square).

But if it is Black to move, he is unable to retain the opposition: 1... Kb7 2 Kd7 Kb6 3 Ke8 etc.

The black king has insufficient space to retain the opposition. White wins by 1 Kd4! Ka7 (1... Kb7 2 a5) 2 Kc7 etc.

With Black’s pawn on its initial square, holding the opposition no longer saves him. Play reduces to the finish of example 251: 1 Kd8 Kb8 2 b6 etc. An exception will be provided by the position where the white king is at the side of the board (cf. the finish to example 253).

Positions where there is a greater distance between the pawns or the kings normally reduce to the concluding positions already examined.

The existence of a reserve tempo (pawn at d3 instead of d4) assures White of a win: 1 Kd4 Kd7 2 Kc5 Ke7 3 d4, or 1... Kc7 2 Kc5 Kd7 3 Kb6! Kd6 4 d4 (254).
Two Pawns against One

258. But here White to move is unable to win: 1 Kb4 Kd5 2 Kc3 c5 etc.

259. The attempt to take the opposition by 1 Kb5? proves unsuccessful, since 1 ... c6 + allows Black to bring the pawns together while not giving the white king any good retreat square: 2 Kc5 Kc7, or 2 Kc4 Kb6, or 2 Ka5 Ka7. Draw.

The only way to win is by 1 Kc5! c6 (otherwise 2 Kc6) 2 b5!

With Black to move in an analogous position, Fine (1941) suggested 1 ... Kc6, which in the event of 2 b5 + Kb6 does indeed lead to a draw without any trouble. But 2 Kc3 creates some complications, e.g. 2 ... Kb5 3 d5, and if Black plays the careless 3 ... Kb6? (3 ... c6? 4 d6), then 4 Kc4 wins. The only way to save the game is by 3 ... Ka6! 4 Kc4 Kb6.

Therefore with Black to move the simplest continuation is 1 ... c6! 2 Kc5 Kc7 (248) or 2 b5 Kb6! Draw.

260. This is an ancient (1843) theoretical position, through the complex labyrinths of which it is easy to find one’s way, knowing the preceding analysis. The basis of the play is the struggle for the c4 square.

If it is White to move, he wins by taking the opposition: 1 Kb3!

A draw results from 1 Kc3? Kc5! (1 ... c5? 2 d3 or 2 b3; 1 ... c6? 2 d4), when even the existence of reserve tempi does not allow White to occupy c4, e.g.:

(a) 2 b4+ Kb5! (the correct defence demands that the black king should step onto the file of the more advanced pawn; if 2 ... Kd5?, then 3 Kd3! c6 4 Kc3, and as before 4 ... c5 is hopeless because of 5 b5) 3 Kb3 c6!, and on 4 Kc3 or 4 d3 there follows 4 ... c5, while 4 d4 also leads to a draw (253).

(b) 2 b3 Kb5! (2 ... c6? 3 b4+! Kb5 4 Kb3 or 3 ... Kd5 4 Kd3) 3 d4 c6 or 3 b4 c5.

1 ... Kc5 (1 ... c6 2 d4 or 1 ... c5 2 Kc3 c4 3 b3 fails to save the game) 2 Ke3 c6.

From previous analysis it is clear that if 2 ... Kd5, then 3 b4!, while if 2 ... Kb5, then 3 d4! (symmetric variations will subsequently be omitted).
Isolated Pawns

3 b4 + Kb5 4 Kb3, and White wins, e.g. 4...
  ... Ka6 5 Kc4 Kb6 6 d3! Kc7 7 Kd5 Kd7 (7...
  ... Kb7 8 b5) 8 Kb6 Kd6 9 Kb7 Kd7 10 d4 Kd6
  11 K8 Ke6 12 Kc7 Kd5 13 Kd7.

  With Black to move the draw is most easily
  secured by 1 ... Kb4 2 d3 (or 2 b3) 2 ... c5!,
  but also possible is 1 ... Ke4 2 h3 + Kb4 3 d4
  (otherwise 3 ... c5) 3 ... c6 4 Kb2 Ka5! 5 Kc3
  Kb5.

  Fine, 1941

Grigoriev, 1936

262. 1 Kc6 Ke7 2 Kd5 Kd7 (in trying to
  bring his king to the blockading square f6,
  from the very start White prevents ... f6;
  now, with the same aim, it remains for him to
  avoid the position Ke4/Ke6 with him to
  move) 3 Kd4! Ke7 4 Ke3 Kd7 5 Kf4 Ke6 6 Ke4
  Kd7 (6 ... f6 is hopeless: 7 e×f6 K×f6 8
  Kf4) 7 Kf5 Ke7 8 g4 Ke8! 9 Kf6! (9 e6? Kc7 or
  9 ... Kf8) 9 ... Kf8 10 g5! and wins (251).

261. With his king at d4 White wins simply
  by 1 e4 and 2 e5. Fine gives an interesting
  example of an incorrect attack: '1 e4 Kc6 2
  Ke5 Kc5 3 Kf6? Kd4 4 e5 Kd5! 5 c3 Ke4! (but
  not 5 ... Kc4 6 Ke7 Kd5 7 K×d7 K×e5 8
  Kc6, winning) 6 c4 Kd4 7 Ke7 K×e5'. He
  does not give a winning move instead of 3
  Kf6, but in fact there isn't one (3 c3 Kc4; 3
  Kf5 Kd4 4 e5 Kd5! 5 c3 Kc4 or 5 Kf6 Kd4).
  Hence after 2. Ke5? the game is already a
  draw, and 3 Kf6 is no better and no worse
  than other continuations. '4 ... Kd5!' is also
  inaccurate in the analysis—4 ... Ke4! is
  simpler.

  When the kings are at some distance from
  each other, and also from the pawns, the
  theoretical motifs considered appear in rather
  more complex form.

Ebersz, 1942

263. 1 Ke3 (1 Kd3? Kc5 2 Kd2 Kc4 3 Kc2
  Kb5l with a draw—259) 1 ... Kd7 (1 ... Kc5
  2 Kd3) 2 Kd3! (2 Kd4? Ke6 with a draw—
  258) 2 ... Ke7 3 Kc4 Ke6 4 Kd4, and wins.

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264. White loses after 1 Kd2? d4! 2 e3 (or 2 e4 Kb5! 3 Kd3 Kc5) 2 ... Kd5 3 Kd3 d × e3, or 1 e3? Ke5 2 Kd2 Kc4 3 Ke2 Kc3 4 Kf2 Kd3 5 Kf3 f5 etc.
Correct is 1 Kf2! Kd6 (no better is 1 ... d4 or 1 ... Kc5 2 Kf3 etc.—260) 2 Ke3! (2 Kf3? Ke5 3 e3 Kf5, or 3 Ke3 f5 4 Kf3 d4) 2 ... Ke5 3 Kd3! d4 4 e3 or 4 e4. Draw.

265. 1 a3? is premature: 1 ... Kc6 2 Ke6 (2 Kd4 Kd6 3 c3 Kc6) 2 ... Kc5 3 Kd7 b4 4 a4 Kb6.
Correct is 1 Ke6! Kc6 2 Ke7! Kc7 3 a3 Kc6 4 Kd8, winning.

266. Black, with a tempo in reserve, does not himself take the opposition, but waits for White to take it, after which he will seize it by ... d6. Therefore White too avoids taking the opposition.

With Black to move there would follow: 1 ... Kc4! (1 ... Kd4? 2 Kf5 Ke3 3 e5 or 2 ... d6 4 Kf4—draw) 2 Ke3 (2 Kg4 d6! or 2 Kf5 Kd4 3 Kf4 d6) 2 ... d6! 3 Ke2 Kd4 4 Kf3 Kd3 5 Kf4 Ke2, and by seizing the critical squares of the e4 pawn, Black wins.

The play is determined here by the following crucial positions: Kf5 Kd4, Kf4/Kc4 and Kg4/Kc5. In other words, the squares corresponding to f5, f4 and g4 (g5 being inaccessible) are d4, c4 and c5 (d5 being inaccessible). On these corresponding squares White is saved from taking the opposition, which in this case would be fatal (and Black is unable to manoeuvre on the adjoining squares to the rear—his f6 pawn is under threat!).

From this the solution is clear: 1 Kg4! (1 Ke3? Kd6! 2 Kf4 Ke6) 1 ... Kc4 (1 ... Kd6 2 Kf5) 2 Kf4! Kd4 (2 ... d6 3 Kg4; 2 ... Kd3 3 e5) 3 Kf5! d6 (after this White now defends by the method of opposition) 4 Kf4 Kd3 5 Kf3 Kc4 6 Kg4. Draw.

267. Here there is the same idea, but with interesting manoeuvring on the rear squares adjoining the critical zone: b5, c5, c4 for Black, and f4, f5, e5 for White.
1 Kg4! (White attacks two squares of the main zone, forcing Black into the corresponding defence; insufficient is 1 Kg3? Ka4 2 Kf4 Kb5!—draw) 1 ... Kb6! (weaker is 1 ... Ka4 2 Kf5 Kb3 3 Ke6! KC4 4 Ke5, winning) 2 Kg5! (2 Kf5? Kc5) 2 ... Kc6 3 Kf6! (threatening 4 Ke5, and Black is a long way from c4; he is forced to step into the main zone, and White soon seizes the decisive correspondence in it) 3 ... Kb5 4 Ke6! Kc5 5 Kf5! (the end result is already clear, but the continuation is instructive) 5 ... Kb5 6 e3 Kb6 7 Kf6 (taking the opposition on the main file) 7 ... Kb5 8 Ke7 (a by-pass!) 8 ... Kc6 9 Ke6 (distant opposition has been transformed into close opposition) 9 ... Kc5 10 Kd7, and White wins.

Certain features of manoeuvring on the ranks are demonstrated by the following examples.

268. The basic solution expounded by Dedrle in 1925 was inaccurate.

The white king cannot step immediately onto the 7th rank, since Black will take the distant opposition and it will be a draw. White’s approach to the critical squares of the d4 pawn must be along the 8th rank, observing rectangular correspondence, which will soon be transformed into a decisive opposition.

1 Kg8 (the corners of the a6–a8–g8–g6 rectangle are all of the same colour!) 1 ... Kb6 2 Kf8 Kc6 3 Ke8 Kd6 4 Kf7.

Since the black king has stepped onto an unfavourable file, where the retention of the opposition is prevented by its own pawn, White can set about seizing the critical squares. Dedrle made the incorrect assertion that 4 Kd8 does not win due to 4 ... Kc6 5 Ke7 Kc7; but instead of 5 Ke7?, correct is 5 Kc8! Kd6 6 Kb7, when White wins as in the main variation.

Dedrle, 1921

4 ... Kd7 5 Kf6 Kd6 6 Kf5 Kd5 7 Kf4 Kd6 8 Ke4 Kc5 9 Ke5 Kc4 10 Kd6, and wins.

The main rank in this example is the 5th, but to approach with his king White first had to win control of the 7th rank—by the same by-passing method. The decisive factor in this was the possession of rectangular correspondence, since it determined who would have the opposition with the white king at d8.

The same position with Black to move may help to explain this: 1 ... Kb6 2 Kg8 Kc6 3 Kf8 Kb6! (3 ... Kd6? 4 Kf7 4 Ke8 Kc6 5 Kd8 Kd6 (keeping the white king on the 8th rank), or 5 Ke7(17) Kc7(17). Draw.

If in the initial position the white king were at g8, the result would again be decided by
the turn to move: White wins only if it is the opponent's move.

Let us now consider another defence after 1 Kg8, namely 1 ... Ka5 (1 ... Kb5 2 Kf7!) 2 Kg7! (in similar fashion White has to gain control of the 6th rank) 2 ... Ka4.

Black immediately gives up the hopeless task of trying to defend the 6th rank. In the first instance this 'confused' the composer of the position, who asserted that only 3 Kf6 was correct now, whereas the mechanical taking of rectangular correspondence by 3 Kg6 would be an irreparable mistake.

But in fact both moves win: here the rectangular correspondence loses its significance, since the black king is on an unfavourable rank, where its own pawn is a hindrance (e.g. 3 Kf6 Kb4 4 Ke6 Kc4 5 Kd6, winning). After 3 Kg6 a possible continuation is 3 ... Ka5 (3 ... Kb4 4 Kf6! Kc3 5 Ke5) 4 Kg5! (on the main file it is essential to take the opposition), and now either 4 ... Ka6 5 Kf4 Kb5 6 Kf5!, or 4 ... Ka4 5 Kg6! etc. From this it is clear that 3 Kf6 is simply stronger than 3 Kg6, since it takes the king more quickly to its goal—that of winning the critical squares.

Dedrle, 1921

Kc6! 2 Kg6 (2 Kf6 Kd6 or 2 ... Kb6) 2 ... Ke5 3 Kg5 Kd4(b4). Draw.

If instead 1 ... Kd6(b6)?, then 2 Kf6 Kd5(b5) 3 Kf5 Kd4(b4) 4 Ke6!, and wins.

Mandler, 1929

270. 1 Kg6 Ka6 2 Kg7! Ka7 3 Kg8! Ka8 (the king has been fatally lured onto the 8th rank, and at the same time further away from its own pawn) 4 c5! (a combination which is possible only when the opponent's pawn is not far advanced) 4 ... d x c5 (4 ... Kc7 is not possible) 5 e5 etc.

Herberg, 1936

269. The black king is unfavourably placed, but White's too is a long way off: 1 ...
5.3 DOUBLED PAWNS

It stands to reason that doubled pawns are much weaker than connected or isolated pawns, but it would be wrong to exaggerate this, since their weakness often proves to be imaginary. Grigoriev wrote in 1935: ‘Doubled pawns, even when isolated, often constitute a great strength. At any rate, greater than is customarily thought. They can deprive the opponent of important squares, and they can also harbour reserve tempi... Even though they appear powerless, they provide many winning chances’.

For a long time this type of ending remained an unexplored area of theory, and it was only a number of analyses by Grigoriev (1931, 1932 and 1935) that laid a basis for the study of this ending.

The character of the play changes, of course, depending on distinctions in the pawn structure: the pawns may be passed, and if this is not so the doubled pawns may be on the same file as the enemy pawn, or on an adjacent one. All these instances will be considered separately.

5.31 All pawns passed

These cases lend themselves least well to a general evaluation, especially since here an ending with queen and pawn against queen can easily arise. But if play is decided in the pure pawn ending, the most typical features of doubled pawns are almost always revealed—reserve tempi and the fact that squares are inaccessible to the enemy king from the side.

272. This example illustrates how doubled pawns are able to defend themselves: 1 a5 Kc5 2 a4 Kd6 3 Kd8 c5 4 a6! Kc6 5 a5 c4 6 Kc8 e3 7 a7 c2 8 a8 = Q+, and wins.

Grigoriev, 1935

273. Correct is 1 Kf1! Kc4 2 Ke2 Kd4 3 g4 Ke4 4 g3 etc. (36). But not 1 g4? Kc4 2 Kf1 Kd3! 3 Ke1 Ke4 4 g3 Ke5 5 Ke2 Ke4! 6 Kf1 (hoping for 6... Kd3? 7 Ke1, winning) 6... Ke5 7 Ke1 Kf6, with a draw.

Grigoriev, 1935
274. The idea of the defence is to force the black pawns to advance to c3 and c4 (36): 1 Kd2! Ke6! 2 Ke3 Kd5 3 Kf4 Ke6.

Here or on the 7th move Black does better to go into a queen ending by 3 . . . c4 d4 4 Kf3 c3 e6 Kd6 6 Kf6 c2 7 e7 c1 = Q 8 e8 = Q Qf4 + 9 Kg6 Qe5 etc., since the pawn ending does not give him any winning chances.


275. White fails to win by 1 Ke5 Kb7 2 b3 Ka6 (or 2 . . . d5) 3 b4 Kb7 4 Kd5 Kc5 d4.

He must provoke the advance of the d4 pawn, so as to give up his b6 pawn for it. But in doing so he must not advance his second pawn, since with the pawn at b3 it will not be possible to gain control of its key squares.

1 Ka6! d6!

Black loses without a struggle after 1 . . . d5? 2 Kb5 Kb7 3 Ke5, or 1 . . . Ka8? 2 b7+ Kb8 3 Kb6 d5 4 Ke5. But after 1 . . . d6! the following continuations are insufficient:

(a) 2 b7? d5 3 Kb5 K × b7 4 Ke5 Ka6!
(b) 2 b3? d5 3 Kb5 Kc5 d4 5 K × d4 K × b6 6 Kc4 Kc6.
(c) 2 Kb5? Kb7 (White is in zugzwang) 3 Ka5 (3 b3 d5) 3 . . . d5 4 Kb5 d4.

2 Ka5!

The start of a 'triangulation'. In the event of 2 . . . Kb7 3 Kb5 Black ends up in zugzwang. The rest is simple: 2 . . . Ka8 3 Kb4 Kb8 4 Ka4 Ka8 5 Ka5 Kb8 6 Ka6, and White wins as in the note to Black's 1st move.

In conclusion—an example of self-stale-mating, which has served as the basis for a number of studies.

Kramer, 1922
Doubled Pawns

276. 1 Kf6! g4 2 Kg6 g3 3 h7 g2 4 Kh6 g1 = Q(R)—stalemate (cf. 328 and 329).

reserve tempo 6 c4 is decisive (7 Kb6 and 8 c6).

5.32 Pawns on adjacent files

With the doubled pawn structure it is of course more favourable if the pawns are on a file adjacent to the enemy pawn, rather than the same file, since the stronger side can hope to exchange one of his pawns and win with the remaining one. Therefore the king is best placed on the file of the enemy pawn, with the aim of occupying the blockading square.

The main hindrance is usually the extreme vulnerability of doubled pawns to attack from the rear.

Grigoriev, 1935

277. White fails to win by 1 Kd2 Kc7 2 Kd3 Kd7! (2 ... Kc6? 3 Kc4) 3 Kd4 Kc7 4 Kc4 (4 Kd5 Kd7) 4 ... Kc6 5 Kb4 b6. Grigoriev makes the interesting comment that, instead of 5 ... b6, there is also a draw by 5 ... Kd5 6 Kb5 Ke4!! (an important theoretical drawn position!).

Correct is 1 Ke2! (the shortest path to the b-file) 1 ... Kc7 2 Kb3 Kd7 3 Kb4! Kc6 (the b5 square!) 4 Kc4 Kd7 5 Kb5 Kc7, and now the

Grigoriev, 1935

278. 1 Ke2! Kb7 2 Kb3 Kc7 3 Ka4! Kc6 4 Kb4 etc.

279. 1 e3 Kd3 2 e5 Ke4 3 Kd6 Kf5 4 Kd5 Kg4! (Grigoriev’s theoretical position—277). Draw.
Two Pawns against One

Crum, 1913

5... Kh5 6 g3 Kg6 7 Kg4, and White wins as in example 277 (cf. also 441).

This play for zugzwang is additionally explained by the comparatively simple examples 282 and 286, which can be regarded as subsidiary to the more complicated positions 287 and 288.

Grigoriev, 1935

280 1 e6! fxe6 (1... f6 2 Kc5) 2 e5!, winning (48).

Grigoriev, 1935

281

282. White fails to win by 1 Kf5? Kf3 2 Kg6 Kg3 3 h5 Kh4, or (which leads to the same finish) 1 h5? Kf3 2 Kf5 (2 Ke6 Kf4!) 2... Kg3 3 Kg6 Kh4, since he is in zugzwang.

But were it now Black to move, he would lose. To attain this, White must reach g6 not in two moves, but in three: 1 Ke6! Kf4 2 Kf7 Kg3 3 h5 Kh4 4 Kg6, and it is Black who is now in zugzwang.

Selezniev, 1913

281. 1 Kg3 Ke2 2 Kf4 Kf2.

Now only a draw results from either 3 g4 Kg2 4 g6 f x g6 5 g5 Kh3!, or 3 Kf5 Kg3 4 Kf6 Kf4! 5 g3+ Kg4.

3 Ke5! Kg3.

Forced in view of the threat of 4 Kf6, but now White puts his opponent in zugzwang, with unexpected results.

4 Kf5! Kh4 5 Kf4!

In the rear the black king is causing too much trouble, so it is driven into a more acceptable position.
Doubled Pawns

283. 1 Kf5 (1 h6? g×h6 2 Kf5 h5! 3 Kg5 Kg4 4 K×h5 Kf5—draw) 1 ... Kg3 2 h6! g×h6 3 h5 etc.

Jandera-Prokes
Prague, 1928

284. White lost after 1 Ke4 Kb4 2 Kd4 K×a4 3 Ke4 h3 (3 ... Ka3 4 Kc3 a4 would also have won) 4 g×h3 h4 etc. A more interesting variation is 1 Kg5 h3! (the only move) 2 g×h3 Kb4 3 K×h5 K×a4, winning.

285. 1 Kd5 Kb4! 2 Ke6 Ka4 3 Ke5 Kb3 4

Moravec, 1938

286. After 1 Kf5! Kg2 2 h5 Kh3 White does not play 3 Kg6? (because of 3 ... Kg4!), but first 3 Kg5!, forcing the black king to make a fatal retreat.

287. Hopeless is 1 Kg5? Kf3 2 h4 Kg2 3 h5 Kh3 4 Kg6 Kg4! 5 h3+ Kf4, or 1 Kg4? Kf2 2 h4 Kg2 3 h3 g6.

Grigoriev, 1932
Two Pawns against One

1 Kg3! Ke3.
1 ... Kf1, an attempt to attack from the rear, was refuted by Grigoriev with the following combination: 2 h4 g6 (otherwise 3 h5 and 4 h4) 3 Kf4 (but not 3 Kg4?, in order to avoid the check on the 4th move) 3 ... Kg2 4 h5! g x h5 5 h4 etc. He also showed that, if the position was moved one file to the left, the combination would not even not work, but would even lose after 5 ... Kg3 6 Kf5 Kh4, and he concluded that the position of the study would not allow being moved.

But after 1 ... Kf1 2 h4 g6 White can also win without a combination—by 3 h3, and if 3 ... Kg1, then 4 h5 g x h5 5 h4, while if 3 ... Kc2, then 4 Kf4 Kf2 5 h5 g x h5 6 h4, and in both cases Black does not have a by-pass to the left (as in example 52). This solution is a general one, i.e., it leads to a win on all files, whereas the combination is an exception, which is possible only with rook's pawns.

After 1 ... Kc3 the win is also not difficult:
2 h4 Ke4 3 Kg4 Ke5 4 Kg5 Ke4 5 h5! Kf3 6 Kf5!

Grigoriev, 1932

... K x b2 2 Kc4 Ka3 3 Kc3; 1 ... a5 2 Kc5 2 Kc6 a5 3 Kb7 a4 4 Ka6.

Flohr-Ragozin
Moscow, 1936

289

289. Black defended impeccably: 1 ... Ke4
2 Kg5 Ke3 3 Kh6 Kf4 4 g5 Kf5 5 Kh5 Kf4 6 Kh4 Ke3 7 Kg3 Ke4 8 Kg4 Ke3 9 Kf5 Kf2 10 Kf6 Kg3 11 Kg7 Kg4 12 Kh6 Kh4. Drawn.

Ebersz, 1942

288

288. White is unsuccessful with 1 Kc5? a5
2 Kb5 a4 3 Ka5 a6, or 2 Kc6? Kc4 (286).
Only 1 Kd5! gives a draw, e.g., 1 ... Kb4 (1

290

290. 1 Ka6? (or 1 Kb4?) 1 ... Kd4! 2 Kb5
Kd3 is insufficient, as is 1 b4? Kd4 2 Kb5
Kd3.
Doubled Pawns

Correct is 1 Kb5! Kd5 (1... Kd4 2 Kc6) 2 b4 c6+ (2... Kd4 3 Kc6 Kd3 is now too late due to 4 Kd5!—286) 3 Ka5!, winning (3... Kc4 4 Ka4 and 5 b5), but 3 Ka4? would have led to a draw: 3... Kc4 4 Ka3 Kd3! (4... Kd5? 5 Kb3 Kd6 6 b5) 5 Kb3 Kd2! etc.

Ebersz, 1942

291

292

Mutual zugzwang: the turn to move is unfavourable

291. 1 Kb3 Kc1 2 Kc3 Kd1 3 Kd3 Ke1 4 Kc3 Kf1 5 Kf3 Kg1 6 f6 gxf6 7 f5 Kh2 8 Kg4 etc. The immediate 1 f6? gxf6 2 Kb3 does not work in view of 2... Kc1 3 Kc3 Kd1 4 Kd3 Ke1 5 Kc3 Kf1 6 Kf3 f5! with a draw.

5.33 Pawns on the same file

Here the play is more varied. Berger, who gave on this theme only one example, and that an untypical one, assumed that the stronger side wins only in the form of an exception. But analyses by Grigoriev showed that, if these are indeed exceptions there are a considerable number of them, and as a result it is even unclear whether a win in these endings should be regarded as an exception or the rule.

The possibility of a win depends entirely on whether or not the enemy pawn can be won. Therefore the degree to which the pawns are converged is important (reserve tempil), as is the placing of the kings.

In endings with blocked pawns the idea of a by-pass from the side predominates; the struggle for control of the critical squares strongly resembles the ending of pawn against pawn. But if the pawns are not blocked, one of the most effective defensive measures (as in the ending with pawns on adjacent files) is an attack from the rear, creating numerous zugzwang positions. These endings are considerably more complicated, since with the advance of either pawn the system of critical squares changes; endings of the first type (with blocked pawns) enter into them as a potential possibility.

292. White’s chances are on the K-side (the invasion at h6). To the left of the f6 pawn White cannot achieve anything, if the black king is on the same rank as its pawn (e.g. in the position Kd4/Kd6 Black has the move... Kc6, neutralizing White’s reserve tempil; but in the position Kd5/Kd7 the advance of the f2 pawn is decisive).

White is two squares away from h5, whereas Black is four away from g7; therefore Black can only hope to save the game by a counter-attack on the f5 pawn.

White to move is unable to win, e.g.: 1 Kg4 Ke4! 2 f3+ Ke5!, or 2 f4 Ke3!; 1 Kg3 Ke5! 2 Kg4 Ke4; 1 f3 Kd6!, and nothing is achieved either by 2 Kg4 Ke5!, or by 2 Kg3 Ke7! (the tempil lost by White have allowed the black
king to approach g7) 3 Kh4 Kf7 4 Kg4 Kf8
(with a reserve square available, Black can
successfully defend both wings—293) 5 Kh5
Kg7.

But if it is Black to move, he is unable to
save the game: 1 ... Kd4 (on 1 ... Kd6 White
has the immediately decisive 2 Kg4 Ke5 3 f3)
2 f3! Kd5 3 Kg3! Kd6 4 Kh4! Ke5 5 Kg4 and 6
KKh5.

By coincidence, position 292 served as the
basis for studies by both Bianchetti (1924)
and Grigoriev (1932). In Biachetti’s position
the kings were at Kd1/Kh7, and the introdunc-
tory play consisted of a simple approach of
the kings: 1 Ke2 Ke6 2 Kh3 Kd5 3 Kf4 (clearly
not 3 Kg4? Kc4). Grigoriev’s king placing
Ke4/Kc6 creates a more intricate position,
since it emphasizes the question of gaining
the decisive correspondence: 1 Kf3! Kd5 2
Kf4! (Grigoriev pointed out that with the
black king at d6 or c5 a win would not be
possible).

Grigoriev, 1933

293. 1 Kf5! Kd6 (1 ... Kd4 2 f4 Kd5 3
Kg4!) 2 f4 Kd7 3 Kg4 Ke8 4 Kh5! Kf8 5 Kg5.
Having no reserve square, Black loses: 5 ...
Ke8 6 Kh6; 5 ... Kg8 6 Kf5 Kf8 7 Ke5 Ke8 8
Kd6 Kd8 9 f5 or 6 ... Kh7 7 Ke4 Kh6 8 Kd5
Kg6 9 Ke5 and 10 Kd6.

If the position is moved to the left the win
is easier, but if it is moved down or to the
right there is no win.

Horwitz & Kling, 1851

294. If he begins, Black draws by 1 ... d6!
followed by maintaining the opposition on
the main files.

White wins by 1 d6! Ke8 2 Kc4 (the shortest
path to a6), but he also succeeds with 2 Ke4
Kf8 3 Kd5 Kf7 4 Kc4!

The result does not change if the position is
moved down the board, but a win with White
to move is possible only on the central files,
otherwise the by-pass is unsuccessful.

It is the same in the following position.

Prokop, 1925

295. 1 Kd3 Kd8 2 Ke4! (2 Kd4? or 2 Ke3?—
2 ... Kc7!) 2 ... Ke8 3 Ke3! Kd8 4 Kf4, or 3
... Kf8 4 Kd4, winning.

The following positions illustrate the at-
tack on doubled pawns from the rear.
296. This is a position by Grigoriev, in which an artificial lengthening of the solution has been removed (Ke8–f7 and Kd6–e5).

After 1 Kg6 (1 Kxg7? Kf5) the position takes on a natural appearance: 1 ... Ke6 2 g4 Ke5 3 g3 and 4 Kxg7 is hopeless for Black, and so: 1 ... Ke4 2 g4 Kf4 (2 ... Ke5 3 g3; 2 ... Ke3 3 Kf5! followed by the advance of the pawn to g6) 3 g5! Kg4 4 g3 etc.

The position after 3 g5 is one of mutual zugzwang. Were it White to move, the result would be a draw: 1 g3+ Kg4 or 1 Kh5 Kg3 (1 ... Kf5? 2 g3) 2 g6 Kf4 3 g4 Ke5.

But if the position with knight’s pawns is moved onto the bishop’s file, White no longer gets into zugzwang.

297. The premature advance of the king to f6 is easily corrected: 1 Kg5! Kd5 (1 ... Kf3 2 f6 and 3 Kh6) 2 f6! Ke5 3 f3! Ke6 4 f4, and wins.

298. Here the white king is unable to break through to the a1 corner, which would ensure a draw. The only chance is an attack on the doubled pawns from the rear: 1 Kf2 Kb4.

A plausible move, but 1 ... Ka4 is stronger, and is considered later.

2 Ke3 Kb3.

If 2 ... Kc4, then 3 Kd2! Kb3 4 Kc1! (4 Kd3? b5! 5 Kd4 b6) 4 ... Ka2 5 Kc2 b5 (to win Black would have needed ... b4) 6 Kc3, with a draw.

2 ... Ka4 is a trappy move, when White can draw by either 3 Kd2 or 3 Kd3, but he loses after 3 Kd4? Kb3 (296).

3 Kd4, and in comparison with example 296 all the zugzwang positions are as though turned inside out. It is now the turn of the stronger side to move, which ensures a draw: 3 ... b5 (3 ... Kb4 can be met by 4 Kd5 or 4 Kd3—the a1 corner!) 4 Kd5! (4 Kc5? b4). E.g.: 4 ... b4 5 Kc5! b6 + 6 Kb5; 4 ... b6 5 Kc6; 4 ... Kb4 5 Kd4! b6 6 Kd5 etc.

The position after White’s 3rd move is a curious parallel to example 288, where the
doubled pawns were not on the knight's file, but the rock's file.

As mentioned above, after 1 Kf2 a stronger try is 1... Ka4 (so that the king does not stand in the way of the pawn) 2 Ke3 b5.

Now care is required of White. The following are equally bad: 3 Kd2? Kb3! 4 Kc1 Ka2 5 Kc2 b4!; 3 Kd3? Kb3! 4 Kd4 b6; 3 Kd4? Kb4! 4 Kd5 Kb3! 5 Kd6 (5 Kc5 b4) 5... Kc4! 6 Kc7 b4 etc.

3 Ke4!! Kb4.

3... b4 can be met by 4 Kd5 or 4 Kd4, while if 3... b6 or 3... Kb3, then 4 Kd5!

4 Kd4, and 4... b6 or 4... Kb3 is met by 5 Kd5 with a quick draw.

Grigoriev, 1935

299. With his pawn at g7 (instead of g6), Black would not advance it, but would sit it out with his king at h8. But here White wins. However, the straightforward advance up the h-file would be met by a rapid advance by Black along the b6-f2 diagonal. Although it loses time, White must approach his king, such that it simultaneously prevents the advance of the enemy king.

1 Kg1! Ke5 2 Kf2 Kd4 3 Kf3 Ke5 (3... Kd3 4 Kf4! and 5 g4) 4 Kg4 Kf6 5 Kf4 g5+ (otherwise 6 Kg5) 6 Ke4 Ke6 7 g4, and wins.

This example suggests the necessity for a detailed analysis of those cases where the kings are in front of the pawns or alongside them.

The following two positions, which illustrate certain variations from the Grigoriev studies given below, are at the same time of independent theoretical significance. With his pawns on one of the central files White wins.

300. Here (the king can also stand at e7) Black must be able to answer Kh4 with ... Kg6.

The black king must keep to the left of White's, otherwise if it is at h6 the white king will seize control of d4 (Black will be unable to answer Ke3 with ... Kd5). The defence is simple, since on Kg3 either ... Kf6 or ... Kf7 is possible, and when the white king is at h3, Black's can be at f6, f7 or g7. As Grigoriev put it, Black has 'an excess of correspondence'.

It is the same in the centre, e.g. 1 Kf3 Kd5 2 Ke3. This looks like zugzwang for Black, but in fact either 2... Ke4 or 2... Ke5 is possible, since on 3 Kf3 Kd5! 4 Kg3 there follows 4... Ke4 (4 Kg2 Ke6).

It is clear that Black would be unable to defend the critical squares, if the position were moved onto one of the central files.

301. The reason for White being unable to win when it is him to move is again that his pawns are on the bishop's file, and not one of the central ones.

The attempt 1 f5+ Kd6 quickly leads to
Correct is 1 Kg4 (a win is also possible after 1 Kg3) 1 ... Kg6 2 f4 f6 (2 ... f5 + 3 Kf3; 2 ... Kf6 3 f5 Ke5 4 Kg5 Ke4 5 Kf6 or 4 ... Kd5 5 f6!—297) 3 f5 + Kf7 (3 ... Kh6 4 f3, then 5 Kf4), and now White has two ways to win: 4 Kh5 Kg7 5 f3! Kh7 6 Kg4, or 4 Kf4 Ke7 5 Kf3! Kf7 (5 ... Kd6 6 Kg4) 6 Ke4.

With Black to move: 1 ... Ke6 2 Kg5 Ke5 (2 ... f6 + 3 Kf4) 3 f4 + Ke4 (3 ... Ke6 4 f3 or 4 f5 +) 4 f5, winning as shown above.

302. If the position were moved onto the g-file, there would be no win. On one of the central files White would win by moving his king to either right or left. But in the given case 1 Ke4 would lead only to a draw: 1 ... Ke6 2 f4 f6! (301).
304. 1 Kg2! Ke6 2 Kg3 (2 f4? f5!; 2 Kh2? Ke5! 3 Kg3 f5!—draw) 2 ... Kf5 3 f4, or 1 ... f5 2 Kg3(h3) Kf7 3 Kh4! Kg6 4 f4, and wins.

Grigoriev, 1936

305. This complicated position, which contains all the variations examined above, is now not difficult to analyze. With the white king at a3 a simple draw would be given by 1 Kb4 (1 ... Kd6 2 c4), while with the black king at d7 White would lose in view of ... c5 and ... Kc6 (303).

White's main task is not to allow the opponent to place his king in front of his pawns. This is easily achieved: 1 Kb3 Kd6 2 Kc4! c5 (forced) 3 Kd3! with a draw, e.g. 3 ... Ke5 4 Kc4 Kd6 5 Kd3; 3 ... Ke6 4 c4 (300); 3 ... Kd5 4 c3 (301).
6. Two Pawns against Two

In endings with a minimal number of pawns the material has been as exhaustive as possible, but beginning with this chapter this will no longer be the aim. However, in view of the intermediate position of these endings between the simplest and those with many pawns, they will nevertheless be covered more fully than those in subsequent chapters.

Increasing the number of pawns does not of course change the basic principles of play. Many of the ideas already considered will be met in these endings too—in a wider orchestration, so to speak. But at the same time, an increase in the material gives rise to a broadening of the range of strategic and tactical ideas, and certain ideas appear which simply could not occur with a smaller number of pawns. Endings with a certain material have their own specific features, and this means that they must be considered separately.

An attempt to make a detailed classification of the material, depending on the pawn structure, in the given endings (to say nothing of those later in the book) runs into significant difficulties in view of the diversity of these structures. Therefore up till now the authors of endgame books have not made such a classification, but at best have limited themselves to picking out endings with passed pawns.

The presence or absence of passed pawns undoubtedly gives the endings a fundamentally different nature, and this has led to a division of the material into two basic groups:

6.1 Endings with passed pawns.
6.2 Endings without passed pawns.

But to restrict the division to this would be insufficient, and each of the basic groups has additional subdivisions, which are necessary for a more clear-cut separation of the types of endings possessing characteristic features. These internal divisions are: connected pawns against connected; connected pawns against isolated; isolated pawns against isolated (including doubled pawns).

6.1 ENDS WITH PASSED PAWNS

It is expedient to divide these endings into two groups:

6.11 Endings with two passed pawns.
6.12 Endings with one passed pawn.

6.11 Two passed pawns

If in the given ending one side has two passed pawns, it stands to reason that the other side also has two passed pawns. These pawns can be connected or isolated, central or wing pawns—in various combinations and with differing degrees of advancement. From this it is clear that any generalizing conclusions are possible only in endings reduced to even narrower groups. Practice has shown, for example, that in the majority of cases isolated wing pawns have the advantage over central connected pawns.

The diversity of features forces us to turn to specific examples.

306. Here the white king is lending powerful support to its pawns. The black a-pawn requires 5 moves to promote, but it can manage only 3–4 moves before White, gaining time thanks to checks, either obtains a queen or gives mate (105).

From the 4th rank White would have required not less than 6 moves to obtain a
Two Pawns against Two

Fine, 1941

306

Y. Behting, 1894

308

queen (and as many as 9 from the 3rd rank). It follows that one can think in terms of a win in such positions, if the pawns are at least on the 4th rank and the relative placing of the kings is favourable, or else there is the possibility of halting the advance of the enemy pawns.

Horwitz, 1879

307

308. In this classic study the difficulty of the solution is caused by the position of the black king at f7. Were it at e8, White would win by 1 Ke5 with the threat of Ke6. This threat constitutes the basic motif of the position: with the black king at e8, White gains the possibility of moving onto the 5th rank, ignoring the advance of the d-pawn.

The straightforward 1 Ke4? leads to a draw: 1 ... c5 2 Kd3 Ke8 3 Ke4 Kf7, and White is unable to gain a tempo.

Correct is 1 Kf3! c6 2 Kf4! c5, and now White transfers his king to c4 in an odd number of moves: 3 Ke4 Ke8 4 Kd5 (4 Kd3 is also possible) 4 ... Kd7 5 Kc4 Ke8 6 K × c5! d3 7 Kd6 Kf7 8 Kd7, and White wins.

309

307. 1 ... Kh6 (1 ... f4 2 d5 Kf5 3 d6 etc.) 2 Kf4 (or first 2 d5) 2 ... Kg7 3 Kg5 Kf7 4 d5 Ke7 5 e6 Kd6 6 Kf6, winning.
309. One textbook gives: ‘1 Ke3 f5 2 Kd4 Ke8 3 Ke5 Kd7 4 Kb6, and wins’.
But instead of 3 ... Kd7?, correct is 3 ... Kb7!, after which White has to continue 4 
Kd5! e3 5 Ke6 e2 6 Kd7 e1 = Q 7 c8 = Q + Kb6 
Qc6 + Ka5 9 Qc5 + Ka4 10 Ke8 Qe6 + 11 
Kc7 f4 12 d7 Qf7 13 Qd4 + Ka3 14 Kb8 
Qb3 + 15 Ka8 Qf3 + 16 Ka7, and wins.

Herberg, 1935
from Behting

5 Kh6) 5 Kf5! e3 6 Ke6 Kg8 (6 ... e2 7 Kf7) 7 
f7 + or 7 Ke7, and wins.

Karstedt, 1921

310. 1 Ke5 Kf7 2 Kd4 c6! 3 Kd3! c5 4 Ke4 
Ke8 5 K × c5 etc.

311. This position arose in one of the 
games from a tournament in Barcelona 
(1932). Here White agreed to a draw.
Dedre (1950) demonstrated a win: 1 Kg3! 
d6 2 Kg4! d5 3 Kf4 Kg8 4 Kg5! Kh7 (4 ... Kf7 
313. This complicated position clearly il-
illustrates the boundary of the enemy king’s 
approach that White can permit.
White cannot get by without the advance 
of his pawns, since they do not present a
barrier (the e4 square!), and are not threatening to promote. E.g. 1 Kc2? Kg4 2 Kd3 Kf4 (Grigoriev shows that this position is also won with Black to move*) 3 Kc2 Ke4 4 Kb3 Kd3, or 4 Kd1 d3 5 Ke1 c2 (5 ... d2 + is also possible) 6 Kb2 (if 6 Kd2, then not now 6 ... Kd4?, but 6 ... Kf4! 7 Kc1 Kf5! 8 Kd2 Ke4 etc.) 6 ... Kd4 7 g4 Ke4 8 g5 Kf3 9 g6 d2, and wins.

1 f3!
The only move: the black king is deprived of the e4 square. If 1 f4 Kg4 2 g3 or 1 g3 Kg4 2 f4, then 2 ... Kf5 and 3 ... Ke4.

1 ... Kh4.
1 ... Kg5? is bad in view of 2 g3! with a draw (but not 2 Kc2? Kf4 3 Kb3 Kg3 4 Kc2 Kf2! 5 Kb2 c2 or 5 ... Ke1 etc.).

2 g4! Kg5 3 Kc2 Kf4 4 Kd3, with a draw in view of zugzwang (Black cannot gain a tempo).

For isolated pawns on the same rank, the rule of the ‘wandering square’ applies (43).

Horwitz & Kling, 1851

314. Here after 1 Kf4! the advance of either of Black’s pawns leads to its loss. Therefore he is forced to stick to waiting tactics, manoeuvring with his king between c7 and b6.

1 ... Kb6.
The composers’ idea was that White wins

* Indeed, if this is regarded as the starting position, Black wins: 1 ... Kg4! 2 Ke2 (2 f3 + Kg3) 2 ... Kf4! 3 Kd3 (3 f3 Kf4; 3 g3 Ke4) 3 ... Kf4, and it is White to move. (J.M.)

315. 1 a5 Kb5 2 e5, or 1 e5 Kd5 2 a5.
The same position, but with the pawns at a3 and e3, is a draw: 1 Kh3! Ke4 2 Kg4 etc. (if the pawns are not forced to advance, they indirectly defend each other).
The following study is instructive.

Rauch

316. If Black can be given the move, then after 1 ... Kd5 2 Ke3 the pawns will promote
of their own accord. Therefore 1 Kf1 Kd3 2 Kg1 Kd4 3 Kg2 Kd3 4 Kf1 Kd4 5 Ke2, and wins.

317

317. 1 Kc4 Kh8 2 Kc3 Kg7 etc.

318

318. 1 Kc5 Kg7 2 Kd6 a4 3 Ke7 a3 4 h8=Q+ etc.

319

319. Both sides are in zugzwang, and therefore the turn to move is a disadvantage. But if the black c-pawn is moved to c4, and the white king to c3, then 1 Kc2 wins.

320

Stoltz-Nimzowitsch
Berlin, 1927

320. 1 ... f4! 2 g×f4+ Kd6!!, and the passed pawns ensure Black a win.
A different idea is illustrated by the following study.

Walker, 1841

321

321. After the queening of the pawns and the exchange of queens by Qb8–b5+, White wins by a2–a4.
322. The win is achieved by advancing the pawns in turn: 1 f4! (1 Kg2? Kb3 or 1...d5—
draw) 1...Kb4 2 h4! (2 Kg2? a5!, winning) 2...d5 3 f5 Kc5 4 h5 (4 Kg2? Kd6! leads to a
draw) 4...d4 5 f6 (here there is an alternative
win by 5 Kg2 Kc4 6 f6 d3 7 f7 d2 8 f8 = Q
d1 = Q 9 Qf7 + etc.) 5...Kd6 6 h6 d3 7 f7
Ke7 8 h7 d2 9 f8 = Q + K×f8 10 h8 = Q +,
and wins.

The second solution is not removed by the
addition of a white pawn at g2, since on the
5th move the king can also go to g1, e.g. 5
Kg1 Kc4 6 Kf2 Kb3 7 f6 d3 8 f7 d2 9 f8 = Q
d1 = Q 10 Qf3 + etc.

The addition of a white pawn at f3, sug-
gested by Halberstadt (1951), also does not
help, since 5 Kg2 is still possible, with a won
queen ending after 5...Kc4 6 h6.

Chéron managed to correct the study, but
his position has a slightly different balance of
forces.

323. 1 f4! (1 h4 d5 is unsuccessful) 1...
Kb4 2 h4.

The only way! If 2 f5, then 2...Kc5 3 h4
Kd6 4 h5 Ke5 5 h6 Kf6 6 K×h2 c5 7 Kg3 c4
8 Kf4 c3 9 Ke3 d5 10 Kd3 d4, with a draw.
2...d5 3 f5! Kc5 4 h5 d4 5 f6! Kd6 6 h6 d3 7
f7 Ke7 8 h7 d2 9 f8 = Q + K×f8 10 h8 = Q +,
and White wins.

324. 1 f5 Ke5 2 h5 g3 3 Ke1! d4 4 f6 Kd6 5
h6 g2 6 Kf2 d3 7 f7 Ke7 8 h7 g1 = Q + 9
K×g1 d2 10 f8 = Q + K×f8 11 h8 = Q +,
and wins.

325. The black pawns do not have time to
occupy the saving position on the same rank,
and are bound to fall. But the play is compi-
lcated by Black’s unexpected use of a curious
defensive resource.

1 h3 c5 2 Kb1 c4 3 Ka2.
Endings with Passed Pawns

Grigoriev, 1937

2 Ke5 a5 (after 2 ... f5 3 d5 etc., Qe8+ is again decisive) 3 h4 f5 (3 ... a4 4 Kb4 f5 5 h5 f4 6 h6 f3 7 h7 f2 8 h8= Q f1= Q 9 Qe5+ etc.) 4 h5 f4 5 h6 f3 6 h7 f2 7 h8= Q f1= Q 8 Qe8+, winning.

Selezniev, 1918

Under normal circumstances this attack on the vulnerable leading pawn would immediately decide matters, but here Black's latent defensive resource comes into play.

3 ... c3! 4 Kb3! (if 4 K x a3?, then 4 ... Kg3! 5 f5 Kf4 6 f6 Ke3 7 f7 c2 8 f8=Q c1=Q+, with a draw) 4 ... a2 5 K x a2 (what White has achieved is that the c-pawn will queen without check) 5 ... Kg3 6 f5 Kf4 7 f6 Ke3 8 f7 c2 9 f8 = Qc1 = Q 10 Qh6+, and White wins.

Jelinek, 1944

326. 1 d4 Ke4 (or 1 ... f5 2 d5 Ke5 3 Kc5 f4 4 d6 etc., winning the queen by a 'skewer')

Kubbel, 1922

327. 1 Ke6 Kd8 2 Kd5 K x d7 3 Ke4! (3 Ke5? Ke7 4 Kd4 Kf6 5 Ke4 g4 6 Kf4 g3 7 K x g3 K x f5, winning) 3 ... Kd6 (3 ... Ke7 4 Kf3 Kf7 5 Kg3 Kf6 6 Kg4—draw) 4 Kf3 Ke5 5 Kg4 Kf6 6 Kh5 K x f5—stalemate. An instructive finish!
Two Pawns against Two

328. 1 Kd4 d6! 2 Ke3! (forcing Black’s reply and obtaining the c5 square) 2 ... d5 3 Kd4! b4 4 K × d5 b3 5 Ke6 Kb8 (otherwise 6 a7) 6 K b6 h2 7 a7 + Ka8 8 Ka6 bl = Q — stalemate.

Gorgiev, 1950

With the pawn at h7 — draw

But the immediate 1 a6 + ? is premature: 1 ... K × a6 2 K × c6 Ka5 with a draw (cf. position 75; the pawn is at g3 rather than g2).

Exploiting the fact that the black king is tied down, White can advance his pawn on the opposite wing — 1 g4!, winning.

With the black pawn at h7, after 1 g4 Ka6! it is a draw (Peretsman, 1957).

329. 1 h6 Kf8 2 h5 Kf7! 3 Ke3!! (3 Kf4? f5 4 Kg3 Kg8 etc.) 3 ... Kf8! 4 Kf4! f5 5 K × f5 g3 6 Kf6! Kg8 7 Kg6 g2 8 h7 + Kh8 9 Kh6. Draw.

Nimzowitsch-Tarrasch
San Sebastian, 1911

330. 1 a5 2 Ke4 f5 + !!, and White resigned in view of 3 Kd4 f4!, after which both white pawns fall.

331. 1 ... a5 2 Ke4 f5 + !!, and White resigned in view of 3 Kd4 f4!, after which both white pawns fall.

332. An essential condition for realizing the outside passed f-pawn is not to allow c3 — c4.

6.12 One passed pawn

Here we must distinguish between the cases where both sides have a passed pawn, and where only one of them has. In the first case what is significant is which of the pawns is stronger (proximity to the queening square, whether it is an outside or a protected passed pawn, and so on). In the second case the possession of a passed pawn is normally a decisive advantage, but sometimes difficulties are involved in realizing it, and even drawn exceptions are possible.

330. In this textbook example White has an outside passed a-pawn, the advance of which will divert Black’s king from the defence of his c-pawn. The blockade of the c-pawn with his king is also a favourable factor for White.
1... Ke5? (1... d2! is correct, as shown by Manteyfeli; see below) 2 a4? (only 2 K×d3 ensures a draw) 2... Kd5? (2... d2 would still have won, but now a draw is inevitable) 3 K×d3 Ke5 4 c4 f5 5 Ke3 f4 (in the absence of other moves, the pawn has had to leave the winning zone—209) 6 Kd3 Kb4 7 Ke4. Drawn, since the white king reaches c1 in time.

Black could have won by 1... d2! 2 K×d2 Ke4 3 a4 (3 Ke2 a4 4 Kd2 f5 5 Ke2 f4 etc.) 3... f6! (but not 3... f5? 4 Ke2 f4 because of 5 c4 with a draw) 4 Ke2 f5 5 Kd2 Kf5! 6 Kd3 (6 c4 Ke4) 6... f4, and the game is decided by the pawn queening with check.

333. The apparent simplicity of the position can easily lead to a mistake.

It is clear that a draw results from 1 Kh6? Kf7 2 g5 (2 Kh7 Kf6) 2... Kg8 3 Kg6 d5 4 Kf5 Kg7.

But it is not less obvious that a draw also follows from 1 Kg6? Ke7 2 Kf5 Kf7 3 Ke4 Kg6, when the pawns fall simultaneously (White would win if, in reply to K×d6, Black were to capture on g4 not immediately, but with a delay of one move, since then his king would not reach e8 in time), or 2 g5 d5 3 Kh7 d4, when the queens are obtained simultaneously (but if the black king were at e6, White would win by queening with check).
334. Black's king is on the ill-fated a3-f8 diagonal. 1 Kf6! puts him in zugzwang; after 1 ... c5 2 bxc5+ Kxc5 3 f4 in certain variations the f-pawn queens with check, while if 1 ... h5, then 2 Kg5 Ke5 (2 ... Ke6 3 f4) 3 f4+ Ke4, and now not 4 f5? h4!, but 4 b5! (to clear the diagonal), winning.

Grigoriev, 1925

335. The white king has to work enormously hard, so as in the end to gain a draw thanks to the outside passed h-pawn. 1 Kd4! Kb4 2 Kd3 Kb3 3 Kd2 Kc4! (3 ... Kb2? 4 h4) 4 Kc2 Kd5 5 Kb3 K × e6 6 Ka4 Kf5 (if 6 ... Kd5, then 7 h4 e5 8 h5 Ke6 9 K × a5 Kf5 10 Kb5—draw) 7 K × a5 e5 8 Kb4! Kf4 (8 ... e4 9 Kc3 etc.) 9 Kc3! (9 h4? e4! 10 h5 e3! 11 Kc3 Kf3!, winning) 9 ... Kf3 10 Kd2 Kf2 11 Kd3. Draw.

In the following examples the outside passed pawn is easily neutralized.

336. 1 Ke7 Kb5 2 Kd6 a5 3 c4 + ! (getting rid of this pawn, which would cause a loss in the ending of queen against pawn) 3 ... K × e4 (otherwise 4 c5) 4 Ke6 a4 5 K × f6 a3 6 Ke7! a2 7 f6 a1 = Q 8 f7. Draw.

Grigoriev, 1931

337. 1 Kd5 Kd2 2 Ke4 Kc3 3 d4 K × f4 4 d5 Ke5 5 Ke5 a4 6 d6 Ke6 7 Kc6 a3 8 d7 a2 9 d8 = Qa1 = Q 10 Qc8 + Kf6 11 Qh8 + , winning.

According to analysis by Grigoriev, an analogous position could have been reached in a game Capablanca-Dake (1931), but moved one file to the right, in which there would no longer have been a win.
338. 1 Kd4 Kf3 2 e4 K x g4 3 e5 Kf5 4 Kd5 b4, and Black does not lose his queen.

Maizelis, 1954

339. 1 Kf4 Kd6 2 Kg3 Ke5 3 f6 Ke6 4 Kh4 h6 5 K x h5! h x g5 6 Kg6 etc.

With Black to move: 1 ... h4 2 Kf4 Kd6 3 Kg4 Ke5 4 f6 h3 5 K x h3 Ke6 6 Kh4 h6 7 Kh5, and wins.

340. The presence of the protected passed pawn ensures a win. By a by-pass to the left White either forces the advance of the h-pawn, after which it is lost, or else gives mate.

1 Kf4(f3) Ke7 2 Ke4 Ke8 3 Kd5 Ke7 4 Ke5 Ke8 5 Kc6 Ke7 (5 ... Kf8 6 Kd6) 6 Kd5 Kf8 7 Kd6 Kg8 8 Ke7 Kc7 (8 ... h5 9 K x f6 h4 10 g7 h3 11 Kg6 h2 12 f6 h1 = Q 13 f7 mate) 9 Ke6 h5 10 Kd5 Kh6 11 Ke4 Kg7 12 Kf4 Kh6 13 Kg3 Kg7 14 Kh3 Kh6 15 Kh4, and White wins.

Horwitz, 1879

The solution can be shortened by one move by 4 Kc6 Ke8 5 Kd6 Kf8 6 Kd7! Kg8 (6 ... h5 7 Ke6) 7 Ke7 Kg8 8 Ke6 etc.

Zinar suggests the following modification of Horwitz's study, to exclude the dual solutions.

341. 1 Ke3 (White has to hurry; if the black king reaches b7 it will be a draw) 1 ... Ke8 2 Kb4 Kd7 3 Ka5 Kd8 4 Ka6 Ke8 5 Ka7! e5 (5 ... Kd8 6 Kb7 Kd7 7 Kb6 e5 8 Ka5) 6
Two Pawns against Two

Kb6 Kd7 7 Ka5 Ke6 8 Kb4 Kf6 9 Kc4 Ke6 10
Kd3 Kf6 11 Ke4(e3) Ke6 12 Kf3 Kf6 13 Kg4
Ke6 14 Kg5, and White wins.

Grigoriev, 1930

Philidor, 1777

342. The position illustrates a case where, in spite of the possession of a protected passed pawn, the stronger side is unable to win the isolated enemy pawn.

The black king defends the g-pawn without stepping outside the 'square' of the c5 pawn, e.g. 1 Ke3 Ke5 2 Kf3 Kf5 3 Kg3 Ke5! (3 . . . Kf6? 4 Kg4) 4 Kg4 Kf6. Draw.

With Black to move, later analyses showed that a draw is given by 1 . . . Ke5 2 Ke3 Kd5!, but not 2 . . . Kf5? 3 Kd4 g4 4 Kd5! g3 5 c6, when White wins the queen or exchanges it. 4 Kd5! succeeds most quickly, but it is of fundamental importance that 4 Ke3 also wins (by the method shown in example 343). From this it follows that in the position (Ke3, pawns b4, c5/Ke5, pawns b5, g4) White wins irrespective of who it is to move.

The situation under consideration (with the isolated pawn on the bishop's file rather than the knight's, which does not essentially change matters) is encountered in the main variation of the following study.

343. 1 b4! Ke7!? (1 . . . a x b4 2 a5) 2 b5! Kd6
3 Ke2 Ke6 4 Kf3 Ke5 5 Kg4! Ke4 6 b6 f3 7

Kg3! Ke3 8 b7 f2 9 b8 = Q f1 = Q 10 Qe5 +
Kd2 11 Q x a5 + Kd1 (the exchange of queens is forced in all variations) 12 Qd5 + Kc1 13
Qc5 + Kd1 14 Qd4 + etc.

If in example 342 the g-pawn is moved to h5, or the position is moved one rank up the board, White wins without difficulty.

344. 1 Kg3 Ke5 2 Kf3 Kf5 3 Kg3 Ke5 4 Kg4
Ke6 (4 . . . Ke4! is even better) 5 Kf4 Kd5, with a draw.

An analogous position, but with an extra pawn for White, is reached in the following interesting and complicated ending.
345. 1... g5 cannot be allowed, but 1 g5 does not win. Therefore: 1 f4! g5! 2 f5! e4.

This move is necessary either now or on the following move, since it is obvious that the defence of the e5 pawn from d6 is hopeless. The composer's solution gives 2... Kd7!, but the transposition of moves is methodologically more convenient for the final determination of the pawn configurations and for showing more clearly the manoeuvring of the kings.

3 Kg2! Kd7.

From zugzwang position Ke3/Ke5 the correspondence of the squares f2/d6 and e2/d5 is clear, and this explains the following play: White's aim is to reach the position Kc4/Ke5 with the opponent to move.

4 Kf1! Kc6 5 Ke1! Ke5 6 Kd1!

This manoeuvre takes account of the additional correspondence of the squares d2/d6 and c2/c6, and in addition c3/d5 and b3/c5, resulting from the zugzwang position Kc4/Ke5.

6... Kc6 7 Kc2! Ke5 8 Kb3! Kd6 9 Kb4! (the decisive approach) 9... Kd5 10 Ke3! Ke5 (in example 344... Kc5 was possible, but here White has an extra pawn, and therefore f5-f6 wins) 11 Kc4!, and wins.

346. 1 d5+ Kd6 2 Kd4 g6 (2... g5 3 Ke4 h5 4 Kf5 h4 5 Kg4 and 6 f4; 2... h5 3 Ke4 h4 4 Kf4 K×d5 5 Kg4 Ke5 6 K×h4 Kf4 7 Kh5) 3 Ke4 h5 4 Kf4 K×d5 5 Kg5 Ke5 6 f4+ Ke4 (or 6... Ke6) 7 K×g6. Draw.

Grigoriev, 1923

347. The basic content here is the struggle for the critical squares of the passed pawn: 1 Kc2 Ka5 2 Kb3 a2 3 Kb2! Kb4 4 Ka1! K×a4 (4... K×c4 5 K×a2—draw) 5 c5! d×c5 6 K×a2. Draw (65).

No less instructive is the next, more complicated position.
Against correct defence by the opponent, White is unable to attain a zugzwang position in the following example.

348. 1 g7 Kf7 2 Kf5! (2 Kxe5? Kxg7 3 Kf5 Kf7—draw) 2 ... Kg8! (2 ... Kxg7 3 Kxg5, and wins) 3 Kg4! (on 3 Kxg5? there follows 3 ... e4! 4 dxe4 Kxg7—draw) 3 ... Kf7 (3 ... e4 4 dxe4 Kf7 5 Kf5 Kg8 6 Kf6 g4 7 e5 etc.) 4 Kxg5! (with the king at f7 the capture is now possible: if 4 ... Kxg7, then 5 Kf5 e4 6 Kxe4!) 4 ... e4 5 Kh6!! Kg8 6 dxe4, and wins.

The following examples reflect the struggle for a zugzwang position.

349. If 1 Kc3 Kf7 White cannot play 2 Kd4? Ke6!, while after 2 Kd3 Ke7 the result is a draw. But he can reach d4 not in two moves, but in three: 1 Kb4! Kf7 2 Ke5 Ke6 3 Kd4, and wins.
351. Here White has to overcome certain difficulties by accurate play. The analysis is by Shusterovich (Saratov).

1 Kf2 Kh3 (or 1 ... g3+ 2 Kg1, as in the main variation) 2 Kg1 (2 f5? Kh2) 2 ... Kh4 3 Kg2 (3 Kh2? g x f3) 3 ... g3 4 Kg1! (4 Kh1?? Kh3!, and Black gives mate with his h-pawn) 4 ... g2 5 Kh2! g1 = Q + 6 K x g1, and wins.

We will now consider positions where only one of the sides has a passed pawn. As a rule, the stronger side wins by elementary means. Disregarding these cases, we will dwell on more complicated examples.

Grigoriev, 1934

352. Black's only chance is to transfer his king to f6 and play ... h5, undermining his opponent's pawn chain. White must therefore defend his f-pawn from e4. Hence the zugzwang position Ke4/Kf6, which determines the immediate manoeuvres of the two sides.

1 Kg2 Kg7 2 Kf3 Ke7! 3 Ke3! Ke7 4 Kd4
Kf6 (otherwise 5 Ke5) 5 Ke4 Ke7 6 Ke5 Kf7 7 f6 h5 (there is nothing else) 8 g x h5 g4 9 h6 g3 (9 ... Kg6 10 Ke6 g3 11 f7 etc.) 10 h7 g2 11 h8 = Q g1 = Q 12 Qh7 + Kf8 13 Qe7 + Kg8 14 f7 +, and White wins.

353. The win appears straightforward, but in reality White encounters serious difficulties, associated with the fact that the distance between his pawns is insufficiently large, and also that one of them is a rook’s pawn. He fails to win, for example, by 1 Kf4 Kf7 2 Kg5

354. The win appears straightforward, but in reality White encounters serious difficulties, associated with the fact that the distance between his pawns is insufficiently large, and also that one of them is a rook’s pawn. He fails to win, for example, by 1 Kf4 Kf7 2 Kg5
Kg7 3 d6, since after 3 ... Kf7 the black king has time to take the passed pawn and then occupy f8 (instead of 3 d6?, White must play 3 Kf4).

Another unpleasantness awaiting White is seen in the following variation: 1 Kd4 Kf7! (1 ... Ke7? 2 Ke5 Kf7 3 d6 or 2 ... Kd7 3 Kf6 with an easy win, which reveals the zugzwang position Ke5/Ke7, but Kd4/Kf7 is also a zugzwang position, as will now become clear) 2 d6? (this leads by force to a draw, as however does 2 Kc5? g5; it was not yet too late to return to the initial position) 2 ... Ke6 3 Kc5 Kd7 4 Kd5 g5 5 h × g5 h4, and the pawn queens with check. This means that the white king must avoid d5.

This variation gives the key to the solution: we now know the basic zugzwang positions, i.e. the correspondence of the squares e5/e7 and d4/f7. The third pair of squares comprising the 'main zone' is, of course, e4/f6.

And so: 1 Ke4! Kf7 (1 ... g5 does not work due to 2 h × g5 + K × g5 3 Ke5) 2 Kd4! Ke8 (weaker is 2 ... Ke7 3 Ke5 or 2 ... Kf6 3 Kc5 g5 4 h × g5 +) 3 d6 (Black cannot now reply 3 ... Ke6) 3 ... Kd7.

White faces his last test: 4 Kd5 is of course bad, but to which move should he give preference—4 Kc5 or 4 Ke5? Concrete calculation is required. After 4 ... g5 etc. a queen ending results, in which White, as in example 352, will play Qf7 +. It follows that the black king must be deprived of c6.

4 Kc5! g5 5 h × g5 h4 6 g6 h3 7 g7 h2 8 g8 = Q h1 = Q 9 Qf7 +, and mate in 2 moves.

355. This is position 354 with colours reversed and the defender's king placed outside the critical zone (f3, f2, e2 for White, and e5, d5, e4 for Black). In the previous example the stronger side's king immediately seized the correspondence, but here it is altogether unable to do this.

1 Kg2! (so as to answer 1 ... Ke5 or 1 ... Kd5 with 2 Kf3 or 2 Kf2 respectively) 1 ... Kd6 2 Kf1! d3 (2 ... Kd5 3 Kf2; 2 ... Ke5 3 Ke1!) 3 Kf2 Kd5 4 Ke3 Kc4 5 Kd2 Kd4 6 g4. Draw.

356. It is clear that the c- and f-pawns will be exchanged, and that an ending of pawn against pawn will arise (similar to example 74). White does not win if K × c4 is answered by ... K × f4. Therefore he is unsuccessful with 1 Ke2? Kd6 2 Kf3 Kd5 3 Ke3 Ke6! 4 Kd4 Kf5, or 4 Ke4 Kf6 5 f5 c3!

Following the general rule, the c-pawn should be blockaded: 1 Kd2! Kd6 2 Kc3 Kd5 3 f5 (3 Kb4? Ke4!) 3 ... Ke5 4 K × c4 (4 f6? K × f6—draw) 4 ... K × f5 5 Kd4! etc. (74).
357. 1 Ke3 Kd5 (1 ... b5 2 Ke4 b4 is weaker in view of 3 d4+!, 4 c x b4 and 5 d5) 2
Kd2! b5 3 Ke2 Ke5.

In the event of 3 ... b4 4 c x b4 Kd4 5 Kd2
b6 6 Kc2 Ke7 Kc3 8 Kc2 Kf4 White wins
by 9 Kb2 or 9 Kb3 (14).

4 Kbh3 b6 5 d4+ Kd5 6 Kb4 Kc6 7 Ka3!
7 d5+? K x d5 8 K x b5 Ke4! would have
led to a draw. Having immobilized the enemy
pawns, White transfers his king to the centre,
manoeuvring such that he can meet ... Kd5
with Kb3, and later—Kd3.

7 ... Kd6 8 Kb2 Kd5 9 Kb3 Kd6! 10 Kc2 Ke6
11 Kd2! Kd6 12 Ke3 Kd5 13 Kd3 Kd6 14 Ke4
Ke6 15 d5+ Kd6 16 Kd4 Kd7 17 Ke5 Ke7 18
d6+ Ke8!

On 19 Ke6 there would now follow 19 ... 
Kd8 20 d7 b4 21 c x b4 b5 with a draw. White
has to give his opponent the move.

19 Kd4 Kd8 20 Ke4 Ke8 21 Ke5 Kd8 22 Ke6
Kd8 23 d7+ Kd8 24 Kd6, and wins.

Grigoriev regarded the following position
as a more complete expression of this study.

358. 1 Kf2 Kb5 2 d3 Kc5 3 Ke3 etc. But, as
was shown by Kopyayev (1953), 1 Kf2 is more
strongly met by 1 ... b5!, when both 2 Ke3 b4
3 c x b4 K b5 4 d4 K x b4 5 Ke4 Kb5! 6 Kd5
Ka4 7 Kc5 b5, and 2 d4 Kb6! 3 Kc3 Kc6 4 Ke4
Kd6 5 d5 Kd7 6 Ke5 Ke7 7 d6+ Ke8 (or 7 ... 
Kf7) lead to a draw.

In the second variation, in contrast to the
similar play in example 357, the c6 square
remains inaccessible to White, since Black
has refrained from playing ... b6.

In the following examples the black pawns
are also deprived of their mobility.

Halberstadt, 1930

359. 1 Kf2? does not work: 1 ... Kc6! 2
Kc3 Kb5 3 Kd3 Ka4 4 K x c3 K x a3 5 Kc4
Kb2 with a draw. White has to provoke ... 
c4.

1 Ke2 c4 2 Kf3!

In the event of 2 Ke3? the a-pawn has to be
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prematurely advanced, e.g. 2 ... Ke5 3 a4 Kd5 4 a5 Ke5 5 a6 Kb6 6 Kd4 K × a6. Draw.

2 ... Kd5 3 Kf4! Kd4 4 a4 Kd5 (4 ... Kc5 5 Kc4 Kb4 6 Kd4 K × a4 7 K × c4 etc.) 5 Ke3 Ke5 6 a5 Kd5 7 a6 Kc6 8 Kd4, and White wins.

6.21 Connected pawns against connected.
6.22 Connected pawns against isolated.
6.23 Isolated pawns against isolated (where 'isolated' can also mean doubled pawns).

6.21 Connected pawns against connected

If the pawns are completely or partially blocking one another, the decisive role is normally played by how deeply the king can penetrate into the enemy position, or more precisely—the possibility of seizing control of the critical squares of the more backward pawn by the methods of opposition or correspondence.

In the majority of cases these endings are drawn, especially if only one pair of pawns is blocked and there is the possibility of an exchange. But exceptions occur, most often in cases where the pawns are not completely blocked.

6.2 ENDINGS WITHOUT PASSED PAWNS

If in the ending of two pawns against two there are no passed pawns, it follows that the white and black pawns must stand in pairs opposite one another on the same file or on adjacent files. In the event of the exchange of one pair of pawns, the familiar ending of pawn against pawn arises, while if one of the pawns is won, play reduces to the ending of two pawns against one. But although these simpler endings form the basis of the endings with two pawns against two, they have their own specific features, giving rise at times to complicated king manoeuvring.

The examples given below are grouped according to the nature of the pawn structure:

360. 1 Kf4! Kh7 2 Kg5 Kh8 3 Kg6 Kg8 4 h7 + Kh8 5 Kh6 b6 6 Kg6 b5 7 Kh6 b4 8 Kg5 K × h7 9 Kf6! Kg8 10 Ke7! Kg7 11 Kd6 Kf6 12 Ke5, winning.

361. In this textbook example Black meets 1 Kd5 with 1 ... Kf7! 2 Kd6 Kf6 or 2 Ke5 Ke7. Here the defence of the critical e6 square is simple, but Black loses, of course, if in the position Ke5/Ke7 it is him to move.

362. With this pawn structure (the set-up g3,h4/g6,h5 is even less favourable for White) Black does not lose even if it is him to move. He can play 1 ... h5 immediately with a draw, but the preliminary 1 ... Kf7 2 Kf5 is also
possible, and now either 2 ... h5 3 K × g5 h × g4 4 h × g4 Kg7, or simply 2 ... Kg7 3 Ke6 Kg6.
This last method is the only one, if the position is moved one file to the left.

Teed, 1885

363. White, to play, wins by 1 Kf7! h5 (otherwise 2 Kg6) 2 h4! g × h4 (2 ... h × g4 3 h × g5) 3 g5, since Black loses a tempo after 4 g6+.

364. This is the conclusion of study 523. Mattison has the king at e5. It was moved to e6 by Prokes (1944), and it transpires that it is bad to play 1 Kf7? Kh7 2 g4 g5 3 Kf6 h5, or 1 g4? g5 2 Kf7 (2 Kf6 h5) 2 ... Kh7! (2 ... h5? 3 h4) 3 Kf6 h5.
Correct is 1 Kf6! Kh7 2 g4 g5 (2 ... h5 3 g5) 3 Kf7, and wins (363).

Salvioli, 1887

365. This position is a double exception: White would be unable to save the game, if the pawns were moved down the board, or if they were moved to the left. Nevertheless the example is of interest, as it illustrates a tactical device by which White approaches the g7 pawn with gain of tempo.
1 Kf4 Kh4 2 Ke5! K × h5 3 Kf5 Kh4 4 Ke6
Two Pawns against Two

Kg3 5 Kf7 h5 6 K × g7 h4 7 Kf6 h3 8 g7 h2 9 g8 = Q + Kf2. Draw.

Horwitz & Kling, 1852

366

366. After 1 Kf7 White can win a pawn, but not the game: 1 ... g5! 2 Kf6 g4 (the simplest, but 2 ... Kh5 also does not lose) 3 h4 Kh5 4 Kf5 h6 (375), and if 5 Kf4, then 5 ... Kg6 6 K × g4 h5+, while if 5 Ke4 Kg6 6 Ke5, then 6 ... Kh5! (6 ... h5? 7 Ke6) with a draw.

Grigoriev, 1937

367

367. In the initial position of the study the kings stand at a6 and a2, and the pawn at g2. After 1 g4! the kings approach the pawns, with White all the time maintaining the opposition. The example illustrates the advantage of White’s pawn formation (his pawns are less vulnerable).

1 Ke5! Kf2 2 Kf6! Kf3 3 Kf5 Kg2 4 Kg6 Kh3 5 Kh5, and wins.

There is an unexpected conclusion to the play in the following example.

Halberstadt, 1929

368

368. 1 h5? Kd2 2 h6 Ke2 3 Kd7 Kf2 4 Ke7 K × g2 5 Kf7 Kf3 6 Kg7 g3 7 K × h7 g2 8 Kh8 g1 = Q 9 h7 loses to 9 ... Kg4!

Correct is 1 Kd7 Kd2 (or 1 ... Kd3 2 Ke6 Ke4 3 Kf6 Kf4 4 h5 with a draw, since Black has lost a tempo) 2 Ke6! Ke2 3 Kf5 g3 4 Kg4 Kf2 5 Kh3 h5 — stalemate.

Pachman-Ilivitsky
Prague, 1955

369
369. Black looks to be in danger: his king is a long way from his pawns. But he turns out to have quite adequate defensive resources: 1 ... Kg3 2 Ke3 Kd2!
A precise move! Black is not able to take his king to the support of his pawns, so he sets himself a different task: to attack the enemy pawns from the rear.

If now, for example, 3 Kd4, then 3 ... Kf3 4 Kd5 Ke3 5 Kd6 Kd3 6 Ke7 Kc2 7 Kxb7 (7 a4 b6!, but not 7 ... Kxb2? 8 Kxb7 a5 9 Kb6, winning) 7 ... a5! 8 a4 (8 Kb6 a4) 8 ... Kb3!, and the draw is obvious. Equally little is promised by 3 b4 Kf1 4 Kd2 Kf2 5 a4 b6! 6 Kd3 Ke1 7 a5 bxa5 8 bxa5 Kf2! etc.

3 a4 b6 4 b4 (4 Kd4 Kf3 5 Kc4 Ke3 6 Kb5 Kd3 7 Ka6 Kc4 8 Kxa7 Kb4 with a draw) 4 ... Kf1 5 Ke4 Ke2 6 a5 bxa5 7 bxa5 Kf2! Drawn.

370. Here a by-pass from the rear is decisive: 1 Kf6! Ke8 2 Kg7 Ke7 3 g3! (3 g4? f5) 3 ... Ke6 (3 ... Ke8 4 g4!) 4 Kf8 Kf6 (if 4 ... f6, then 5 Kg7 Kf5 6 Kf7) 5 g4 Ke6 6 g5! and White wins, e.g. 6 ... f5 7 h5 f4 8 h × g6 etc.
The following position has a natural appearance.

371. 1 Ke7! (1 Kd7? c5 2 d5 b5) 1 ... Ka7 (1 ... Ke7 2 e4) 2 Kd7? e5 (2 ... Kb7 3 Kd6 b5 4 Kd7 Kb6 5 Kc8) 3 d5 b4 4 d6 b4 5 Ke8! b3 6 d7 b2 7 d8 = Q b1 = Q 8 Qa5 mate.

372. This position is a valuable contribution to theory.
1 Kd5 Kf8!
The best defence. 1 ... Kh6 loses quickly to 2 Ke5 Kg7 3 Kd6 Kf8 4 Kd7 Kg7 5 Ke8 Kg8 6 Ke7 Kg7 7 f5 g5 8 Ke8. In instead of 3 ... Kf8 Black plays 3 ... Kh8, then 4 Kd7 Kh7 5 Kd8! winning, e.g. 5 ... Kh8 6 f5; 5 ... Kg7 6 Ke8; 5 ... Kg8 (or 5 ... Kh6) 6 Ke7.
Two Pawns against Two

2 Kd6 Ke8 3 f5 g5 4 Ke7! (4 f6? Kd8 5 Kc5 Kd7 6 Kf5 Kd6 7 K x g5 Ke6—draw) 4 . . . Ke7 5 Ke8! Kd6 (5 . . . Ke8 6 f6) 6 Kd8 Ke5 7 Ke7 f6 8 Kf7 Kf4 9 K x f6 K x g4 10 Kg6 etc.

It should be noted that position 372 becomes drawn if it is moved to the right or down the board, but remains won if it is moved one or two files to the left.

Duras, 1927

373. The decisive zugzwang position here is Ke6/Kf4. If it is White to move in this case, he cannot win: 1 e3+ Ke4 2 K x e7 K x f5, whereas if it is Black to move he loses: 1 . . . Ke4 2 e3 K x e3 3 K x e7. The second decisive position is Kd5/Kc3 (in the event of 1 . . . K x e2 or 1 . . . Kf4 White wins by 2 Ke6, but if it is move the position is a draw).

Now the manoeuvring of the kings will be readily understandable: 1 Kb6 Kc3 (with the aim of penetrating via d4 to e5; if 1 . . . Kd2, then 2 Kc6 K x c2 3 Kd7 Kc3 4 K x e7) 2 Ke5 Kd2 3 Kc6! (3 Kd5? Ke3) 3 . . . Ke3 4 Kd5! Kf4 5 Ke6 Ke4 6 e3, and White wins.

374. Here White has to transfer his king to c6 and make the b5-b6 break. The struggle initially revolves around the d5 square (the zugzwang position is Kd4/Ke6). On achieving the position Kd5/Kd7, White will have to use up a pawn tempo, in order to occupy c6. Then in the position K6/Kc8 he will have to

374. Grigoriev, 1933

375. An analogous position occurred in one of the variations of example 366, where
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Black was saved by his king being at the side of the board (the possibility of stalemate). Here he does not have this resource, but as before a draw results from 1 Kd4 Ke6 2 K x e4 f5+. To win, White has to give his opponent the move.

1 Kd4! (other moves are answered by 1 ... Kg4) 1 ... Ke6! 2 Kc5! Kf6 3 Kd6 Kf5 4 Kd5 f6 5 Kd4 Ke6 6 Kc5 Kd7 7 Kd5 (7 f5? Kc7 8 Kd5 Kd7 9 K x e4 Kd6 10 Kd4 Ke6 11 Kc4 Kd6—draw) 7 ... Ke7 8 K x e4, and wins.

Position 375 is the finish to the following study.

Bianchetti, 1925

376. 1 Kg6 Kc5 2 Kf5 Kd5 3 Kf4 c6 4 Ke3 Kc4 5 Ke4 e5 6 Ke3 Kd5 7 Kf4! etc.

Ebersz, 1941

377. 1 Kb5? loses to 1 ... Kd6! 2 Kc4 Ke5 3 Kd3 Kf4 4 Ke2 Kg3 5 Kf1 K x h4 6 Kf2 Kg4 and 7 ... Kg3.

White is saved by a by-passing advance from the rear: 1 K b7! Kd7 (1 ... Kf6 2 Kc6 g5 3 h x g5+ K x g5 does not achieve anything due to 4 Kd6 etc.) 2 K b8! (2 K b6? Kd6 3 Kb7 Ke5, and Black first picks up the g-pawn) 2 ... Ke6 (2 ... Kd6 is no better: 3 Kc8 Ke5 4 Kd7 Kf4 5 Ke6 Kg3 6 Kf6) 3 Kc7 Kf5 4 Kd6 Kg4 5 Ke5 K x h4 6 Kf6(f4) g5 7 Kf5 g4 8 Kf4. Draw.

6.22 Connected pawns against isolated

Other things being equal, connected pawns are stronger than isolated or doubled pawns, but the decisive factor is nevertheless the existence of a positional advantage—the placing of the kings (their proximity to the invasion points), the possession of the opposition, and so on.

378. If it is White to play, the 1 c6 breakthrough is decisive (the a-pawn becomes passed). If Black begins, after 1 ... c6 2 Ke4 Ke6 he controls the critical squares of the c5 pawn.

In the position Kf5/Kf7 the move 1 ... c6 would give only a draw.
379. The players agreed a draw, although Black has an easy win thanks to the poor position of the white king: 1 Kf8 Ke6 2 Ke8 Ke6 3 Kd8 Kd6 4 Ke8 Ke5 5 Kd7 Kb4 6 Kc6 d4 7 Kd5 Ke3 8 b4 K×c2 9 K×d4 Kb3 10 Ke5 Ka4.

If it is Black to play, he wins by 1 ... Kf6, but 1 ... d4! is even simpler (but not 1 ... b4?!—draw).

380. 1 Kb3! Ka5 (1 ... b5 2 Kb4) 2 Ka3 b5 3 a×b5 K×b5 4 d6! Draw (65).

381. 1 Ke2! Ke4 (or 1 ... Kd4 2 Kd2 Kc4 3 c3) 2 f4! e×f4 3 Kf3 Kc3 4 K×f4 K×c2 5 Ke5. Draw.

Szasz-Fuster
Budapest, 1937

382. After 1 ... Kf8? White won by 2 Kg5 etc.

Black should have played 1 ... h4! 2 g4 g3! with a draw: 3 Kf4 f6 4 Kg3 Kf7 5 K×h3 Kg6 6 Kh4 f5 7 g5 f4 8 Kg4 f3.

383. Nothing is achieved by 1 Kg5? Kc7! 2 Kf4 Kd6 3 Ke4 Kd7 4 Kc5 Ke7 or 1 Kg6? Kc6. White must seize the opposition on the
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Ebersz, 1942

383

6th and 7th ranks, so as to carry out a by-pass along the 8th rank.

1 Kb6! Kb7 2 Kh7! Ke6 (2 ... Kb8 3 Kg6 Kc7 4 Kg7; 2 ... Kb6 3 Kg8 Kc6 4 Kf8 Kd6 5 Ke8, and then as in the main variation) 3 Kg6! (3 Kg8? Kd6 4 Kf8 leads to a draw after 4 ... e5 5 Ke8 Ke6) 3 ... Kc7 4 Kg7 Kd6 5 Kf6! Kd7 6 Kf7 Kd6 7 Ke8! e5 8 Kd8 (8 Kf7? e4) 8 ... e4 9 dxe4 Ke5 10 Kd7 Kd4 11 e5 Kxe5 12 Kc6 Kd4 13 Kb5, and wins.

Horwitz & Kling, 1851

385

384. While in the previous example success demanded holding the opposition on two

ranks, here it is necessary to do this on three (5th, 6th and 7th).

White has to approach the d5 pawn, and he wins by 1 Kg7!, since Black cannot reply 1 ... Kc7 or 1 ... Ka7. The continuation might be 1 ... Kc6 (1 ... Kc5 2 Kf7) 2 Kg6 (taking the opposition on one of the main ranks) 2 ... Kc7 3 Kf5 Kb6 4 Kf6 Kc5 5 Ke7 Kc6 6 Ke6 Kc5 7 Kd7, and wins.

Some valuable analyses of endings with connected pawns against doubled pawns were put forward in 1932 by Grigoriev. Up till then only the following two elementary positions were known in literature.

Halberstadt, 1932

384

385. The kings were initially at c3 and h7, and the diagram position was reached after 1 Kd4 Kh6 2 Ke5 Kg6. The black king could not move onto the f-file, due to the loss of the g5 pawn. Since his king remains imprisoned, Black loses in view of White’s reserve tempo.

1 Ke6 Kh7 2 Kf5 Kh6 3 h3 g6+ 4 Kf6 Kh7 5 Kxg5 Kg7 6 h4 etc.

386. White wins by 1 Kf8! h6 2 Kf7. It is easy to see what 1 Kf7? would have led to a draw.

By the examples given below, Grigoriev showed that the presence of doubled pawns does not definitely lead to a loss. To win, the
able: 1 Ke4 Kg5! 2 Ke5 Kh6! 3 Kf6 (or 3 Ke6) 3... Kh5 4 Kg7 Kg5. Draw.

Bondarevsky, 1952

388. Here Black is not in zugzwang (even with the move White is not able to win), but the defence requires accuracy. For example, 1... Kh7 loses to 2 g5, as does 1... g5, since White has a reserve tempo (385). Depriving White of this tempo—that is Black’s immediate task!

Therefore: 1... Kg5! 2 f3 Kh6! 3 f4 g5 4 f5 Kh7 5 Ke7 Kg8 6 Ke8 Kh8 7 Kf8 Kh7 8 Kf7 Kh6! 9 Kg8 g6 10 f6—stalemate.

387. If it is Black to move, he loses after 1... Kh5 2 Kf4, or 1... g5 2 Kf6. But if it is White to move, Black’s position is impreg-
389. White has a big advantage, but even so he cannot win.

1 Kf7 Kh6 2 Kg8 (if 2 g3, then 2... Kh7!, while regarding 2.g4, cf. position 388) 2... g5
3 g4 (otherwise Black plays... g4) 3... Kg6 4 Kf8 Kf6! 5 f3 Kg6 (387) or 5 Ke8 Ke5. Draw.

Grigoriev, 1932

390. The straightforward 1 Kd6(d5) does not succeed here. Black replies 1... Kg7, and
then plans his defence in accordance with White's moves. For example: 2 g4 Kf7! (with
the king at d5—2... Kf6!) 3 Kd7 Kf6 4 f3
Kf7; 2 Ke6 g4; 2 Ke5 Kf7. Finally, if White begins with 1 Kc6, there follows 1... g4!,
creating the drawn position 387.

The correct move is 1 g4!, retaining the possibility of choosing a move, depending on
where the black king goes to. If, for example,
1... Kg8, then 2 Kd5! Kf7 3 Kd6 Kf6 4 f3
Kf7 5 Kd7 Kf6 6 Ke8!, winning.

However, 1... Kg7 is more tenacious, when the only way to win is by 2 Kc6! Kf6 3
Kd5! Ke7 4 Ke5 Kf7 5 Kd6 Kf6 6 f3, when
White wins as already examined.

The following position by Grigoriev is an instructive example—the pawn formation is
the same, but the placing of the kings is different.

391. Here 1 g4? would be an irreparable mistake, since the black king cannot be put in
zugzwang. For example: 1... Ke7 (the sim-
plest: after g3–g4 Black must try to take the
vertical opposition) 2 Kd5 Kd7; 2 Kd4 Kd6;
2 Kc5 Ke6.

Correct is 1 Kd4! (not allowing... g4, and
keeping g3–g4 in reserve) 1... Ke7 2 Kd5!
Kf7 3 Ke5! or 2... Kf6 3 g4!, winning.

Regarding position 390, Grigoriev also
showed that, if the g3 pawn is replaced at g2,
any move onto the d-file wins for White,
thanks to his extra pawn tempo. E.g. 1 Kd5
Kg7 2 Ke5! Kf7 2... Kh6 3 g4) 3 g3! Ke7 (or
3... Kg7) 4 g4! etc.

The following example unites a number of
motifs from the preceding positions, and its
solution is now much easier for us.
392. Here, in comparison with example 385, the black king has greater freedom of movement, but White paralyzes it using the horizontal opposition.

1 Kh5! Kg6 (if 1 ... g6, then 2 Kc6 Kg7 3 f3 or 3 Kc5) 2 Kc6! Kf7 3 Kd7 Kf6 4 Kd6 Kf7 5 Ke5! g6 6 Kd6, or 5 ... Kg6 6 Kc6 (385).

The reader’s attention is also drawn to examples 226 and 227 in section 5.22.

6.23 Isolated pawns against isolated

At the basis of the internal systematization of this group of endings is the concept of ‘closely’ and ‘widely’ separated pawns. Certain features, characterizing the closer and the wider pawn formations, become quite clearly apparent, and the more so, the greater the distance between the pawns (in those cases, for example, where positions with an interval of one file between the pawns are compared with positions where the interval reaches 5–6 files). But when the distance between the pawns is 2–4 files, the strategic and tactical content of the positions is so rich and diverse, that they no longer lend themselves to generalization. Therefore the examples are grouped, as far as possible, on the basis of the ideas connecting them, rather than in order of increasing number of files between the pawns—in the given case such a basis would be formal and untypical.

393. The result in endings of this type is determined, as usual, by the superior king position, the existence of reserve pawn tempi, and, as a result, the possibility of seizing the critical squares (the invasion points).

At first both sides bring their kings into play: 1 Kd2 Ke6 2 Ke3 Kf5 3 Kf3 e4+.

Forced, otherwise comes 4 Ke4, and White’s reserve tempo (e2−e3) decides the game in his favour.

4 Kg3 e3! (the most tenacious; 4 ... Kg5 loses to 5 e3 Kf5 6 Kh4 etc.) 5 Kf3 Kf6!

The charming point of the study! It transpires that after 6 K × e3 Ke5! 7 Kd3 Kf5! 8 e3 (8 Kc3 Ke4 9 Kd2 Ke3 or 9 Kb3 Kd4 with a draw) 8 ... Ke5 9 Kd2 Ke4 10 Ke2 Kf5! 11 Kf3 Ke5 12 e4 Kd4 13 Kf4 K × c4 14 e5 Kb3 15 e6 c4 16 e7 c3 17 e8 = Q c2 Black attains a draw.

6 Kf4! Ke6 7 Ke4 Kd6 8 Kf5, winning the c-pawn.

Horwitz & Kling, 1851

394. 1 Ke4 d5 + 2 Kf4! (2 Kd4? Kd6 3 f3 Kc6—draw) 2 ... Kd6 3 Kf5 Kc5 (3 ... Ke7 4 d4 Kf7 5 f4) 4 K × f6 Kd4 5 Ke6 K × d3 6 K × d5 and 7 f4, or 1 ... f5 + 2 Kd4 d5 3 f3 Kd6 4 f4 Ke6 5 Kc5, and wins.

With the pawn at f3 it is a draw: 1 Ke4 d5 + 2 Kf4 f5 3 Kg5 Ke5 4 f4 + Kd4.
Endings without Passed Pawns

Salvioli, 1887

395. 1 Ke3!

This gives White better chances of seizing the critical squares of the g5 pawn than does 1
Kf3. It is not a question here of the oppo-
sition.

1 ... Kf6 2 Kd4 Ke6 3 Ke4 Kf6 4 Kd5 e6 +
5 Kd6 Kf7 6 e4, and wins.

Averbakh, 1983

396. A classic example of two invasion
squares (e6 and b7). The basic correspon-
dence squares are clear: a6/b8, b5/c8, c5/d8
and d5/e7. But due to lack of space, Black has
no square corresponding to a5.

Therefore: 1 Ke5 Kd8 2 Kb5 Ke8 3 Ka5!
Kb8 (3 ... Kd8 4 Ka6 Kc8 5 Kc7 or 4 ... Ke7
5 Ka7) 4 Ka6 Ka8 (now the opponent’s king
is a long way from e6, and White heads for
there) 5 Kc5 Kc7 6 Kc5 Ka6 7 Kd5 Kb6 8 Ke6
K × e6 9 K × f6, and White wins.

Leick, 1942

397. White fails to win by the straightforward
1 Ke5 Ke8 2 K × f5 Kf7! 3 Kg5 Kg7! 4
f5 Kf7 (238).

Correct is 1 Kd4!, putting Black in zugz-
wang. If now 1 ... Ke8, then 2 Ke5 Kf7 3 Kb6
Kf6 4 Kb7! Kf7 5 Ke8 Ke6 6 Ke7, while on 1
... Ke8 White has 2 Ke5 Kd8 3 K × f5.

Zinar, 1982

139
We know from section 5.22, 1 \( K \times h6 \) is pointless, since the pawn is inside the drawing zone.

Correct is 1...h5! Kg8 2 Kf6! Kf8.3 Ke6! Kg7 4 Ke7! Kg8 5 Kd6 Kf7 6 Ke5 Kf6 7 Kb4 Kg5 8 K \times a3 K \times h5 9 Kb4 Kg5 10 a4, and White wins.

Relying on Bähr's rule (209), Black easily gains a draw in the following position.

\[ \text{Bähr, 1936} \]

399. 1 Ke3.

After this move, winning the battle for the critical squares, Black loses a pawn, and he has to decide which one to give up.

1...Kf7! 2 Kd4 Kf6! 3 Ke5 Ke6 4 Kb6 Kd6 5 K \times a6 Kc7 (5...Kc6 6 Ka7 Kc7 7 a6 Ke8 8 Kb6 Kb8 is also possible) 6 Kb5 Kb7, and Black gains a draw: after capturing the a-pawn, his king reaches f8 in time.

In the following examples, with bishop's and rook's pawns, one of the sides inevitably loses material. His task is to obtain a theoretical ending (225), in which the extra pawn on the bishop's file does not bring success.

In position 400 this proves possible.

400. Here White's superior king position proves insufficient for a win: 1...Ke6 2 Ke5 Kd7 3 Kd5 Kc8 4 Ke6 Kb8 5 Kd7 (5 a6 Kc8 6 c5 Kb8 7 Kd7 Ka8) 5...c5! 6 Ke6 Kc8 7

\[ \text{Drimer-Södeborg} \]
\[ \text{Reykjavik, 1957} \]

K \times c5 Kc7 8 Kd5 Kd7, and White cannot win, since his pawn is already at a5, and not a4 (230).

It is the opposite picture in Hooper's study. Here, by subtle king manoeuvres, White succeeds in obtaining a favourable situation.

\[ \text{Hooper, 1961} \]

401. The immediate attack on the pawn is unsuccessful: 1 Kd5 (1 Kd4 Kg8) 1...Kg7! 2 Ke5 Kf7! 3 K \times f5 Ke7! 4 Ke5 Kf7 5 h5 Ke7, with a draw.

It will be readily apparent that here there is
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a system of corresponding squares: e5/f7, e6/g6(e8), f6/f8 and d5/g7. With his greater freedom of manoeuvre, White wins the battle for the correspondence.

1 Ke5! Kg8 2 Ke6! (2 Kd6 Kf8! 3 Kd7 Ke7 would be pointless) 2 ... Kg7 (2 ... Kf7 3 Kd7) 3 Kd5 Kf8 (3 ... Kh6 4 Ke5 Kg6 5 Ke6, winning) 4 Kd6! (4 Ke6 Ke8! 5 K × f5 Ke7!) 4 ... Kg7 (4 ... Ke8 5 Ke6 Kf8 6 Kf6 Kg8 7 K × f5, and then as in the main variation) 5 Ke7?

The decisive move. Nothing is achieved by 5 Ke5 Kf7 6 K × f5 Ke7?!
5 ... Kg8 (5 ... Kg6 6 Ke6 Kh5 7 K × f5 K × h4 8 Ke6 h5 9 f5, winning) 6 Ke6 Kf8 (6 ... Kg7 7 K × f5 Kf7 8 h5 Ke7 9 Ke5 Kf7 10 Kd6) 7 Kf6! Kg8 8 K × f5 Kf7 9 h5 Ke7 10 Ke5 Kf7 11 Kd6!, and White wins (230).

The following study shows an interesting possibility of saving the draw in an ending with pawn against queen.

Antipov, 1967

402. After 1 Kb3? c5 2 c4 Kd3 K × a3 Ke3! 4 Ka4 K × c4 White loses due to the poor position of his king.

The way to draw is by 1 Kb4! c5 + (or 1 ... Kc2 2 K × a3 K × c3 3 Ka4 c5 4 Kb5 c4 5 a4 with a draw) 2 Kb3 Kd3 3 c4 Ke2.
3 ... Kd4 4 Kc2 K × c4 5 Kd2 Kd4 6 Kc2 Ke3 7 Kc3 Ke2 8 Kc2 Ke1 9 Kc1 leads to a well-known draw. But now White finds an unexpected resource.

4 Ka4! Kc2 (4 ... Ke3 5 K × a3!) 5 Kb5 Kd2 6 K × c5 K × a2 7 Kd6! Kh3 8 e5 a2 9 c6 a1 = Q 10 c7 Qa8 11 Kd7. Draw.

Grigoriev, 1929

403. White must inevitably lose one of his pawns, and can draw only by capturing a black pawn in return. If Black heads for the h6 pawn, White has time to take on e6; if instead he makes for the e5 pawn, White has to be able to meet ... K × e5 with Kg5, obtaining access to the h7 pawn.

Thus the basic zugzwang position is Kg5/Kc5. As the kings manoeuvre, particular importance is acquired by the squares adjacent to e5 and g5, namely d5, d4 and e4 for Black, and h5, h4 and g4 for White. With the black king at d5 or d4, White's can occupy either h5 or h4, but on ... Ke4 he must reply Kg4 (in other words, only the squares e5/g5 and e4/g4 strictly correspond).

Why this is so, is shown by the following incorrect defensive try: 1 Ke2? Kc6! 2 Kf3 Kd5 (White has not managed to reach h4, and ends up in zugzwang) 3 Kf4 Kd4 4 Kg4 Ke4! 5 Kh4 Kf4 6 Kh5 Kf5 7 Kh4 Kg6! (7 ... K × e5? 3 Kg5—zugzwang). By means of a by-pass Black won the 'other' pawn.
Two Pawns against Two

It will now be clear that White's task is to reach the h-file by the shortest path (along the diagonal): 1 Kf2! Ke6 (after 1 ... Ke7 White uses the other diagonal: 2 Ke3 Kf7 3 Kd4 Kg6 4 Ke5, and if 4 ... K×h6 5 Kd6, and if 4 ... Kg5 5 Kd6 Kf5 6 Ke7) 2 Kg3 Kd5 3 Kh4! (at just the right time) 3 ... Kd4 4 Kh5! Ke4 5 Kg4! K×e5 6 Kg5. Draw.

As often happens, this splendid study stimulated some imitations.

Isenegger, 1940

404. The idea here (following Grigoriev) was to win the b3 pawn by a by-pass via the K-side (the direct attack on it does not work due to the counter-attack on the e3 pawn). After 1 Kg2 Kf6? 2 Kh3! etc. White does indeed succeed in winning by seizing and maintaining the opposition, but in the same year Leick showed that by 1 ... Kd6! and 2 ... Ke5! etc. (Grigoriev's manoeuvre) Black could attain a draw.

A creative development of Grigoriev's idea has also been seen.

405. 1 Kg2! Ke6 (1 ... Ke5 2 Kf3 Kf5 3 e4+ d×e4+ 4 Kg3—546) 2 e4! d×e4 3 Kh3! Kf5 4 Kg3 Kg5 5 e3 Kf5 6 Kh4 Ke5 7 Kg4 Kd5 8 Kf5 Ke5! 9 Ke5! (9 K×e4? Kc4) 9 ... Kb5 10 Kd5 Kb6 11 K×e4, and wins.

Zhigis, 1929

406. 1 Kh7! Kd7 (nothing is changed by 1 ... Ke7 2 Kg7 Ke6 3 Kg6 etc.) 2 Kh6 Kd6 3 Kh5 Ke5 (4 K×h4 was threatened) 4 Kg5! Ke4 5 Kg4 Ke5 6 Kf3 Kf5 7 Ke3 Ke5 8 K×d3 (White has carried out the idea of capturing the 'other' pawn) 8 ... Kf4 9 Ke2 Kg3 10 d4 K×h3 11 d5 Kg4 12 d6 h3 13 Kf2 etc.

407. A rather complex position. It is clearly unfavourable for Black to advance his pawn, since this quickly leads to Grigoriev's position (403). That only leaves manoeuvring with the kings. It is not difficult to establish the correspondence of the squares f4/f6, e3/e5 and hence also f3/e6. The establishment of
this main zone provides the necessary guide.

1 Kg3! (waiting for the black king to step into the main zone) 1 ... Ke6 2 Kf3! Ke5 3
Ke3 Kd6 (3 ... Kf6 4 Kf4 or 4 Kd4) 4 Kf4! (4
Kd4? e5+) 4 ... e6 (4 ... Kc6 5 Ke5) 5 Kg4!
(5 Kf3? Kc5 6 Kf4 Kd4) 5 ... Ke5 (5 ... Ke5
6 Kf3 Kd4 7 Kf4) 6 Kg5! Draw.

1 Kf3? loses after 1 ... Ke6! 2 Kf4 Kf6 3
e5+(3 Kf3 Kg5!) 3 ... Kg6 4 Kg4 e6 5 Kf4
Kh5 6 Kf3 Kg5 7 Ke4 Kg4 8 Ke3 Kf5 9 Kd4
Kf4 10 Kc4 Ke4 11 Kb4 Kd4 12 Kb5 Kd5 13
Kb4 Kc6 14 Ka5 Kc5 and 15 ... K ... b6.

409. White should avoid the hasty advance of his b-pawn: 1 Kc2! Kb4 2 Kc1!
But not 2 Kd2? Kb3 3 Kc1 Ka2 4 Kc2 b4
etc. But now on 2 ... Kb3 there follows 3
Kb1 b4 4 Kc1, when 4 ... Ka2? is bad for
Black in view of 5 Kc2 Ka1 6 Kb3, winning.

2 ... Ke5 3 Kd1 (retaining the possibility of
taking the opposition to defend the critical
squares d3 and e3) 3 ... Kd5 4 Kc1! (the e4
square is inaccessible to Black) 4 ... Kd4 5
Kd2 Kc4 6 Kc2 Kb4 7 Kc1. Draw.
410. In spite of certain squares being inaccessible to Black's king, which should lead to him losing the opposition, he is saved by tactical threats.

1 Kg7 (if 1 Kf7, then 1 ... Kd6 or 1 ... Kf7, while if 1 Kh6, then only 1 ... Kd6, when 2 Kh7 can be met by 2 ... Kxc6) 1 ... Ke7 2 Kg8 Ke8 3 Kh7 (3 Kh8 Kf8!, 3 ... Kf7! (a position of mutual zugzwang) 4 Kh6 f5! with a draw: 5 g5! Kg8! 6 Kh5 Kg7 7 g6 f4.

If it is Black to move, he loses: 1 ... Ke7 2 Kg7 Ke6 3 Kf8 etc.

Bähr, 1936

411. The composer erroneously thought that 1 Kh6 would win, since the reply 1 ... Kd6 is not possible. For example: 1 ... Kh7 Ke8 3 Kg6 etc. But instead of 2 ... Ke8?, Black can draw by 2 ... Ke6!, while 1 Kh6 can also be met by 1 ... Ke5 (2 Kg7 f5).

Black to move draws by 1 ... Ke5 2 Kf7 f5.

Such instances, where a conspicuous main idea hides other motifs, are a typical cause of many mistakes.

412. The idea of this position is simple: the a7 pawn is doomed, on Kxb7 there follows ... Ke7, and then comes f4-f5, after which the white king escapes to freedom.

413. Of interest here is the winning of a tempo, with the aim of hindering the approach of the black king: 1 Ke6 (of course, not 1 Ke6? Kg5) 1 ... Ke5.

Since 1 ... Kxf5 fails to save the game, Black aims to stalemate the enemy king in the corner. A position of zugzwang has been reached: 2 Kb7 does not win, but Black to move would lose. Therefore, to give his opponent the move, White carries out a triangulation manoeuvre.

2 Ke7! (2 Kd7? Kxf5) 2 ... Kd5 3 Kd7! Ke5 4 Ke6!, and wins.
Other ideas are contained in the following position.

Moravec, 1941

414. 1 Kf4 Kg8 2 Ke5 Kf8 3 Kd6! Ke8 4 Ke6 Kf8 5 Kd7 Kg8 6 Ke7 Kh8 (only after this can the a3 pawn be picked up) 7 Kd6 Kg8 8 Kc5 Kf8 9 Kb4 etc.

Adamson, 1925

415. The natural 1 K×e7 K×b2 2 Ke6 Kc3 3 Kd5 Kd3 4 h4 c5! does not succeed.
Correct is 1 Ke6! c5 2 Kd5 c4 3 h4 e6+ 4 Ke5! e5 5 h5 e4 6 Kd4 e3 7 K×e3 K×b2 8 h6, and wins.

416. The manoeuvring of the kings, which at first sight here is complicated, has been insufficiently well explained in literature, sometimes even by the method of corresponding squares, whereas the solution becomes extremely simple if the method of opposition is correctly applied.

1 Ke4 Kg4.

On 2 Ke5 there now follows 2 ... Kg5, but not 2 ... Kg3? 3 Kf5 etc. 2 Kd4 is also pointless in view of 2 ... Kh4!, but again not 2 ... Kg3? 3 Ke5 Kg4 4 Kf6, winning.

It is clear that the key squares here are f4, f5 and f6, and that the 5th rank is the main one.

2 Kd5 Kh5!

But not 2 ... Kf5?, since after 3 Kd4 Black is prevented from retaining the opposition by his own pawn.

Now White's problem is to gain the opposition on the main rank. 3 Kc5 is insufficient to do this, in view of 3 ... Kg5, when on 4 K×b5 Black replies with the capture on g2 (4 ... Kg4 5 Kc5 Kg3 6 b5 K×g2—draw).

As we have already seen, White also achieves nothing by having his king on the 4th rank.

3 Kc6!

It transpires that Black cannot make the necessary reply 3 ... Kg6, because of 4
K × b5! Kg5 5 Kc5 Kg4 6 Kd4! Kg3 7 Ke4 etc. Black's king is forced to remain on the 5th rank, and hence he loses the opposition.

3 ... Kg5 (if 3 ... Kg4 or 3 ... Kh4, White immediately begins the by-pass: 4 Kd6 etc.) 4 Ke5!

The rest is a matter of technique: 4 ... Kg4 5 Kd6 Kh5 6 Kd5 Kh4 7 Ke6! (7 Kd4 Kh5 8 Kd5 would be a loss of time) 7 ... Kg5 8 Ke5 Kg4 9 Kf6, and wins.

Dedrle, 1926

Grigoriev, 1933

417. 1 Ke6! (1 K × b6? Kg4; 1 Kd6? Kh6!; 1 Kd7? Kh5!—rectangular correspondence—2 Kc6 Kg5!, with a draw since the c5 square is inaccessible) 1 ... Kg4 (1 ... Kh5 2 Kd5!) 2 Kd6! etc.

We will now consider endings with pawns on adjacent files.

418. 1 Ka6 (1 b6 Kb7 is premature) 1 ... Kb8.

If 1 ... f4, then 2 b6! c6 3 b7 + Kb8 4 Kb6 etc. But now 2 b6? is again impossible due to 2 ... Kc8! with a draw.

2 g3! Ka8.

On 2 ... Kc8 there would have followed 3 Ka7 Kd8 4 Kb8! Kd7 5 Kb7 Kd8 (5 ... Kd6? 6 Kc8) 6 Kc6 Kc8 7 Kd5 Kd8 8 Ke5 Kd6: 9 K × f5 K × b5 10 g4 c5 (had the pawn not been fixed in its time at f5, and had the white king stood now at f4, Black would have

achieved a draw by 10 ... Kc6) 11 g5 c4 12 Ke4! and wins (the black king is lured into a check at b3).

3 b6 Kb8! 4 Kb5! (4 b7? c5! 5 Kb5 K × b7 6 K × c5 f4 7 g × f4 Kc7 — draw) 4 ... Kb7 5 b × c7 K × c7 6 Ke5, and White wins (the trappy 6 ... Kd8 is correctly met, of course, by 7 Kd6!, but not 7 Kd5? or 7 Kd4? because of 7 ... f4!—66).

Grigoriev, 1931

419. 1 Kg3 (1 d4? Kf2) 1 ... c5 (1 ... Ke2 leads to a drawn queen ending after 2 d4!! Ke3 3 Kg4 etc.) 2 Kf3! Kf1! (retaining the possibility of attacking either of the pawns.
but White has a reply ready) 3 Ke4! (if now 3 ... Kg2, then 4 Kf5, or 3 ... Ke2 4 Kd5) 3 ... Kf7! (a last attempt) 4 Ke5! Kf3 5 Ke6 Kf4 6 Ke7! (6 Kf6? Ke3) 6 ... Ke5 (6 ... Kf5 7 Kd6) 7 Kd7! Kf4 (7 ... Kd5 8 Ke7) 8 Ke6! Kf3 9 Ke5 Kf2 10 Ke4 Kf1 11 Kf3 Ke1 (11 ... Kgl 12 Kg3 Kh1 13 Kg4) 12 Ke3 Kd1 13 Ke4 Ke2 14 Kd5 K × d3 15 K × c5 Ke4 16 Kd6. Draw.

420. White must aim to exchange the K-side pawns, but only by a by-pass to the f5 pawn from the rear (allowing the black king to take the g-pawn).

1 Ke3 Kg6 2 Kf4 Kf6 3 b5! Kg6.

After 3 ... Ke6 4 Kg5 Ke5 5 g3! (5 ... f4 must not be allowed, as is evident from the note to Black’s 2nd move in example 418: the black pawn has to be fixed at f5) 5 ... Ke6 6 Kg6 Ke5 7 Kf7 White wins.

4 Ke5 Kg5 5 g3! Kg6 6 Ke6 Kg5 7 Kf7 Kh5!

8 Kf6 Kg4!

A zugzwang position has been reached: 9 Kg6 leads to a draw after 9 ... f4. Black has to be given the move.

9 Ke5! Kg5 10 Ke6, and White wins, since on 10 ... Kg4 there follows 11 Kf6, while if 10 ... Kg6, then 11 Kd7.

421. 1 Ke4! (now 1 ... Kg3 is met by 2 Kb5 with a draw) 1 ... Ke4 2 Kb4! Kd4 3 Kb5 Ke3 4 Ka5! Ke4 5 Kb6 Kb4 (in contrast to the analogous position in example 420, Black cannot gain a tempo, since the attack on the g-pawn requires too much time) 6 c6! b × c6 7 K × c6. Draw.

422. 1 Ke2! Kf5 2 Kd3! Kf4 3 Kd4! Kf5 (3 ... Kf3 4 Ke5) 4 Kd5 Kf4 5 Ke6 Kg5 6 Ke5 Kg6 7 Kf4 Kh5 8 Kf5 Kh4 9 Kg6 Kh3 10 Kg5, and wins.
in reserve), when 7... Kh7 is not possible, since after 8 Ke6 etc. the b-pawn queens with check. As a result Black loses the opposition, e.g. 7... Kh6 (7... Kf6 8 Kd6; 7... Kg5 8 Ke7) 8 Kd6 Kh5 9 Ke7 (a by-pass) 9... Kg6 10 Ke6 Kg5 11 Kf7 Kh5 12 Kf6 Kg4 13 b3, and wins.

Grigoriev, 1938

425. The basic winning idea is the same as in example 412. But there the problem was simple, whereas here, with an interval of 2 squares between the b-pawns, it becomes complicated. The point is that, after K×h7 Kf7, White cannot release his king by b4–b5 due to ... b6. It follows that he must achieve an interval of only one square between the pawns (b5/b7 or b4/b6), when he will calmly be able to pick up the h-pawn.

The solution to the study was never explained using the theory of corresponding squares, and therefore it seemed more complicated than it in fact is.

Let us find out which squares correspond to one another.

The first zugzwang position is Kb6/Kb8. If it is Black to move, he loses: 1... Ka8 2 Kc7 Ka7 3 b5 Ka8 4 Kd7 etc.

The second zugzwang position is Kc5/Kc7. For example: 1... Kd7(d8) 2 Kb6 Kc8 3 Ka7; 1... Kb8 2 Kb6; 1... Kc8 2 Kd6 Kd8(b8) 3 b5; 1... b6+ 2 Kd5 Kd7 3 b5.
When the black pawn goes to b6, White seizes control of its critical squares.

It should be noted, incidentally, that Kd5/Kd7 is also a decisive zugzwang position (Kc7 2 Ke5; 1 ... Ke7 2 Ke5 etc.), as is Kc5/Kc7, since on 1 ... Kd7 there follows 2 Kd5, while if 1 ... Kc6, then 2 Ke6 b6 (2 ... b5 3 Ke5; 2 ... Kc7 3 b5 Kd8 4 Kf7 b6 5 Ke6) 3 Kf6. Thus up till now all the zugzwang positions have coincided with the normal opposition.

The correspondence of the squares b6/b8 and c5/c7 also reveals the correspondence b5/c8. This completes the determination of the main zone: for White—b6, b5 and c5 (c6 is inaccessible), and for Black—b8, c8 and c7 (b7 is inaccessible). These triangles are easy to remember.

Let us now examine the nearest rear squares. The white king at c4 attacks the squares b5 and c5 of the main zone, and therefore Black’s king must be at d8, so as to hold the c8 and c7 squares in his main zone. Hence the correspondence c4/d8. And what if the white king is at d4 (attacking c4 and c5)? Then the black king must stand at c8 (defending d8 and c7).

Having revealed all the necessary correlations, we can embark on the solution, which is now fairly simple. First, however, two general remarks should be made: (1) the most natural path for White is to take his king to the centre as quickly as possible, so as to penetrate as deeply as possible into the opponent’s position; (2) White should aim for there to be two files between the kings, since then Black will be unable to answer K × h7 with ... Kf7.

1 Kd3! Kb8.

1 ... Ka7 is out of keeping with the character of the defence. After 2 Ke4! Kb6 3 Kf5! it is true that by 3 ... Kc6 or 3 ... Kc7 Black approaches the f7 square in time (since 4 Kf6 leaves the white king on the same file), but White continues 4 Ke6 or 4 Ke5 respectively, and wins as shown above.

2 Ke4!

Forcing 2 ... Kc7 or 2 ... Kc8, after which White seizes the correspondence. On 2 Kc4? there would have followed 2 ... Ka7!, with a counter-attack.

2 ... Kc8 (or 2 ... Kc7 3 Kc5 and wins; now White begins approaching the main zone) 3 Kd4 Kd8 (3 ... Kd7 4 Kd5; 3 ... b6 4 Ke5, and if 4 ... Kd7, then 5 Kf6 b5 6 Ke5, or 4 ... Kc7 5 b5) 4 Ke4 Ke8 5 Kb5 Ke7 6 Ke5, and wins.

In endings of this type an important role is played by the positioning of the pawns on the ranks (how far up the board they are), and also the distance between the pawns (in terms of files).

426.

426. Here the player to win is the one who seizes the opposition, since the distant opposition can always be transformed into close opposition.

1 Ke2! Kf8 2 Kd3! (a by-pass) 2 ... Kc7 3 Ke3! Ke6 4 Ke4 Kd6 5 Kd4! (5 Kf5? Kd5, and the pawns queen simultaneously) 5 ... Ke6 (5 ... Ke6 6 Kc5) 6 Ke5, and wins.

The attempt after 6 ... Kc7 7 Kd5 Kd7 8 Kc5 Kc7 9 K × b5 Kb7 to reduce play to example 209 (a passed pawn with blocked rook’s pawns) does not succeed, since Black cannot attain the ‘normal’ position. After 10 Kc5 (of course, not 10 Kc4?? Kb6 or 10 ... Ke6 he is forced to play 10 ... Kc7 (10 ... Ka6 11 Kc6), when after 11 Kd5 Kb6 the black king can no longer reach f8 in time.
Two Pawns against Two

If it is Black to move, he wins by 1 ... Kf7. The situation is radically changed if the b-pawns are moved onto the a-file.

427. After 1 Ke2 White still wins the pawn, but not the game, e.g. 1 ... Ke7 2 Ke3 Kd6 3 Kd4 (3 Kf4 Kd5! 4 Kg5 Ke5 etc.) 3 ... Ke6 4 Kc5 Ke5! 5 Kb5 Kd5 6 K × a5 Kg5 7 Ka6 Ke6 8 a5 Kg7 9 Kb5 Kh7 with a draw, since the 'normal' position has been attained. This factor neutralizes the possession of the opposition (whether distant or close).

If the h-pawns are moved onto the g-file, an unusual situation arises, where the possession of the distant opposition does not play a part, and only the close opposition is of importance.

428. In reply to 1 Ke2 Ke7 (1 ... Kd7 is also possible) 2 Ke3 Black plays 2 ... Kd7, forcing the enemy king up to the front line: 3 Ke4 Ke6 or 3 Kd4 Kd6 with a draw, since White has no possibility of a by-pass.

We will now consider positions with pawns on other ranks.

Grigoriev. 1927

429. White's pawns have crossed the middle of the board, and this ensures a win, in spite of the black king's good position. After 1 Ke3! (1 Kd3? Kd5!) Black is not saved either by 1 ... Kf5 2 Kc4, or by 1 ... Kd5 2 Kd3! Ke5 3 Ke4 etc., since White queens first. The attempt to answer K × a6 with ... Kc6 is also hopeless, since the h5 pawn has crossed the middle of the board (209).

With Black to move it is a draw: White answers 1 ... Kf5 with a counter-attack on the a6 pawn, while if 1 ... Kd4, winning the a5 pawn, the general rule (209) shows that Black is unable to win.

If the pawns stand further down the board or further up, with White to move too the result is a draw.

430. 1 Kc3 Kf4 2 Kd4 Kg4 3 Ke4 K × h4 4 Kf4. Draw (427).
431. After 1. Ke3 Kf5 2. Kd4 Kg5 3. Ke5 Black is saved by 3...Kf6! etc. (K×a7 is met by ...Kc7, and the white king cannot escape from the corner).


433. This was the first study by Grigoriev to be published. It is by no means obvious that play here reduces to an ending with a passed pawn and blocked pawns (209).


Positions close to those considered, but nevertheless distinctive, are those in which
each of the kings is in the enemy rear. Taking the opposition on the main files proves decisive.


Grigorlev, 1932

434. 1 Ke7! Ke2 (1 . . . Kd2 2 Kd6! and 3 Kc5; 1 . . . Kf2 2 Kf6! and 3 Kg5) 2 Ke6! (2 Kd6? Kf3; 2 Kf6? Kd3) 2 . . . Ke3 3 Ke5 Ke2 (3 . . . Kd3 4 Kd5!) 4 Ke4! Ke1 5 Ke3! and wins, since 5 . . . Kd1 is met by 6 Kd4!, and 5 . . . Kf1 by 6 Kf4!

Bähr, 1935

436. White fails to save the game by 1 Ka3? Kc3 2 a5 Kc4 3 Ka4 Kc5 4 a6 Kb6 5 Kb4 K × a6 6 Kc5 Kb7 7 Kb5, since the a7 pawn is inside the winning zone (209): 7 . . . Kc7 8 Kc6 Kb6 etc.

Correct is 1 Kc2! Kb4 2 Kd3 K × a4 3 Kc4! Kc5 (3 . . . Ka3 4 Kb5) 4 Kc5 a6.

There is no other way for the black king to free itself, but now the pawn is no longer inside the winning zone, and to draw White only needs to take the 'normal' opposition.

5 Kc4 Kb6 6 Kb4. Draw.

If after 1 Kc2 Kb4 2 Kd3 K × a4 3 Kc4 Black plays not 3 . . . Ka5, but 3 . . . a6 immediately, to attain the 'normal' position White must continue 4 Kc5! (but not 4 Kc3?, Kb5, winning).

437. For which pawn should the white king make? If for the h5 pawn, Black has time to reach the a2 pawn, while the following is a loss of time: 1 h4 Kb7 2 Ke4 Kc6 3 Kd4 Kd6 4 Kc4 Ke5 5 Kb3 Kd4 6 K × a3 Kc3, and we reach Bähr's drawn position (209). No better is 1 h3 Kb7 2 Ke4 Kc6 3 Kd4 Kd6 4 Kc4 Ke5 5 Kb3 Kf4!
Endings without Passed Pawns

Zinar, 1979
from Prokop, 1924

2 Kb3 Ka6 3 Kb4 Ka7 4 K × b5 Kb7 5 a4 Kc7
6 Ka6 Kc6, with a draw (??). although 7 c4
wins easily.

The correct solution was pointed out by
Salvioli: 1 c3 b4! 2 c4 b3!! (to answer 3
a × b3 + with 3 ... Kb4 and 4 ... b5) 3 a3
Ka5 4 K × b3 Ka6 5 Kb4 Ka7! 6 Kb5 Kd7 7 a4
Kc7! (7 ... Ka7? 8 a5) 8 Ka6 Kc6. Draw (250).

The position obtained by moving example
438 one file to the right was at first inadequately
analyzed.

Maizelis, 1954

439. Berger restricted himself to the com-
ment that it was a draw in view of 1 d3 Ka4 2
Kc3 Kb5 3 b3 Ka5 4 Kc4 Kb6.

But this system of defence collapses if,
instead of 3 b3?, White plays 3 Kb3!, keeping
b2–b3 in reserve. All that remains for Black is
the try 3 ... c4 + 4 d × c4 + Ka5, but after 5
Kc3 c5 (5 ... Ka4 6 b4; 5 ... Kb6 6 Kd4) 6 b3!
White wins.

Thus in reply to 1 d3, Berger’s 1 ... Ka4 is
bad. A draw is given by 1 ... c4! 2 d4 Ka4.
The following position is simpler for
White.

440. 1 d3 Ka5 2 Kc4 Kb6 3 b3 and wins. If 1
... c4 the simplest is 2 d4 c5 3 d5, whereas 2
d × c4 + leads to complicated variations
(160).

Fine gives this example, and begins with 1

438. This position was suggested by Hor-
witz, but with an incorrect analysis: ‘1 c3 Ka5
b3, which in the event of 1 ... c4 forces 2 b×c4+. He does not give any analysis, despite the fact that a whole set of new variations is revealed here. Therefore 1 b3 (although it does in fact win) must be considered less clear, in view of the absence of a detailed analysis.

If it is Black to move, he draws by 1 ... c4.

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**Ebersz, 1933**

441. After 1 Kf5 Ke2 2 K×f6 Ke3 3 Ke5! Kf2 4 g5 Kg3 5 Kf5 the position reached has already been examined in example 281 (after White's 4th move).

**Grigoriev, 1932**

442. This position is the finish to a study (Ka4, pawns a5, a6, d5/Ka1, pawns a7, e5, d6).

1 Kb4! Ke4 (1 ... Kc6 2 Kc4 d5+ is hopeless due to 3 Kd4) 2 Kb5! Kd5.

Black has to go back, since 2 ... d5 is met by 3 Kc6, when after 3 ... Ke5 4 Kb7 Kd6 play concludes in stalemate at a8, while 3 ... d4 etc. leads to a queen ending which is drawn due to the remoteness of the black king.

3 Kb4 Ke5 4 Kc4! (now 4 Kb5? loses to 4 ... d5 5 Kc6 d4 6 Kb7 Kd6 7 K×a7 Kc7 8 Ka8 d3 9 a7 d2 10 a6 Kb6!) 4 ... Ke4 (4 ... d5 + 5 Kd3) 5 Kb5! Draw.
7. Three Pawns against Two or Three Pawns

Everything that was said in the introduction to chapter 6 also relates to these endings. To a certain extent this is the last transitional stage to endings with many pawns. The variety of positions here is so great that in the present book they can be covered only selectively. Preference has been given to the endings which are most valuable in the theoretical or practical sense. Many examples reveal new strategic ideas and tactical procedures, resulting from the specific nature of the given balance of forces. Others are of interest for the fact that they transpose into theoretical positions already considered.

One radical difference compared with the previous chapter is the principle on which the material has been systemized. There it was possible to use the pawn structure to a considerable extent, but in the present chapter this is unthinkable, and the basic division has been made into two groups:

7.1 Endings with passed pawns.
7.2 Endings without passed pawns.

Within each of these groups the material is arranged by ideas (themes), but wherever possible the nature of the pawn structure is also taken into account (for example: all the pawns on one wing; pawns on opposite wings; scattered pawns).

7.1 Endings with passed pawns

In characterizing these endings, we can merely repeat those remarks which were made in the corresponding section of the previous chapter (6.1). The game is normally decided not by the number of passed pawns, but by whose pawns are qualitatively superior, i.e. by positional factors (which are examined thematically below).

If the two players' pawns are on opposite wings, the kings have (sometimes not without difficulty) to restrain the advance of several pawns.

Of theoretical and practical interest here is the struggle of a king against three connected pawns. It occurs, of course, only when the enemy king is tied to the opposite wing, and is unable to support its own pawns. To analyze these cases, we can use the following four schematic positions.

443. Black's king has as many tempi as he wants (which is an indirect help to his pawns!), and so on the opposite wing there can be no struggle by the king against the connected pawns—it is hopeless. White can, it is true, try to penetrate with his king to d6, so as to carry out the familiar combination 1 a8=Q+ and 2 Kc7. But this is not a direct solution, but an avoidance of the problem.
facing White, and so the possibility of such a situation on the Q-side will be excluded.

444. Positions of this type are also excluded, since in practice such stalemate positions are extremely rare.

There remain, therefore, the following two schematic positions, which are the most likely in practice (cf. also 465).

445, 446. In positions of this type the black king cannot move, due to the threat of immediate defeat. In all the following examples we will therefore conventionally consider that White wins if he succeeds in halting the enemy pawns, since to halt them also means to eliminate them.

Generalizations and procedures, which would enable the strategy of play to be easily established in various positions with three connected pawns, have not yet been found. Only the following general assessment can be suggested: if the pawns are on their initial squares or are not far advanced, they are almost inevitably lost; if one or two pawns have crossed the middle of the board, the situation becomes dangerous for the king, and for certain pawn structures it even becomes critical (in the sense that the turn to move is decisive, or else the king has few squares from which it can restrain the advance of the pawns).

We will examine these critical positions systematically—depending on the pawn structure.
447. Here the side to move loses. On 1...g3 there follows 2 Kg2 (456), while if 1...f3 or 1...h3, then 2 Kf2 or 2 Kh2 respectively, winning (449).

With White to move, the picture is reversed: 1 Kg2 g3!, while if 1 Kf2 or 1 Kh2, both 1...f3 and 1...h3 are possible. If instead 1 Kf1 or 1 Kh1, then 1...h3 or 1...f3 respectively, and in every case the pawns are victorious.

If the pawns are one rank further down the board, Black wins by any move, the simplest being 1...g2. With White to move, 1 Kh1 is best met by 1...f2 (only not 1...g2 +? 2 Kg1, winning), temporarily stalemating the white king (on the Q-side a schematic position of type 445 or 446 is assumed, so that Black still has time for ...f1 = Q mate; but if the black king is stalemated as in position 444, the result is of course a draw).

448. Here the position has been moved one rank up the board, so that White wins even when it is him to move: 1 Kg3 g4 (1...f4+ or 1...h4+ is met by 2 Kf3 or 2 Kh3) 2 Kf2(h2) f4 (or 2...h4) 3 Kg2, winning (451).

The king can be on any of the squares marked by dots. The only king positions that are fatal are f3 and h3, with Black to move. E.g. (king at f3): 1...h4! 2 Kf2 (2 Kg2 f4—454) 2...h3! 3 Kf3 (3 Kg3 g4!—452) 3...f4! 4 Kf2 g4, winning.

If it is White to move, his king can also be at e1, e2 or e3 (Ke3–f2?).

449. Here the turn to move is unfavourable: after 1...h3 2 Kg3 the pawns fall one after another, while if it is White to move, 1 Kf1(g1) is met by 1...g3(h3). If the position is moved one rank down the board, Black also wins when it is him to move.

450. If position 449 is moved one rank up the board, White wins irrespective of who it is to move, but only if his king is at f3, f2, f1 or g1.
Three Pawns against Two or Three Pawns

For other positions the result depends on whose turn it is to move. For example, with his king at e1 or g2, White wins if it is him to move, but loses if it is Black to move in view of 1... h4 and 2... h3. With his king at h3, White loses irrespective of who it is to move.

452. White wins after 1... h4 = 2 Kf2, but if it is him to move, he loses: 1 Kf2 h4 (447) or 1 Kh2 h4.

453. If it is White to move, he has only one good retreat square—f3, whereas 1 Kh3? loses to 1... h5 (2 Kh2 f3; 2 Kg2 h4)—454.

With the king at g2 and pawns at f4, g3 and h2. Black wins immediately by 1... f3++; if it is White to move 1 Kh1 f3 leads to a situation considered in the notes to example 447.
454. The distinctive feature of this pawn structure (in the form of a triangle open to the invasion of the king) is that one of the critical squares (the one in the middle of the triangle) is cut off from the others.

Here the king has two (non-adjacent) decisive squares — g2 and g4; in these positions, as is clear from previous analysis, the turn to move is fatal.

If the pawns are moved one rank down the board, the position of the king is of considerable importance. If it is at g3 we have the same zugzwang situation; it disappears (the pawns win!) only if the position is moved down a further rank (when the king is at g2). But if the pawns are at f3, g4, h3 and the king stands at g1, Black wins irrespective of whose turn it is to move; in the event of 1 ... g3 2 Kh1 the ideas expounded in the notes to example 447 come into force.

The position moved one rank up the board should be noted.

455. If the king were at g5, we would have the same zugzwang as in the previous position. It disappears only when the position is moved one further rank up the board (pawns at f6, g7, h6); here White wins, irrespective of the turn to move, with his king at g6, f5 or h5 (adjacent squares at last!), or on any of the squares in the e1-e4-h4-h1 rectangle.

This emphasizes the at first sight amazing fact that in position 455 the squares f4 and h4 are losing ones for White if it is the opponent to move. But note that White wins, irrespective of the turn to move, if his king is on any of the marked squares. E.g., with his king at e3 he must meet 1 ... h4 with 2 Kf2! g5 3 Kg1!, winning.

456. This pawn formation is one of the weakest. Even if the position is moved one rank down the board, the pawns win only if it is White to move (it follows that the zugzwang remains in force).
457. If the pawns stand further up the board, as in this example, the king now has many winning squares.

458. The king has two (non-adjacent) critical squares f3 and h3; the result depends on the turn to move.

459. Whoever it is to move, White loses: 1 Kg1 f3, or 1 ... g1 = Q + ! 2 K x g1 f3.

460. Here the king has only three squares, but on the other hand they are adjacent. The strength of such a pawn formation is that the presence of a backward pawn provides reserve tempi. This advantage becomes especially marked if the backward pawn is still on its initial square.

461. White loses irrespective of the turn to move: 1 ... f5 (452); 1 Kh2 f6!

462. Possible variations are: 1 Kg2 g5 (454); 1 Kf2(f1) h3; 1 Kh2(h1) f3. Black to move plays 1 ... g6!
significantly backward pawn

The following position is a practical example.

464. White wins easily (we will restrict ourselves to typical variations):
(a) 1 ... f5 2 Kg2 h5 3 Kg3 g6 4 Kg2! g5 (4
    ... f4 5 Kf3 g5 6 Kf2) 5 Kg1 g4 6 Kg2 h4 7
    Kh2 f4 8 Kg1;
(b) 1 ... g5 2 Kg2 h5 3 Kg3 g4 (3 ... f5 4
    Kg2) 4 Kf4 f6 5 Kg3 f5 6 Kg2(f2, h2).

In all the examples considered the king has
stood in front of the pawns. An important
practical question arises: how far to the side
can White's king be when it is him to move,
so that he can still succeed in stopping the
pawns?

In example 455 we saw that against pawns
at f5, g6, h5 (not having crossed the middle
of the board) the king could be on the squares
d1–d4. At the same time, from position 448 it
is clear that against pawns at f5, g5, h5 the
squares e1–e3 mark the edge of the boundary.
It is natural that, the less advanced the pawns
are, the further to the side the king can be.

By expanding ancient analyses we can
compile the following table (we will suppose
that the schematic position 444 exists on the
Q-side).

With reference to the pawn formations in
the table:
(4) A sample variation: 1 Kd2(d4) h4 2 Ke3
    f5 3 Kf2! g5(f4) 4 Kg1, and the pawns are
    halted.
Table showing how distant the king can be

<table>
<thead>
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<th>pawn formation</th>
<th>king position</th>
</tr>
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<tbody>
<tr>
<td>f5, g5, h5</td>
<td>e1–e3</td>
</tr>
<tr>
<td>f3, g6, h5</td>
<td>d1–d4</td>
</tr>
<tr>
<td>f6, g5, h5</td>
<td>d1–d4</td>
</tr>
<tr>
<td>f6, g6, h5</td>
<td>c1–c5</td>
</tr>
<tr>
<td>f7, g6, h6</td>
<td>c1–c5</td>
</tr>
<tr>
<td>f7, g7, h6( h7)</td>
<td>d1–d6</td>
</tr>
</tbody>
</table>

(5) The position is weakened, since the pawn stands not at g7, but at g6. From c1–c5 the king reaches the key e3 square in two moves. A sample variation: 1 Kd2(d4) h5 2 Ke3 h4 (2 ... h5 3 Kf3) 3 Kf3 f5 4 Kg2! f4 5 Kg1, and the pawns are stopped.

If it is Black to move, White loses with his king at d5: 1 ... f5! 2 Ke5 h5 3 Kf4 h4 4 Kf3 g5, and Black wins.

(6) From the c-file the king is too late. If 1 Kd2(d3), then 1 ... f5 2 Ke3 h5 and wins, since the king cannot reach g2 or g3. If instead 1 Kd4(d5), then it is true that the immediate 1 ... f5 is not possible due to 2 Ke5, but Black wins by 1 ... h5 2 Kc4 h4 and 3 ... f5.

The remaining cases have been examined earlier.

If in positions 4 and 5 there is a situation of type 445 on the Q-side, then with his king at c1 White should carry out the plan of eliminating the pawns; but if it stands further up the board, it is simpler to go to b6.

The following study, which is as though taken from a practical game, is a good example.

465. 1 b4 Kd5 2 h5 Ke6 3 h6 Kf6 4 f5!

The position of interest to us has been reached.

4 ... b5! 5 Ke2 b4 6 Kd3 d5 7 Kc2!

Not 7 Kd4 c5+! 8 Kd3 b3!, when it is Black who wins (454 and 455).

7 ... d4 8 Kc1! c5 9 Kc2! c4 10 Kc1! (447)

and wins.

466. To win, the opponent has to be put in zugzwang on both wings. Since he will not occupy such a position voluntarily, the extra tempo (the advantage of the first move) must be used to advance the pawns.

1 a5 + Ka6 2 c5 h4 + (2 ... Kb5 3 Kg2!; 2 ... Kb7 3 b5) 3 Kh3 f4 (3 ... Kb5 4 Kh2 g4 5 Kg2 f4 6 Kg1) 4 e6 f3 5 b5 + Ka7 6 b6 + (the most energetic, but 6 c7 is also possible) 6 ... Kb8 7 a6 g4 + 8 Kh2 g3 + 9 Kg1 h3 10 a7 + Ka8 11 c7, and White wins.
Endings with Passed Pawns

Chapais, 1780

467. After 1 a5 White achieves the set-up a6, b5 and wins.

Behting, 1900

468. 1 Kg1! h3 (1 ... Ka7 or 1 ... Kc7 immediately provokes the combinational finish) 2 Kh2 f3 3 Kg3 Ka7 4 b8=Q + ! K x b8 5 c6 etc.

The number of zugzwang positions is relatively small (almost all of them have already been indicated).

469. Here Black answers 1 c5 with 1 ... f4, when White is in zugzwang on both wings. The reply to 1 Kh2 or 1 b5 is the same. On 1

Kg1 Black can play 1 ... f4 (but not 1 ... Ka6? 2 c5 Kb5 3 Kh2 g4 or 3 ... f4 4 Kg2, winning—Peret'sman, 1958) 2 Kg2 (2 c5 g4) 2 ... Ka6 3 c5 Kb5 4 Kg1 g4, and wins.

The position on each of the wings does not in itself lead to zugzwang, but the necessity to have (under conditions of complete symmetry!) not the first word, but the last, creates here a position of mutual zugzwang.

Drawn positions also occur.

470. The kings are forced to move between h2–g3 and a7–b6 respectively. If either player risks moving his backward pawn, he inevitably loses, e.g. 1 c3? f6 2 Kh2 Ka7 3 Kg3 (3 c4 Kb6 4 Kg3 f5) 3 ... f5 4 c4 Kb6 5 c5 + Ka7 6 c6 Kb6 etc.

We are now able to analyze very easily the
following ancient position, the first correct assessment of which was given by Zsen, and the first correct analysis by Walker (1840).

471. It is clear from the preceding analysis that the correct strategy cannot consist of an attempt to advance the pawns as quickly as possible—all the same they will be quickly stopped. An advantage must be gained uniformly on both wings, aiming to put the opponent in zugzwang, and if he should avoid it then the extra tempo must be exploited.

1 Ke2 (already stopping the black pawns; it is best, of course, not to allow them to advance far) 1... Kd7 (1... h5 2 Kf3 g5 3 a4 h4 4 Kg4 f5 + 5 Kh3 Kd7 6 c4 Kc6 7 a5, and White’s decisive advantage is obvious) 2 Kf3 Ke6 3 a4 h5 4 e4 f5 (each side is now threatening to advance his rook’s pawn) 5 Kg3 Kb6 6 b4 g5 (or 6... g6) a5+, and wins (466). Here there are numerous other variations, but they are all fairly simple.

The solution can also be begun with 1 a4 Kd7 2 Ke2 (but not 2 a5), as well as by the less strong 1 Ke1. Such a position, with the king at e1, was analyzed by Greco (1612), but he did not solve it correctly. In this position White wins even if it is the opponent to move.

Instances with isolated and doubled pawns, and also with two passed pawns, are examined in the following examples.

Grigoriev, 1936

473. White wins only because he succeeds in advancing his pawn to h7: 1 h5 Ke7 2 h6
Endings with Passed Pawns

$Kf6 \, 3 \, h7 \, Kg7 \, 4 \, Ke4!! \, (K \times c6 - d7 - e7 \, or \, \text{Kd4- e5- e6} \, \text{is bad, since Black obtains a queen at a1 just in time}) \, 4 \ldots \, Kh8 \, 5 \, Kd4 \, a4 \, 6 \, Ke5 \, a3 \,(6 \ldots \, Kg7 \, 7 \, Ke6 \, \text{and} \, 8 \, h8 = Q +) \, 7 \, Kf6! \, a2 \, 8 \, g7+! \, K \times h7 \, 9 \, Kf7$, and wins.

Grigoriev

Grigoriev, 1928

474

1. $Kf2! \, c5$ (if $1 \ldots \, d4$, then $2 \, Kf3! \, c6 \, 3 \, Kf4 \, c5 \, 4 \, Ke4!$) \, 2 \, Ke3! \, c4 \, (2 \ldots \, d4 + \, 3 \, Ke4 \, Kh8 \, 4 \, Ke5 \, Kg7 \, 5 \, Kd6) \, 3 \, Kd4 \, Kh8 \, (3 \ldots \, d6 \, 4 \, K \times d5 \, \text{and} \, 5 \, Ke6) \, 4 \, Ke5! \, Kg7 \, 5 \, Kd6 \, c3 \, 6 \, Ke7 \, \text{etc.}$

Horwitz & Kling, 1851

475

1. $h5+ \, Kh7 \, 2 \, Ke3 \, Kg8 \, 3 \, g6 \, Kh8 \, 4 \, h7$

476. Only stalemate at $h8$ can force the black pawns to lose their invulnerability: 1 $K \times a6 \, Kf8 \, 2 \, Kf6 \, Kg8 \, 3 \, Kg6 \, Kf8(h8) \, 4 \, h3!!$

The chief subtlety: the move $h6-h7$ must not be accompanied by check. Wrong is 4 $h4? \, Kg8 \, 5 \, h5 \, Kh8 \, 6 \, h6 \, Kg8 \, 7 \, h7 + \, Kh8 \, 8 \, Kh6 \, d5 \, 9 \, Kg5 \, K \times h7$. Draw.

4 $\ldots \, Kg8 \, 5 \, h4 \, Kh8 \, 6 \, h5 \, Kg8 \, 7 \, h6 \, Kh8 \, 8 \, h7 \, d5 \, 9 \, Kf5 \, \text{etc.}$

Horwitz & Kling, 1851

477
There is an alternative: to penetrate with the king to e6 (with the idea of $h8 = Q+$ and $Kf7$), but it is easy to see that Black obtains a new queen at $b1$ and averts the mate at $g6$. One is led to conclude that the $b1-h7$ diagonal must be blocked by $e3-e4$, but at what point should this move be made?

**Grigoriev, 1932**

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478. Here White cannot even move his pawns, since the opponent king manoeuvres between $g5$ and $h4$.

479. The attempt to win the seemingly helpless b-pawn does not succeed: $1 \text{Kg3} \text{b5} 2 \text{Kf3} \text{b4} 3 \text{Ke2} f5! 4 \text{Kd3} b3! 5 \text{Kc3} d4+!! 6 e \times d4 f4 7 d5 f3 8 d6 f2 9 d7 f1 = Q 10 d8 = Q Qc1 +! 11 Kb4 Qe1 +!, with a draw by perpetual check.

The idea of the draw is as follows: the king is forced sooner or later to take on $b3$, otherwise it will be impossible to block the checks with the queen; but after $K \times b3$ the queen gives check from below at $b1$ and $c1$, to answer $Kb4$ with ... $Qb1+$, and if $Kc5$ $Qc2+$, or $Ka5 Qf5+$, when the king cannot cross the 6th rank due to ... $Q \times g6+$.

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477. White's king is close enough to be able to lend his pawns decisive support (308): $1 \text{Ke4} \text{Kg4} 2 \text{h4} \text{Kh5} 3 \text{Kf4} \text{Kh6} 4 \text{g4} \text{Kg6} 5 \text{h5+} \text{Kh6} 6 \text{Ke4} \text{Kg5} 7 \text{Kf3} \text{Kh6} 8 \text{Kf4} \text{Kf7} 9 \text{g5} \text{Kg7} 10 \text{g6+} \text{Kh7} (10 \text{h6+} \text{Kh7 leads to a draw}) 10 $ ... $\text{Kh6} 11 \text{Kg4} \text{Kg7} 12 \text{Kg5!} d3 13 \text{h6+} \text{Kg8} (f8, h8) 14 \text{Kf6}$, and wins.

If the white king and the c- and d-pawns are moved one file to the left, the position is still a win; if they are moved two files to the left, it is a draw, as in the following position.
481. Black won by a precise manoeuvre: 1 ... Kb7 2 a3! Ka6 3 c6! (after 3 a4 Black has the quickly decisive 3 ... Ka5 4 c6 K x a4 5 Ke6 Kb5) 3 ... Kb6! 4 a4 Kc5!! 5 Ke6 e4 6 Kd7 e3 7 K x e7 e2 8 Kd7 e1 = Q 9 c7 K x d5 10 c8 = Q Qe6 + 11 Kd8 Q x c8 + 12 K x c8 Kc6! (12 ... Kc5? 13 Kb7! — draw) White resigns.

482. White must correctly parry the advance of the f-pawn: 1 g6 f4 + 2 Kg2! Ke2 3 g7 f3 + 4 Kg3!

The tournament bulletin incorrectly gives 4 Kh2 f2 5 g8 = Q f1 = Q 6 Qc4 + Ke1 7 f4 + 7 Kg2!, and wins.

Q x f1 + K x f1 8 Kg3 Ke2 9 Kf4 Resigns, although a by-pass by the king leads to a draw: 9 ... Kd3! 10 Kg5 Ke4 11 K x h5 Kf5 12 Kh6 Kf6 13 h5 Kf7 14 Kg5 Kg7.

4 ... f2 5 g8 = Q f1 = Q 6 Qc4 + Ke1 7 Q x f1 + K x f1 8 Kf4. Black resigns, since White queens his h-pawn.

The idea of this finish occurred in this earlier ending.

Gusev, 1924
Three Pawns against Two or Three Pawns

Berg-Petrov
Kemeri, 1937

484. With his sealed move White forced a transition into this pawn ending, but he resigned without resuming, since a curious winning path was found for Black, based on a subtle king manoeuvre.

If it were now Black to move, 1 ... g5! would immediately decide the game in his favour. Therefore White must play 1 g4.

Black's task now is to approach the opponent's K-side pawns.

1 ... Kg8 2 Kc2 Kf7 3 Kd3 Ke7.

3 ... Ke6 4 Ke4 b3 5 Kd3 Ke5 6 Ke3 Kf4 fails to win due to 7 g5! Kg4 8 K × b3 K × h4 9 Kc4 K × g5 10 Kb5 Kf5 11 K × a5 g5 12 Kb5 g4 13 a5, when the pawns queen simultaneously.

4 Ke3!

White loses immediately after 4 Ke4 Ke6 (zugzwang) 5 h5 Kf6 or 5 g5 g6. Black has to be able to bring his king out onto the 5th rank (to c5, d5 or e5), and by taking the distant opposition White tries to prevent this.

4 ... Kd7 5 Kd3! Ke7!

The occupation of this square is decisive, since in White's position there is no corresponding square. But the struggle is not yet over, and Black has to proceed with a certain accuracy. Thus on 6 h5 he should play 6 ... Kd6 7 Ke4 Ke7! 8 g5 Ke6! etc.

6 Ke4 Kc6! 7 Kd4 Kd6 8 Ke4 Kc5 9 Kd3 (9 h5 can be met by 9 ... Kd6 or 9 ... Kc4) 9 ... Kd5 10 Ke3 Ke5, and Black won.

Horwitz & Kling, 1851

485.

486. 1 f4! e × f4 (1 ... Kc5 2 f5) 2 d4 Ke6 3 Kg2 Kd7 4 Kf3 Kc7 5 K × f4 Kb6 6 Ke4 K × b5 7 Kd5, winning.

The possession of an outside passed pawn is an undoubted positional advantage. The
following example is typical, and shows the method of realizing this advantage.

Grigoriev, 1930

487. White’s resistance is based on the good position of his king, but he is unable to maintain it: 1 ... Kd6!

Now the following are obviously bad: 2 c3? Kc6; 2 f4? e × f4!; 2 Kd3? Kd5 3 c4 + Kc5 4 Kc3 e4 5 f × e4 (5 f4 e3) 5 ... f × e4 6 Kb3 Kd4, and wins.

2 Kb5 e4! 3 f × e4 f × e4 4 Kc4 Ke5 5 Kc3 Kf4 6 Kd2 Kf3 7 Ke1 Ke3! 8 c4 Kd4 9 Kd2 K × c4 10 Ke3 Kb4!, and Black wins.

But certain exceptions are possible, where an outside passed pawn cannot be realized.

O’Kelly-Mieses
Brussels, 1935

488. 1 Kf4 Kf7 2 Ke5 Ke7 3 f6 + Kf7 4 Kf5 Kf8 5 Ke6 Ke8 6 Kf5 (6 f7 + Kf8 7 Kf6 h5 8 Kg6 a5 — draw). 6 ... Kf7 7 Kg5 Ke6 8 Kh6 K × f6 9 K × h7. Drawn.

Tartakover-Stalda
Venice, 1949

489. 1 ... g5 2 h5 Ke5 3 h6 K × h6 4 K × d4 Kc6 5 Ke5 h6 6 h3 (if White tries to preserve this tempo, all the same a draw is inevitable) 6 ... Kd7 7 Kf6 Kd6 8 Kg6 Ke5 9 Kg7 Kf5! 10 K × h6 Kf6. Drawn.

The following examples are instructive, with three pawns against two on one wing.

Grigoriev, 1934
extension of a study by Mattison
There is a story behind this position. An analogous position, but with an additional black pawn at h6, was published in 1927 by Mattison. The idea of the study was as follows: 1 b4 Kd4 2 Ke6! (not allowing the king to go to d5; if 2 Kf5? Black succeeds in defending by 2 ... Kd5 3 Kg4 Kc6 4 Kh5 Kb7 5 K×h6 Kc6 6 K×h7 c5 7 c3—an enforced loss of time—7 ... c×b4 8 c×b4 Kb7 9 Kg6 Kc6 10 Kf6—White is short of the tempo for Kf6–e6—10 ... Kd5. Draw) 2 ... h5 3 c3+! (this gains the lacking tempo) 3 ... Kc4 4 Kf5, and the defence ... Kd5–c6–b7 no longer works. 4 ... h4 5 Kg4 or 4 ... h6 5 Kf4 is hopeless for Black, since in both cases the king picks up both black pawns within 4 moves.

Mattison also considered 4 ... c5 to be hopeless due to 5 b5. But it was here that Grigoriev discovered a mistake! He found that after 5 ... Kd5 6 Kg5 Black can save the draw by 6 ... c4!, creating a stalemate shelter for his king c5, on condition that Black holds the opposition. For example: 7 K×h5 h6 6 K×h6 Kd6! or 8 Kg4 Ke6 9 Kh5 Kd5 10 K×h6 Kd6! He can also leave the pawn at h7, and answer K×h7 with ... Kd7!

If he holds the opposition, Black cannot lose, e.g. (after 8 K×h6 Kd6): 9 Kg7 Ke7 10 Kg6 Ke6 11 Kg5 Ke5 12 Kg4 Ke6! (in view of the threat of a4–a5, Black must keep within the square of the b5 pawn; the manoeuvring here is similar to that in example 118) 13 Kf4 Kd6! (this would also have been the reply to 13 Kh4) 14 Ke4 Ke6! Draw.

Thus Grigoriev found a fundamentally new system of defence, which made Mattison's study incorrect.

But why is Grigoriev's position (without the h6 pawn) won? Let us follow the solution: 1 b4 Kd4 2 Ke6! h6 (trying to complicate White's task; if 2 ... c5? 3 a5!) 3 c3+! Kc4 4 Ke5! (had the pawn moved to h5, White would have played 4 Kf5!) 4 ... h5 (4 ... c5 5 b5; 4 ... c6 5 Ke4! or 5 Kf6) 5 Kf5! Kd5 6 Kg5 c6 (hoping for 7 K×h5? c5 8 b5 c4 when Black has the opposition) 7 Kb4! c5 8 b5 c4 9 K×h5, and White has the opposition. It is this—what explains all the trickery: Black can rearrange his moves as much as he likes—all the same it is hopeless. The rest is simple: 9 ... Kd6 10 Kf6! (opposition on the main rank—the middle one in the space between the 5th and the 8th) 10 ... Kd7 11 Kg5; 10 ... Kd5 11 Kg7; 10 ... Ke6 11 Kgf6 Kd6 12 Kf6, winning in all cases by a by-pass.

Does Grigoriev's 'stalemate shelter' save the game if the position is moved one rank up the board? The answer to this question is given by an analysis of the following position, which occurred in a practical game.

491. The continuation was 1 b5 Kc5 2 Kg4 Kd6 3 Kh5, and White won.

Black did not even suspect that there was any possibility of saving the game. It is extremely curious that defence à la Grigoriev' does not work here: 1 b5 c6 2 b6 c5 3 Kg4 Kd7 4 Kh5 Kc6 5 K×h6 Kd6, and, despite holding the opposition, Black loses, since due to the necessity for manoeuvring within the small 'square' of the b6 pawn he immediately loses the opposition (6 Kg7 etc.).

But defence 'à la Mattison' succeeds: 1 ... Kd7! 2 Kg4 Kc8 3 Kh5 Kc7 4 K×h6 Ka7 5 Kg4 K×b5 6 Kc6 Kf6 (6 b×c6 is also hopeless) 6 ... c×b5 7 c×b5 Kc8. Draw (165).
Endings with Passed Pawns

Nimzowitsch-Chigorin
Carlsbad, 1907

492. In this theoretically interesting position, after 1 ... Ke6? 2 Kc5 f5 3 h3 f×g4 4 h×g4 d4 5 K×d4 Kd6 6 f5 Black resigned.

In the tournament book Schlechter expressed the opinion that by 1 ... Kc6! Black could have drawn, e.g. 2 h4 Kd6 3 h5 Ke6! 4 Kc5 f5 5 g5 h×g5 6 h6 Kf7! 7 f×g5 f4.

Nimzowitsch did not agree with this assessment. Here is his analysis (1918): 2 h3 Kd6 3 h4 Ke6 (3 ... Ke6 4 Kc5 f5 loses to 5 g5) 4 g5 (if 4 h5, then 4 ... Kd6 5 g5 f×g5 6 f×g5 Ke6! 7 g6 Kf6! with a draw — 108) 4 ... f×g5.

If 4 ... h×g5, then 5 f×g5! Kd6 6 g6 Ke6 7 h5 f5 h6 Kf6 8 g7 Kf7 10 K×d5, winning. But 5 h5 g4 6 Ke3 Kc5 7 h6 d4+ 8 Ke2 d3+ leads only to a draw.

5 f×g5 h×g5 6 h5 g4 7 Ke3! (7 h6 g3 8 Ke3 d4+ 9 Kf3 d3 leads to a simple draw) 7 ... Kc5! 8 Kf4! d4 9 K×g4 d3 10 Kf3 Kc4 11 h6 Kc3 12 h7, and White wins.

However, as was shown by readers of the magazine Shakhmaty v SSR, by playing 11 ... Kb3! instead of the incorrect 11 ... Kc3, Black can draw. Thus Schlechter was right!

After an examination of position 492 it is easy to conclude that the general pawn formation ensures White a comparatively easy breakthrough, obtaining a more distant and hence more dangerous passed pawn; as for whether it is a win or a draw — this depends on various details of the position, and at times (as the given analysis shows) on a single tempo.

Bogoljubow-Fine
Zandvoort, 1936

493. In the absence of blocked pawns, Black easily gains a draw: 1 ... Kf7 2 Kd3 Kc6 3 Ke4 g6! 4 Kd4.

4 f5+? Kd6! leads immediately to a draw.

4 a4 Kd6 5 a5 Kc5 6 a6 Kb6 7 Kd5 is also futile in view of 7 ... g5! 8 f5 (8 f×g5 occurred in the game) 8 ... h5 9 Kc6 h4, and the queens are obtained simultaneously.

4 ... Kd6 5 Kc4 h6 (5 ... Kc6 is also good) 6 Kd4 Kc6 (6 ... g5? 7 Ke4!) 7 Ke4 Kb5 8 Kd5 g5! 9 f×g5 f×g5! 10 Kc5 Ka4 11 Kb5 K×a3 12 Kb6 Kb4 13 K×h6 g4! Draw (the black king goes to f8).

The possession of a passed pawn is normally a decisive advantage only if the opponent has no chance of obtaining a passed pawn in turn.

494. In his comments to the game Marshall-Reti (New York, 1924), analysis by Alekhine leads to this position.

1 g5! (paralysing the enemy forces) 1 ... Kc6 2 Ke5 Kd7 3 Kd5! (of course, not 3 Kf6? K×d6 4 K×f7 Ke5 etc.) 3 ... Kd8 4 Ke6, winning.
Three Pawns against Two or Three Pawns

Alekhine, 1924

van Trotsenburg—Esser
The Hague, 1910

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495

1 Kb2! Ke6 2 Ka3 Kf5 3 Kb4 Kg4 4 Kc5!! K × h4 5 K × e6 Kg5 6 K × c7 Kf4 (6 ... Kf6 7 Kd6) 7 Kd6(e6) Ke3 8 Kc5 Kd3 9 Kb4, and White wins.

496

Things are made slightly difficult for Black here by the fact that he cannot play ... e5 immediately. The subsequent play takes account of the zugzwang position Ke5/Ke7.

1 ... Kf7!

1 ... Ke7 is a loss of time due to 2 Ke5 Kf7 3 Kd4(d6) Kf6 4 Kc5 Ke7! 5 Kd4 (if 5 Kb6, then 5 ... e5 6 K × a6 e4 7 b5 e3 8 b6 e2 9 b7 e1 = Q 10 b8 = Q Qa1 + 11 Kb7 Qb2 +, and after the exchange of queens Black promotes his g-pawn, while after 5 Ke6 e5 6 Kd5 Kf6 7 Kc4 e4 8 Kd4 e3! 9 K × e3 Ke5 he again wins).

2 Kd4 Ke8! 3 Ke4 Kd7 4 Kd4. Here there are two ways to win:

(a) 4 ... Ke6 5 Ke5 Kd5 6 Kf6 K × b4 7 K × g5 a5 8 Kf4 Kc5! 9 Ke5 (after 9 g5 Kd6 the king stops the g-pawn) 9 ... a4 10 g5 a3, and Black's pawn queens with check;

(b) 4 ... Kd6 5 Kc4 e5 6 Kf5 Kd5 7 K × g5 e4 8 Kf4 Kd4 9 g5 e3 10 g6 e2 11 g7 e1 = Q 12 g8 = Q Qf2 +, and with his next move Black wins the opponent's queen.

In the following examples one of the sides decisively creates an outside passed pawn, which proves stronger than the opponent's (cf. also 492).

497. In making his breakthrough, White has to concern himself over shutting the enemy king out of play, and has also to take measures against the 'wandering square' which arises.

1 Kf3! b5 2 e4 b4 3 Ke2! b3 4 Kd1! Kg3 5 e5 d × e5 6 d6 e4 7 d7 e3 (had the king gone to d2 on move 4, this move would have given check and would have led to a draw) 8 d8 = Q, and wins.
Endings without Passed Pawns

Herbstmann, 1929

In the second group the pawns are deployed separately on both wings.

The remaining formations (with mixed or scattered pawns) are assigned to the third group.

Within each group the material is arranged thematically, which in the majority of cases is directly related to the pawn formation. Endings with three pawns against two are given first, and then three against three—but, of course, only if as a result the overall thematic arrangement of the material is not disturbed.

7.21 All pawns on one wing

Smetana, 1942

Lolli, 1763

498. 1 f4 h4 2 f5+! g × f5+ 3 Kf3! Kf7 4 Kg2 Kg6 5 f4, and White wins.

7.2 ENDINGS WITHOUT PASSED PAWNS

Endings without passed pawns are divided into three groups.

In the first group all the pawns are on one wing, opposite one another. First we examine the struggle of connected pawns against connected pawns, then connected pawns against isolated (including doubled pawns), and finally, isolated pawns against isolated.

499. With White to move, the only way to win is by 1 g6! A draw results from both 1 h6? g × h6 2 g × h6 (225), and 1 f6+? g × f6+ 2 g × f6+ Kf7 3 Kf5 Ke8! (225). After 1 g6! Black is not saved either by 1 ... h × g6 2 h × g6 (131), or 1 ... h6. In the latter case, it is true, White cannot immediately play 2 f6+? g × f6+ 3 Kf5 Kf8 4 K × f6 Kg8 with a draw (108), but must first drive the king to h8, e.g. 2 Kd5 Kf8 (2 ... Kd7 3 f6; 2 ... Kf6 3 Ke4 or more simply 3 Kd6 K × f5 4 Ke7) 3 Kd6 Ke8 4 Ke6 Kf8 5 Kd7 Kg8 6 Ke7 Kh8 7 f6 g × f6 8 Kf7.
With Black to move:

(a) 1... h6 2 g6 or 2 g×h6 (only not Berger's 2 f6? due to 2... Kf7).

(b) 1... g6 2 h×g6 h×g6 3 f6+; or, even simpler, 3 f×g6! Ke8 4 Ke6! Kf8 5 Kf6 (36).

(c) 1... Kf7 2 g6+ Kg8 (2... h×g6 3 h×g6+ -131), and now White can play either 3 Ke6 Kh8 4 Kf7 h×g6 (4... h6 5 f6) 5 h6 g×h6 f×g6, or 'à la Salvioli', 3 g×h7+ K×h7 4 Ke6 Kh6 5 Kf7 Kh7 6 Kf8 Kh8 7 h6 (256).

Maizelis, 1954

500

500. Surprisingly, this position is not analyzed in other books. Moving position 499 one or more ranks down the board is in general favourable to White, but certain special features appear.

If it is White to move, he can win by 1 g5 h×g5 2 h×g5!, or 1 f5+, or 1 h5.

With Black to move, 1... h5 can be met by 2 g×h5, 2 g5 or 2 f5+, while after 1... Kf6 either 2 h5 or 2 f5 wins (but not 2 g5+?).

501. The king was moved to c1, by Walker; the composer had it at c3.

1 Kd2 h4 2 g×h4! (Berger recommended 2 g4+? Kf4 3 Ke2, but then 3... Kg3 leads to a draw; this variation is possible for White only in the position moved one rank up the board) 2... g×h4 3 Ke3 etc.

Cozio, 1766
Walker, 1841

501

With Black to move, 1... h4! 2 g×h4 (2 g4+? loses) 2... g×h4 gives a draw.

In the position moved one rank up the board, Black would be unable to draw.

Ilyin-Genevsky v. Abramian
½-Final, 11th USSR Championship, 1938

502

502. 1... f3+? (an incomprehensible mistake, since even 2 g×f3 g3 does not give Black a win) 2 Kf2 g×h3 3 g×h3. Drawn.

1... g3! would have won, e.g. 2 Kf3 (2 Kf1 is hopeless—131) 2... Ke5 3 K×f4 Kd4 4 Kg4 Ke3 etc.
503. This is a drawn position (with White to move) which is not referred to elsewhere. If one supposes that the white pawn is at h2, and the black one at g7, then h2–h4? (h2–h3 is correct) would be a decisive mistake in view of...g6!

With White to move:
(a) 1 f5+ g × f5+ 2 g × f5+. Draw (225).
(b) 1 g5 Kd6 2 f5 g × f5+ 3 K × f5 Ke7 (179), or 2 h5 Ke6 (133), or 2 Kf3 Ke6 3 Kg4 Ke7(f7). Draw.
(c) 1 h5 g × h5 2 g5 h4 3 f5+ Kd6 4 g6 (4 Kf4 h3—draw) 4...h × g6 5 f × g6 Ke6.
(d) 1 Kd4 Kd6 2 Kc4 Ke6 3 Kc3 Ke7, and the white pawns cannot break through.

With Black to move, after 1...Kd6 White does not achieve anything by 2 h5, or 2 g5, or 2 f5 g × f5+ 3 K × f5 Ke7 (188) 4 Kg5 Kf7 5 Kh6 Kg8. Draw (the h-pawn does not stand at h3 or h2).

The win is rather complicated: 2 Kd4 Ke6 3 Kc5 Ke7 (3...h5 4 g × h5! g × h5 5 Kd4 Kf5 6 Kc3 Kg4 7 Ke4 K × h4 8 Kf3 and wins) 4 Kd5 Kf6 (4...Kd5 5 Ke5 Ke7, as in the main variation) 5 Kd6 Kf7 (5...h5 fails to draw due to 6 g5+! Kf5 7 Ke7 etc.) 6 Ke5 Kf7 g5! Kf7 8 Kd6! (8 h5? Ke7!—draw) 8...Kf8 9 Kf6 Ke8 10 Kf6 Kf8 11 h5! g × h5 12 Ke5 h4 (12...Ke7 13 f5! etc., as in example 339) 13 Ke4 Ke7 (13...Kf7 14 f5!) 14 Kf3! Ke6 15 Kg4, and wins.

504. Active play by Black loses here due to the possibility of a breakthrough, e.g. 1...Ke6 2 Kc8 Kd5 3 f5 g × f5 4 g5 Ke6 5 g6, and the pawn cannot be stopped.

Black maintains the balance, by manoeuvring with his king on the corresponding squares: 1...Kf8! 2 Kb7 Kf7 3 Kc6 Ke6 4 Kc5 Kf7 5 Kd5 Kf7 6 Kd6 (similarly, nothing is achieved by 6 Ke5 Kf7 7 f5 g × f5 8 g × f5 Kf7 etc.) 6...Kf6 7 Kd7 Kf7 8 Kd8 Kf8.

Keidanski, 1908

505. 1 Kc3 Ka5 (1...Ka5 2 b4+ Ke6 3 Kd4 b6 4 c5, or 1...a5 2 Kd4 Kc6 3 c5 Kd7 4 Kc4 Kc6 5 b3 b6 6 c × b6 K × b6 7 Kd5, winning) 2 b3 b6 3 Kd4! Kb4 4 a5! K × a5 (4
... b × a 5 5 c 5) 5 K c 3 b 5 6 c 5 b 4 + 7 K d 4, and wins.

But if it is Black to move, he draws after 1 ... a 5 ! (1 ... K c 5? 2 a 5!) 2 K c 3 K e 5 ! 3 b 3 b 6 4 K d 3 K b 4 5 K c 2 K e 5 6 K c 3 K e 6 ! (6 ... K d 6? 7 K d 4 K c 6 8 c 5 !) 7 K d 4 K d 6.

Ke 2 f 4 2 f 3 + K f 5 3 g × f 4 K × f 4 4 K f 2, and then as in position 223.

If Black begins, he can draw: 1 ... K f 3 2 K g 1 (2 K e 1 f 4 3 g × f 4 K × f 4 4 K e 2 K g 4) 2 ... f 4 3 K h 2 f × g 3 + 4 f × g 3 (134).

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Grigoriev, 1925

506. 1 K a 2 ! b 4 2 a × b 4 a × b 4 3 K a 1 ! c 2 (3 ... K b 3 4 K b 1) 4 b 3 + K b 5 5 K b 2 K c 5 6 K × c 2 K d 4 7 K d 2. Draw.

Walker, 1841

507. Contrary to the opinion of the composer, White wins only if it is him to move: 1

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508. In spite of his doubled pawns, White can realize his advantage: 1 K g 3 K f 5 2 h 4 K f 6 3 K f 2 K e 6 4 K e 2 ! K f 6 5 K d 3 ! K f 5 6 K e 3.

Now Black has two possibilities, but neither saves the game:

(a) 6 ... K f 6 7 K e 4 K e 6 8 f 5 + ! g × f 5 + 9 K d 4 ! K d 6 10 f 4.

(b) 6 ... K e 6 7 K e 4 K f 6 8 f 5 ! g 5 (8 ... g × f 5 + 9 K d 5 or 9 K f 4) 9 h × g 5 + K × g 5 10 K e 5 h 4 11 f 6 h 3 12 f 7 h 2 13 f 8 = Q h 1 = Q 1 4 Q g 7 + K b 5 15 Q h 7 + .

509. The composer’s solution demonstrates a by-pass by the king with the aim of an attack on the opponent’s pawn weaknesses: 1 K d 4 K c 6 2 K c 4 K d 6 3 K b 5 ! K d 5 ! (if 3 ... K e 5, then 4 K c 5 K f 4 5 K d 5 K × g 4 6 K e 6, or 4 ... h 5 5 g × h 5 K × f 5 6 K d 5, winning) 4 K b 6 K d 6 5 K b 7 ! K d 7 6 h 5 K d 6 7 K c 8 K e 5 8 K d 7 K f 4 9 K e 6 K g 5 10 K f 7, and wins.

However, as was found by Zinar, White can win more simply — by a pawn breakthrough, after preparing it with a precise king manoeuvre.

Let us consider the corresponding squares.
Endings without Passed Pawns

Neustadt, 1898

The square corresponding to e4 is d6, to f4—e7, and to e3 and f3 there is only one corresponding square, d7. This means that we have a classic case of ‘triangulation!’ By manoeuvring between e3 and f3, White breaks the correspondence in his favour.

And so: 1 Kf3 Kd7 (if 1 ... Ke5 2 Ke3 h5, then 3 Kf3 etc.) 2 Ke3 Kd6 (2 ... Ke7 3 Kf4 Kf7 4 Ke4 Ke7 5 Kd5 Kd7 6 h5) 3 Ke4 Kc6 4 Kf4 Kd6 5 g5, and White wins.

The following study is another example on the breakthrough theme.

Behring, 1905

510. The immediate breakthrough does not succeed: after 1 g4 f x g4 2 f5 g3 the black pawn very quickly promotes.

This means that, by king manoeuvres, White must force the opponent’s king to stand on the g-file, where it will hinder the advance of its own pawn, but which move is correct; 1 Ke2 or 1 Ke1? We will consider them in turn:

(a) 1 Ke2 Kg2.

1 ... Kh2 is bad: 2 Kf2 Kh3 (2 ... Kh1 3 g4 f x g4 4 Kg3) 3 Kf3 Kh2 4 g4, and wins.

2 g4 f x g4 3 f5 g3 4 f6 (if 4 h6, then 4 ... g x h6 5 f6 Kh1 with a draw) 4 ... g x f6 5 h6 f5! 6 h7 f4 7 h8 = Q f3 + 8 Ke3 f2, and it transpires that, thanks to the possibility of advancing his pawn with check, Black has attained a theoretically drawn position.

This means that it is correct to play the king to e1, not allowing Black this possibility:

(b) 1 Ke1! Kg2 2 g4 f x g4 3 f5 g3 4 f6 g x f6 5 h6 f5 (5 ... Kf3 6 Kf1) 6 h7 f4 7 h8 = Q f3 8 Qa8!, and White wins.

Jelinek, 1932

511. Typical ‘tempo’ play with three pawns against two—only the obtaining of a queen at h8 is any good for White: 1 Ke1! f4 (1 ... h5 2 h4; 1 ... h6 2 h3) 2 Kc2! f3 3 g4 h6 4 h3 h5 5 h4, and wins.

177
Three Pawns against Two or Three Pawns

Charousek-Kosterka
Prague, 1897

512

512. This interesting position was saved from oblivion by Dedre (1950). It was reached in a simultaneous display, and after unsuccessful attempts to win, Charousek agreed to a draw. But analysis by Josef and Johann Kvicala demonstrated the possibility of a win.

With the key squares being c6, d6 and e6, the main file is the d-file. If White’s chances lay only in taking the opposition on the main file, he would be unable to achieve anything. He wins only by combining this threat with the appropriate preparation of h3–h4.

1 Ke4! Ke8! 2 Kf3!

Now Black is in a dilemma: after 2 ... Ke7 he loses the opposition, while if 2 ... Kf7 or 2 ... Kd7, White succeeds in playing h3–h4. For example:

(a) 2 ... Ke7 3 Ke3 Kd7 4 Kd3 Ke7 5 Ke4 Kd8 6 Kd4 Ke8 7 Ke5 Kd7 8 Kd5 Ke7 9 Ke6 (this is quicker than 9 Ke5 Kf7–109) 9 ... Kf6 10 Kd6 Kf7 11 Kd7 Kf8 12 Ke6 Ke8 13 f6 g6 14 f7+ Kf8 15 Kd6! (391) 15 ... K × f7 16 Kd7, winning.

(b) 2 ... Kf7 3 h4! (with the threat of 4 h5; with the king at e7 or d7 this does not work due to 3 ... g × h4 4 g5 Kd6 and 5 ... Ke5, but now the king does not have the e6 square) 3 ... g × h4 4 g5 Ke7 5 Kg4 Kd6 6 K × h4 Ke5 7 Kg4 Kd6 8 Kf4 and 9 g6 (131).

(c) 2 ... Kd7 (the king is now so far away that White has time for a necessary preparatory move) 3 Kg3! Ke7 4 h4 g × h4 5 K × b4 Kf6 (otherwise 6 Kg5) 6 Kh5 and wins.

A valuable analysis!

Najdorf-Kotov
Stockholm, 1948

513

513. 1 h4 Kf7.

Here the simplest win is by 2 Kd6 Kf8 3 Ke6 Ke8 4 Kf6 Kf8 5 f4 Kg8 6 Ke7 Kg7 7 h5! g × h5 8 f5 h4 9 f6+ Kg6 10 f7, but also possible is 2 f4 Ke7 3 h5! g × h5 4 f5 Kd7 4 ... h4 5 Kf4) 5 Kf4 Kd6 6 Kg3 Ke5 7 f6 Ke6 8 Kh4 h6 9 K × h5! h × g5 10 Kg6.

Najdorf failed to find the win, and the game ended in a draw!

Pillsbury-Mason
London, 1899

514

514. 1 Ke7 Kf6

(a) 2 f4 Ke6 3 g4 Kd6 and the king is far away.

(b) 2 f4 Ke6 3 g4 Kd6 and the king is far away.

(c) 2 f4 Ke6 3 g4 Kd6 and the king is far away.
514. 1 h5 Kg8?
A mistake. 1 ... g × h5 2 g × h5 Ke8! would have led to a draw.
2 Ke7 Kh8 3 Kf8 Kh7 4 Kf7 g × h5 5
g × h5 Kh8 6 Kg6! (6 h6 Kh7?!) 6 ... Kg8 7 h6,
and White won.

Vaganian-Tal
Moscow, 1977

515. The game went: 1 Ke4 Kf6 2 h4 Ke6
3 f3+ g × f3+ 4 Kd4 Kd6 5 h5 Ke6 6 f4 Kd6
7 h6.

In this way White succeeds in winning a pawn, but ... after 7 ... Ke6 8 Kc5 Ke7! 9
Kd5 Kd7 the players agreed a draw, since the extra pawn does not lead to a win.

But a reader of the magazine Shakhmaty v
SSSR showed that White failed to win, only
due to his incorrect plan. He should not have
taken the opposition (that opposition again!),
but threatened a by-pass.

Correct was 1 Kg4! Kf6 2 h4 h6 3 f3! h5+
(3 ... Ke6 4 f5 +! g × f5 + 5 K × h5) 4 Kh3!,
and then as in example 508. If instead 2 ... Ke6 3
Kg5 Kf7, then 4 f5 g × f5 5 K × f5 Ke7 6 f4
Kf7 7 Ke5 Kf7 8 h5!, and White wins (230).

516. Black loses due to the weakness of his
f-pawn: 1 Ke2 Kf6 2 Kd3 Ke5 (2 ... Kg5 3
Kc4 Kh4 4 K × f4 Kh3 5 Ke3! K × h2 6 f4,
with a win in the queen ending) 3 Ke4 h4!
(threatening after ... h3 to create a stalemate
shelter at h4) 4 h3! Ke6 5 Kd4 Kf5 6 Kd5 Kg5

517. 1 Ke5! (1 Kf7? g6! 2 Kg7 Kh4! —
draw) 1 ... Kh4! 2 g5! K × g5 3 Ke4 Kh5! 4
Kf3! (4 Kf4 Kh4 5 Kf3 g5 — draw) 4 ... Kh4
(4 ... g6 5 K × g3 g5 6 Kf2! Kh4 7 Kf3 Kh5 8
Kg3 etc.) 5 Kf4 g6 6 Ke3 Kg5 7 Kf3 Kh4 8 Kf4
g5 + 9 Kf3, winning.
518. White played 1 g×f6? (evidently hoping for 1 … g×f6 2 Kg4 Ke4 3 Kh5 K×f5 4 Kh6 Kg4 5 K×h7 f5 6 Kg6 with a draw), but on noticing a mistake in this variation, he resigned two moves later.

Some commentators stated that instead of 4 … Kg4 the only correct continuation was 4 … Ke6! 5 K×h7 f5 with a win in the queen ending, but 4 … Kg4! 5 K×h7 is in fact the simplest, and now not 5 … f5, but 5 … Kh5!, winning.

Instead of 1 g×f6?, Maroczy showed that White could have drawn by 1 Kg4! Ke4 2 g6! h6 3 Kh5! K×f5—stalemate.

519. 1 b4! (fixing the ‘hole’ at c5) 1 … c5 (after 1 … c6 a by-pass and the reserve tempo prove decisive: 2 d4 Kd7 3 Kf6 Kd6 4 Kf7! Kd7 5 c3 etc.) 2 d4! c×d4 (or 2 … c×b4) 3 Kf6, winning.

Kottmayer-Thomas
London, 1947

520. Black’s advantage is obvious: 1 …
g5! 2 Kf1 h5 3 Kg1 f6.

Excessively slow. Since 3 … h4 did not work immediately due to 4 g4, the logical move was 3 … f5!, and now either 4 h4 g×h4 5 g×h4 Ke2! 6 Kg2 f4 7 Kg1 f3, or 4 Kf1 h4 5 g×h4 g×h4 etc.

4 Kf1 f5 5 h4 g×h4 6 g×h4 Kg4! (the previous winning method is no longer possible) 7 Kg2 K×h4 8 Kf3 Kg5? (8 … Kh3! 9 Kf4 Kg2 would still have won) 9 Kg3 f4 + (9 … Kg6 10 f4) 10 Kh3. Drawn.

The defensive resources in such positions are shown by the following two examples.

521. Things seem to be bad for Black: the opponent’s king will break into his position and win a pawn. But after 1 … Kb4 2 Kd3 Ke5 3 Ke4 Kd6 4 Kf5 Ke7 5 Kg6 Ke6 6 K×h6 the clever 6 … Kd6! enables Black to maintain control of the key squares f5, f7 and f8. For example: 7 Kh5 Ke5! 8 h4 g×h4 9 K×h4 Kf4, or 7 Kh7 Kd7! 8 Kh8 Kd8! 9 Kg8 Ke8! 10 Kg7 Ke7 11 Kg6 Ke6.
Averbakh, 1983

521

[Image]

Black's system of defence is very simple—maintaining the distant and close opposition.

Honfi-Tröger
Hamburg, 1965

522

[Image]

522. 1... h5 2 Kf1 f5 3 Kg1 (if 3 Ke1, then 3... f4 4 Kf1 f3 5 Ke1 Kc2 etc.) 3... Ke2 4 Kg2 g6 5 g4!

The last chance! After the passive 5 Kg1 Kf3 6 Kf1 f4 Black wins without any difficulty.

5... f×g4?

A mistake, after which Black has no chance of winning. Correct was 5... h×g4! 6 Kg3 Kd3! (6... Kf1 7 f3! g×f3 8 K×f3 with a draw) 7 Kf4 (7 f3 Ke3! 8 f×g4 f4+) 7... Kd4!, and Black wins, e.g. 8 Kg5 (8 Kg3 Ke4, or 8 f3 g3! 9 K×g3 Ke3) 8... Ke4! 9 K×g6 f4 10 h5 g3 11 f×g3 f×g3 12 h6 g2 13 h7 g1=Q + 14 Kf7 Qd8 15 Kg8 Qd8+ 16 Kg7 Qe7+ 17 Kg8 Kf5! 18 h8=Q Kg6, winning.

6 Kg3 Ke1.

On 6... Kd3 there follows 7 f3 with a draw, whereas now 7 f3 loses to 7... g×f3 K×f3 Kf1.

7 Kg2 Kd2 8 Kg3 Kc2 9 Kf4 Kd2 10 Kf3 with a draw.

Mattison, 1929

523

[Image]

523. It can be assumed that the black king has had to eliminate a pawn at a4. Nevertheless, if the opponent delays Black will be able to bring his king back, seriously hindering the white king. White fails to win, for example, after 1 Kf3? Kb5 2 Ke4 Kc6, or 1 Kg4? Kb5 2 h4 Kc5 3 h5 g×h5+! 4 K×h5 Kd5 5 Kg6 Ke4 6 g3 h5!! 7 K×g7 (7 K×h5 Kf5 8 Kh4 g6 9 Kh3 g5) 7... Kf5 8 Kh6 Kg4 9 f5 h4! 10 g×h4 K×f5.

Correct is 1 f5! Kb5 2 Kf4! Kc6 3 Ke5 Kd7 4 f6 Ke8 5 f×g7 (5 Ke6 Kf8 6 g3 or 6 h3 is also possible—Pospisil, 1956) 5... Kf7 6 g8=Q + K×g8 7 Kf6, winning (364).

524. White rapidly approaches g5 with his king, keeping all three pawn tempi in reserve. There is no other way to win, e.g. 1 f4? Kg7 2
Kf3 h4! 3 g × h4 Kh6!, and against the attempt at a by-pass Black replies by eliminating the h4 and f4 pawns; 1 Kf2? Kg7 2 Ke3 Kf6 (now White gains no advantage by 3 Kf4 g5+ 4 Ke3 Ke5, or 3 f4 g5! 4 Kd4 h4!, or 4 h4 g × h4 5 g × h4 Ke6) 3 Kd4 Kg5! (not 3 ... Ke6? 4 f4 with a by-pass thanks to the reserve tempi, or 3 ... g5? 4 Kd5) 4 h4+(4 Ke5 h4!) 4 ... Kf6 5 f4 (5 Kd5? f4) 5 ... Ke6 6 Ke5 Ke7!, and Black maintains the opposition.

Correct is 1 Kh3! Kg7.
1 ... g5 achieves nothing due to 2 f4! g × f4 3 g × f4 Kg7, and now 'à la Bondarevsky' 4 Kh4! Kg6 5 h3 Kh6 6 Kg3 h4+ (or 6 ... Kg6) 7 Kh3!, winning. It should be noted that after 4 ... Kh6 White wins by 5 Kg3 Kg6 (5 ... h4+ 6 K × h4−243) 6 Kf3 h4 7 Ke3!, and if 7 ... Kh5 8 h3!, or 7 ... h3 8 Kf3! etc.

2 Kh4! Kf6 (if 2 ... f4, then not 3 g4? Kh6, but 3 g × f4 followed by the transfer of the king to e4−508) 3 f4 Ke7 (now White exploits his remaining pawn tempi) 4 Kg5 Kg7 5 h3! Kf7 6 Kh6 Kf6 7 h4, and wins.

525. The solution is similar: 1 Ke2 Kd7 2 Kf3 (2 Ke3? g5! 3 h4 f4+ 4 g × f4 g × h4) 2 ... Ke6 (2 ... g5 3 h4) 3 Kf4 Kf6 4 h4 etc.

526. Here White has only two reserve tempi, so that the advance of the king to h4 is doomed to failure. But on the other hand, with the black pawns blocked there is a possibility of a by-pass and the seizure of the opposition on the main rank (the 7th, since the key points are g6, g7 and g8).

1 Kf2 Kf7 2 Ke3 Ke6 (2 ... Kf6 3 Kd4 g5 4 h4!−519) 3 Kd4 Kd6 4 h3! Ke6 5 Kc5 Ke7 6 Ke6!

Only a draw results from 6 Kd5 Kd7 7 Ke5 Ke7 8 h4 Kf7 9 Kd6 Kf6 or 9 Kd5 Ke7. The last tempo is needed for seizing the opposition on the main rank.
6...Ke6 7 h4! Ke7 8 Ke7! Ke6 (8...Ke8 9 Kd6) 9 Kd8, and now a possible conclusion is 9...Kd5 10 Ke7 Ke4 11 Kf6 Kf3 12 K×g6 K×g3 (12...Kg4 13 Kf6) 13 Kg5! or 9...Kf7 10 Kd7 Kf8 11 Ke6 Kg7 12 Ke7 Kg8 13 Kf6 Kh7 14 Kf7 Kh6 15 Kg8, winning.

Grigoriev, 1938

527. Grigoriev died on 10th October 1938, and this study was printed in the October 1938 issue of the magazine Shakhmaty v SSR. Its solution was only published after the composer’s death, in 1939. The published solution was as follows: 1 a4 Kb7 2 Kg5 Kc7 3 Kf6 Kd6 4 b3! Kd7 5 Kf7 Kd6 6 Ke8!, winning as in example 526. However, this solution is incorrect: instead of the erroneous 3...Kd6?, Black maintains the balance by 3...Kd7! 4 Kf7 Kd6, when there is no way for White to exploit his spare tempo.

And even so, White can win. The correct continuation is 2 Kh5!! (instead of 2 Kg5), e.g. 2...Kc6 (if 2...Kc7 3 Kg6, or 2...Kc8 3 Kg5) 3 Kg5! Ke7 4 Kg6 5 b3! Kc7 6 Kg7 Kc6 7 Kf8 Kd7 8 Kf7 Kd6 9 Ke8 etc. Thus this Grigoriev gem continues to live!

528. White wrests the opposition, thanks to the enemy king’s lack of manoeuvrability on the edge of the board.

1 Ke7 Kg7 2 Ke6 Kh7 3 Kf7 Kh6 4 Kg8 g5 5 Kf7 g×f4 6 Kf6 and wins (cf. also 552).

529. This position was reached in a consultation game between the Riga Chess Society and a Polytechnical School (Riga, 1892). Here we have the same pawn formation as in example 526, but the placing of the kings is more favourable for the weaker side, which in this case is White.

White lost after 1 Ke2? (1 h5? is also insufficient due to 1...g×h5 2 Kg2 h4?) 1...Kc4 2 Ke3 (other moves also fail to draw) 2...Kc3 3 Ke2 Kc2 4 Kc3 Kd1 5 Kf3 h5!

But it was shown by Behting that White could have drawn by 1 Kg2!, e.g. 1...Kc4 2 h5! g×h5 3 Kb3 Kd4 4 Kh4 Ke4 5 K×h5 Kf3 6 Kg5 (6 Kh4 Ke4 7 Kg5 Kf3) 6...K×g3 7 K×f5. It should be added that after 1 Kg2! it is hopeless to play 1...h5 in view of 2 Kf2!, when White has the opposition.
530. An instructive position. In spite of the symmetric arrangement of the pawns, taking the opposition by 1 Ke8 does not achieve anything, since Black is not forced to continue 1...a5? 2 a4. After 1...Kc7 2 Ke7 he has the tactical resource 2...b5!, creating the possibility of a stalemate (518). For example: 3 Ke6 b4 4 a4 Kb6 etc.; if instead 4 a×b4 c×b4 5 Kd5, Black draws by 5...a5! 6 Kc5 a4. The move...b5 (after 1 Ke8 Ke7) cannot be prevented by 2 a4: all the same Black plays 2...b5 (3 a5 b×c4 4 b×c4 Kd6; 3 Ke7 Kb6 4 Kd6 b4).

Therefore: 1 a4! Kd7.

Now 1...b5 would merely weaken irrevocably the c-pawn (2 a5 Kd7 3 Kf7).

Now 2 a5 is forced (2 Kf7? a5!), but at the same time it is the basic solution.

2 a5! Kd6!

On 2...b5 or 2...b×a5 there follows 3 Kf7, while if 2...Ke6(d8), then 3 a×b6 and 4 b7, as in the main variation.

Now 3 a×b6 Kc6 4 Ke7 K×b6 5 Kd6 is premature due to 5...Ka5! (again stalemate).

3 Kf7 Kd7.

Or 3...Ke5 4 a×b6, while 3...b×a5 is also hopeless due to 4 Kf6 Kc6 (4...a4 5 b×a4 a5 6 Kf5) 5 Kf6 Kb6 6 Kd6 a4 7 b×a4 Ka5 8 K×c5, and White queens first.

4 Kf6 Kd6 5 Kf5 Kc7.

Relatively best. If 5...Kc7, then 6 a×b6! Kd6 7 b7! Kc7 8 Ke6 K×b7 9 Kd7, as in the main variation, or 5...Kd7 6 Ke5 Kc6 7 Ke5 b5 (7...Kc7 8 Kd5 Kb7 9 Kd6) 8 Ke5 Kc7 9 Kd5 b×c4 10 b×c4 and 11 K×c5 (224).

6 Ke6! (6 Ke5 is a loss of time in view of 6...Kd7) 6...Kc6 7 Ke7 Kc7 8 Ke8! Kc8.

If 8...Kc6 (8...Kd6 9 a×b6!), then 9 Kd8! b×a5 10 Ke7! Kc7 11 Ke6 Kc6 12 Ke5 Kc7 13 Kd5 Kb6 14 Kd6 Kd7 15 K×c5 Kc7 16 Kd5 Kd7 17 c5 Kc7 18 c6 and wins (227).

9 a×b6! Kb7 10 Kd7 K×b6 (10...a5 11 Kd6 K×b6 12 Kd5) 11 Ke8!

The decisive zugzwang! With White to move there is no win, e.g. Kb8 a5! or Kd7 Ka5.

11...Kc6 (or 11...Ka5 12 Kd7 Kb4 13 K×a6 etc.) 12 Kb8!, and White wins: 12...Kb6 13 Ka8 a5 14 Kb8 a4 15 b×a4 Ka5 16 Kb7(c7) K×a4 17 Ke6 Kb4 18 Kd5.

531. Here the a3, c3 pawns could also be at a2, c2. It is in White's interests to block the enemy pawns since his king can easily gain the opposition on the 7th rank (the black king is hindered by its own pawn at b6). He must also prevent...c4, which could lead to a draw.

1 a4! Kb7.

Black threatens to create a stalemate shel-
Endings without Passed Pawns

ter at a5 by ... b5-b4. In addition (in reply, say, to 2 Kg5), 2 ... a5 is also possible with the threat of ... a4, when if 3 a4 Kc7!, and Black has the opposition (529).

2 a4! Ke6.

The black pawns are paralysed: after 2 ... h5 3 a5 the c5 pawn is lost, while 2 ... a5 3 Kh6! gains White the opposition, since the opponent does not have 3 ... Kb6.

3 Kg5! (of course, not 3 Kg6? a5—draw) 3 ... Kc7.

No better is 3 ... Kd6 4 Kf5! or 3 ... Kd7 4 Kf6! Kd6 5 a5! (530), but not 4 a5? in view of 4 ... Ke6! 5 Kf4 b x a5 6 Kc4 a4 7 b x a4 a5 with a draw.

4 Kf6!

Not 4 Kf5? a5! with a draw, since White can no longer gain the opposition on the 7th rank.

4 ... Kd7 (with the threat of meeting 5 Ke5 with 5 ... a5, drawing) 5 a5!, and White has achieved a winning position, as in example 530 after White's 4th move.

Both these examples (530 and 531) illustrate how the black pawns are weakened by the pawn being at a5 instead of a7.

Maizelis, 1955

533. After 1 ... Ke7 2 Kc6 or 1 ... Kd7 2 Kd5 etc. the reserve tempo 3 g3 proves decisive.

But if it is White to move, the position is drawn, e.g. 1 Kc6 Ke7 2 Kc7 Ke6 3 Kd8 Kd5 4 Ke7 (a by-pass is futile—the f4 pawn is not defended!) 4 ... Ke4 5 Kf6! (5 g3? loses) 5 ... K x f4 6 K x g6 Kg4 7 g3 K x g3 8 K x f5 K x h4 8 Kf4. Draw.

534. White's insignificant positional advantage proves insufficient for a win: 1 h3 Kd7 2 h4 Ke7 3 f3 Kd7 4 Kf4! (White's plan is to
place his king at g5 and play h5–h6) 4 ... Ke7
5 Kg5 Kf7.

Averbakh-Karpov
Riga, 1970

It is dangerous to play 5 ... Kd6 6 h5 Ke5
7 h6 g6 (7 ... g × h6 + 8 K × h6 Kd4 9 K × h7
Ke3 10 g4 Kf4 11 Kh6 e5 12 g5 K × f3 13 g6,
winning) 8 f4 + Ke4 9 Kf6 Kf3 10 Kg7
K × g3 11 K × h7, and White wins.
6 g4 Ke7 7 h5 Kf7.

Just how safe is Black’s position is shown
by the variation 7 ... h6 +?! 8 Kg6 Kf8 8
Kh7 Kf7 10 f4 Kf6! (10 ... Kf8? 11 g5 h × g5
12 f × g5 e5 13 h6 g × h6 14 g6, winning) 11
Kg8 g5 12 h × g6 K × g6 12 Kf8 h5, with a
draw.
8 h6 g6 (not 8 ... g × h6 + 9 K × h6 Kg8 10
g5 Kh8 11 g6) 9 Kf4 Kf6 10 g5 + Kf7 11 Ke5
Ke7 12 f4 Kf7 13 Kd6 Kf8. Drawn.

Doubled pawns are normally a serious
weakening, but to disclose this, other factors
favouring the stronger side are necessary.

535. Here White succeeded in carrying out
a by-pass with his king: 1 ... Kd7 2 Kb7! Kd6
3 Ke8! Kd5 4 Kd7 Ke4 5 Ke6 Kf3 6 K × f6
K × g3 7 Kg5, and White won.

It is readily apparent, that, if the f6 pawn
had been at g6, White’s manoeuvre would
not have been successful.

Averbakh-Kogan
Moscow, 1974

Goldenov, 1967

536. Here White can quickly attack the
opponent’s pawns, but for which pawn
should he head with his king—h5 or f7? Let
us first consider the first path:
(a) 1 Kf4 Ke4 2 Kg5 Kd4 3 K × h5 K × e4
4 Kg5! Kf3 5 g4 Ke4!

This is stronger than 5 ... Kg3 6 h5 g6 + (6
... Kf3 7 Kf5) 7 Kf5 Kh4 8 Kf4 f6 9 Kf5 Kg3
10 K × f6 K × g4 11 Kg6, when White wins.

6 h5 Ke7 7 Kb6 f5!

An unexpected chance. If now 8 g × f5,
them 8 ... K × f5 9 K × h7 Kf6! with a draw.
The only possibility of playing for a win is 8 g5 f4 9 K×h7 f3 10 g6 f2 11 g7 f1=Q 12 g8=Q, leading to a queen ending with chances for White, but the result of it is far from clear.

Instead of 6 h5 White should play 6 Kf6 Kf4 7 h5 K×g4 8 h6 Kf4 9 Kg7 (9 K×f7 Kf5 10 Kg7 Ke6 with a draw), but then position 234 is reached, where Black is able to draw.

Correct is the second path, a highly instructive one, where Black does not gain any counter-chances:

(b) 1 Kf4 (1 Kd4 is also possible, followed by 2 Ke5 and 3 Kf6) 1...Ke4 2 Kf5 Kd4 3 e5 Kd5 4 Kf6 Ke4 5 K×f7 K×e5.

It is very important that the black king is at e5 (and not e4), and is not able to attack White’s pawns immediately.

6 Kg7 Kf5 7 K×h7 Kg4 8 Kg6 K×g3 9 K×h5, and White wins.

3...f4! was also possible, e.g. 4 e×f4 h4! 5 Kf2 f3 6 Kg2 Ke3 7 Kh2 Kf3 8 g×h4 K×f4, or 4 g×f4 h4 5 Kf2 (if 5 Kd2, then 5...Kf3 6 Kd3 Kg3 7 Ke2 K×h3 8 Kf3 f5, with a draw) 5...Kd3 6 Kf3 f5! 7 Kf2 Kd2, with a draw.

4 Kf3 Ke5 (after this White is forced to use up his last tempo) 5 h4 Kd5 6 Kf4 Ke6 (7 e4 f×e4 8 K×e4 f5+). Drawn.

Grigoriev (1935) showed that 1...Kd5? would have lost to 2 Kd2! Ke5 (2...h4 3 g×h4 f4 4 e×f4 Ke4 5 h5; 2...Ke4 3 Ke2! h4 4 g×h4 f4 5 h5! Kf5 6 e×f4 or 3...Kd5 4 Kf3 Ke5 5 h3! Kd5 6 Kf4 Ke6 7 h4, winning) 3 Ke1! Kd5 4 Kf2! Ke4 5 Ke2, and White wins.

Grigoriev also emphasized that, apart from 1...Kc5, 1...Kf7 was also possible, e.g. 2 Ke2 Kg6 3 Kf3 (3 Kf2 Kh6) 3...Kg5 4 h3 h4! 5 g×h4 K×h4 6 Kf4 (6 Kg2 f4 7 e×f4 f5--draw) 6...K×h3 7 K×f5 Kg3. Draw.

### Flohr-Capablanca

Microsoft, 1935

537. Here White is aiming to transfer his king to f4; a defensive resource available to Black is...h4, devaluing the white pawns. The decisive zugzwang position is Ke2/Kc4.

1...Ke5 2 Ke2 (in the event of 2 Kd2 Capablanca would have continued 2...h4! 3 g×h4 f4! 4 h5 f×e3+! with a draw) 2...Ke4! 3 h3 (3 Kf2 h4!) 3...Kd5.

Grigoriev, 1935

538. 1 Kd5 Kd7 2 h3 Ke7 3 Kc6 Ke6 4 h4 f4!! (Black loses after 4...Ke5! 5 Kd7 Ke4 6 Ke6 K×e3 7 K×f5! or 6...Kf3 7 K×f6! K×g3 8 K×f5 K×h4 9 Kf4! Kh3 10 e4 etc.) 5 g×f4 Kf5 6 Kd5 Kg4 7 Ke6 K×h4. Draw.

Also possible is 1 Ke5 Ke7 2 Ke6 Kg6 3 h3 (3 h4 has already been considered; if 3 Kc7, then 3...Ke5 4 Kd7 Ke4 5 Kd6 K×e3 6 K×f5 Kg3 7 K×f6 h4!—draw) 3...Ke5 4 Kd7 h4 5 g×h4 f4. Draw.
Three Pawns against Two or Three Pawns

It should be added that a draw also results from 1 e4 f×e4 2 K×e4 h4 3 g4 h3 4 Kg4 Kg7 5 Kg3 Kg6 6 K×h3 f5.

Dedrle, 1942

539. 'It has not been possible to find the composer's solution, but, taking account of Grigoriev's analysis of position 537, it can be assumed that there is no winning solution to Dedrle's study'.

These words were written by Muizelis in 1956. It was later established that Dedrle's solution to the position was as follows (the exclamation marks are his): 1 Kd4!! Kd6 2 Ke3!! Kd5 3 Kd2! Ke5 4 Ke1!! Kd5 5 Kf2!! Ke4 6 Ke2 Kd5 7 Kf3, and White wins.

It is not difficult to see that, on the basis of the previous position, Black can answer 1 Kd4 with 1 ... Ke7! 2 Ke5 Kd7 3 Kd5 Ke7 4 Kc6 Ke6, or 2 Kd3 Kf7? 3 Ke2 Kc6 4 Kf3 Kg5 5 h3 h4!, with a draw.

But, of course, 1 ... Kd6 also does not lose. On 2 Kc3 Black should not continue 2 ... Kd5? 3 Kd2! or 2 ... Ke5? 3 Kd3!, but 2 ... Ke6(e7)!, so as to answer 3 Kd3(d2) with 3 ... Kf7! 4 Ke2 Kd6 etc.

540. The aim of this instructional position was to illustrate the weakness of isolated pawns: 1 Ke4 Kd6 2 Kd4 c5 + 3 Kc4 Kc6 4 c3 Kb6 5 Kd5 c6 + 6 Kc4, and wins.

But A. Feinstein (Tallinn) showed that after 1 Ke4 Black can draw by 1 ... c5!, e.g. 2 Kd3 Kd5 3 Kc3 c6 4 Kd3 c4 + 5 b×c4 + Ke5! and 6 ... c5.

With the kings at Ke4/Kd6 the example is correct.

Pospisil, 1952

541. White has to reckon with the threat of ... f4, e.g. 1 Kd4? f4! 2 g×f4 Kh6 3 e4 Kh5 4 e5 Ke3 K×h4 6 e6 Kh3 7 e7 g2—draw.

But 1 Ke3? is insufficient in view of 1 ... Kh6! 2 Kf4 Kh5 3 e3 Kh6 4 Ke5 g5! 5 h×g5+ K×g5, with a draw.

1 e3! Kh6.
The best defence. If 1 \ldots Kg7, then 2 Kd4(c4) Kf6 3 Kd5 Ke7 (3 \ldots Kf7 4 Ke5 Ke7 5 e4 fxe4 6 Kxe4 Kf6 7 Kf4 Kg7 8 Kxg4 Kh6 9 Kf3 Kh5—9 \ldots g5 10 Kg4—10 Kf4 Kh6 11 Kg4, winning) 4 Ke5 Kf7 5 e4 g5! (the last chance) 6 h x g5! f x e4 7 K x e4 Kg6 8 Kf4 Kh5 9 g6 Kh6 10 Kf5 Kg7 11 Kg5, and White wins.

2 Kd4 Kh5 3 Kd5!

On 3 Ke5? there would have followed 3 \ldots g5 4 h x g5 K x g5! with a draw, whereas now 3 \ldots g5? loses to 4 h x g5 K x g5 5 Ke5 Kg6 6 Ke6 Kg5 7 Kf7 and 8 Kf6.

3 \ldots Kh6! 4 Ke6 Kh5! 5 Ke7!!

After 5 Kf6(f7)? Black is saved by 5 \ldots f4!! 6 e x f4 g5 7 f x g5 or 7 h x g5—stalemate.

5 \ldots Kh6.

Now 6 Kf7 would be a loss of time due to 6 \ldots Kh7, when White has to win back the opposition (528).

6 Kf8! Kh6! (with the aim of answering 7 Kg7? with 7 \ldots f4!) 7 Kg8! Kh6 8 Kh8! Kh5 9 Kh7, and White wins both after 9 \ldots f4 10 e x f4 g5 11 f5! g x h4 12 f6 h x g3 (or 12 \ldots h3) 13 f7 g2 14 f8=Q g1=Q 15 Qh6 mate, and after 9 \ldots g5 10 h x g5 f4 11 g x f4 g3 12 g6 g2 13 g7 g1=Q 14 g8=Q, when the checks can be avoided: 14 \ldots Qb1+ 15 Kh8 Qa1+ 16 Qg7 Qa8+ 17 Kh7 Qe4+ 18 Kg8! Qa8+! 19 Qf8 Qg2++ 20 Kh8! Qb2+ 21 Qg7 Qb8+ 22 Kh7 Qb1+ 23 Kg8 Qb8+ 24 Qf8 etc.

7.22 Separated pawns on both wings

542. Since only pawns can be moved, an assessment of who will first run out of moves is required well in advance.

Together with the three subsidiary positions (543—545), this example exhausts all conceivable pawn formations of two against two, when they stand on their initial squares.

The turn to move is a disadvantage here. On 1 c4 Black must reply 1 \ldots a5!, not allowing 2 b4 (which would allow white to make a breakthrough), then 2 c5 a4 or 2 b3 d6. If 1 c3, correct is 1 \ldots a6! 2 b3 d6. If 1 \ldots d5, then after 2 b3 a5 one of the white pawns can again break through to queen, albeit later than the opponent's (this is to be understood in a theoretical sense, since on the K-side a great variety of positions is possible; in the given case White would be mated at g4).

If it is Black to move, 1 \ldots a6 (1 \ldots a5 2 c4) is met by 2 c3 d6 3 b3 d5 4 b4, winning.

In the following three subsidiary positions the situation on the K-side is the same as in example 542, since in practice the kings are more often in zugzwang when there is a large number of pawns.

543. Here one must simply imitate the opponent's moves: 1 b3 b6! or 1 b4 b5!
544. The principle of the play is exactly the same: 1 e4 a5! or 1 c3 a6!

545. The connected pawns win, irrespective of who it is to move, with the b-pawn always moving first: 1 b4! or 1... c5 2 b3! etc.

If the distance between the pawns is less than 4 ranks, or if any of the pawns have been advanced from their initial squares, other results and exceptions of various types are possible, with an important role sometimes being played by a pawn (or pawns) retaining the right to a double move.

546. This position illustrates a typical procedure.

1 e6 + b x c6 + 2 Ke5! (2 Kc5? h4!) 2... h4 (this is better than the 2... Ke7 given by the composers, which is met not by 3 c5 h4, but by the more decisive 3 h4, with an easy win) 3 c5 Ke7 4 Kf5 Kf7 5 Kg5 Ke6 6 K x h4 Kd5 7 Kg5 K x c5 8 h4 Kd6! (the threat of... Ke7-f8 forces White to lose a tempo) 9 Kf6! c5 10 h5 c4 11 h6 c3 12 h7 c2 13 h8 = Q c1 = Q 14 Qd8+, winning.

547. 1 e5+ d x e5 + 2 Kg4! Kg6(e6) 3 e4 Kf6 4 Kh5 etc.

548. This position arose in a game from a junior tournament at Hastings (1949). Here the players agreed to a draw.

George Thomas pointed out a win: 1... e4+! 2 f x e4 + Ke5! (2... Ke5? 3 Ke3—draw) 3 Ke3 e5 etc.

190
The only move. In the event of 3 Kg2? Ke3 Black wins the f-pawn, after which his king heads for the a-pawn, without trying to pick up the h-pawn which would allow White to save the game.

3 ... Kg3 4 Ke3! K × h3 5 Kf2! Kh2 6 f4 h3 7 f5 Kh1 8 f6 h2. Drawn (now it is White who is short of a tempo for a win in the queen ending).

Salvioli, 1887

550. 1 g5 h × g5 (1 ... h5 also fails to draw) 2 h × g5 Kb5 3 Kd5 Kb6 4 Kc4 Ka5 5 Kc5 Ka4 6 Kb6 Ka3 7 Kb5 Kb2 8 K × b4 K × e2 9 Kc4, and wins.

Some interesting features are revealed in this simple ending if the positions of the kings are reversed.

Grigoriev, 1932

551. 1 ... f3?! A clever try. In the event of 1 ... Kd4? 2 Kf3 Kc4 3 K × f4 Kb4 4 g4! h × g4 5 K × g3 the white pawn queens first.

2 g × f4+ Kf4! (2 ... Kd3 is weaker in view of 3 f4 Ke4 4 f5 K × f5 5 Kf3, and Black is saved only by the fact that the h-pawn is outside the winning zone—209) 3 Ke2!
551. If the K-side pawns were blocked, White would win on condition that it was Black to move, but it is precisely this that he is unable to achieve.

1 g4! h4!

But not 1 ... h × g4? 2 h × g4 Kc5 (2 ... Kc3 3 Kb5) 3 Ka5 Kc4 4 Kb6, winning.

2 Ka5 Kc5! with a draw (but not 2 ... b3? 3 a × b3 = K × b3 4 Kb5 Ke3 5 Kc5 Kd3 6 Kd5 Ke3 7 Kc5 Kf3 8 Kf5 Kg3 9 K × g5 K × h3 10 Kf4!, with a win in the queen ending).

It is interesting that other moves lose for White, e.g. 1 h4? g4! 2 Ka5 b3, or 1 Ka5? g4! 2 h × g4 h × g4 3 Ka4 Kc3 and 4 ... b3.

If in the initial position White's king were at a5, all the same the result would be a draw: 1 g4! h × g4! (the only way) 2 h × g4 Kc5. (The composer's solution was not published, and the content of this position has not been fully covered in literature).

from a practical game, 1936

552. 1 ... Kd4! 2 Ka5 (2 Kb3 Kd3) 2 ... Kc3 3 Ka4 Kb2 4 g3 Ka2, and wins.

Although of a different type, the following positions are also of great practical interest.

553. 1 Kc2 Kd5 (1 ... Kb5 2 Kd3!) 2 Kb3! Kc6 (2 ... Kd6 3 Ka4! Kc6 4 Ka5) 3 Kc4 Kd6 4 Kb5 Kd5 5 Kb6! Kc4 6 Kc6 (the successive taking of the opposition at c2, c4 and c6 is amusing) 6 ... K × c3 7 K × c5, and White wins.

Black to move draws by 1 ... Kb5! (the only move) 2 Kb3 c4+ etc.

554. 1 Kb5 Ke7 (1 ... Kd8 2 d6, and either 2 ... Kc8 3 Ka6, or 2 ... Ke8 3 Kb6 Kf7 4 Kb7 etc.) 2 Kc5 Kd8 3 Kd6 Ke8 4 Kc7 Ke7 5 Kc8 Kd6 6 Kd8 K × d5 7 K × d7 Ke5 8 Ke7 Kf5 9 Kf7 Kg5 10 Ke6, and wins.

555. 1 Kf2 Kf5! (in the event of 1 ... f5 White occupies f4 with his king, having the g2–g3 tempo in reserve) 2 Kf3 Ke5! 3 g4
Botvinnik, 1945

555

\[ h \times g4 + 4 \, K \times g4 \, Ke4 \, 5 \, h5 \, f5 + 6 \, Kh3! \, f4 \, 7 \, h6 \, f3 \, 8 \, h7 \, f2 \, 9 \, Kg2 \, etc. \]

Botvinnik composed this study on the basis of an ending which occurred in a simultaneous display by him, although it is true that he also took account of a mistake he found in an analysis by Fine (1941).

Euwe, 1940

557

556

Knoch-van Scheltinga
Amsterdam, 1936

556

558

Grigoriev, 1938

556. 1 \ldots \, Kf5 \, 2 \, Kf3 \, Ke5 \, 3 \, Kg4 \, (3 \, h5? \, Kf5) \n3 \ldots \, Ke4 \, 4 \, h5 \, f5 + \, 5 \, Kg3 \, (a \, mistake; \, as \, shown \, by \, Botvinnik, \, 5 \, Kh3! \, was \, correct, \, 5 \ldots \, Ke3 \, 6 \, h6 \, f4 +. \, Drawn \, (the \, continuation \, 7 \, Kg4 \, f3 \, 8 \, h7 \, f2 \, 9 \, h8 = Q \, f1 = Q \, 10 \, Qe5 + \, Kd2!!

11 \, Qd4 + \, Kc2 \, 12 \, Q \times b4 \, was \, evidently \, also \, not \, taken \, into \, account \, by \, Fine). \n
Botvinnik accompanied his study with a joke: 'It will probably be demonstrated that the study is not original—in our time it is so difficult to devise something new in chess'. A response to this comment was made by Nedeljkovic (1951), who found the following example.

557. 1 \, a4 \, a5 \, 2 \, c4 \, c5 \, 3 \, b3 \, b6 \, 4 \, h4 \, Kf5 \, 5 \, Kf3 \, Ke5 \, 6 \, Kg4 \, Ke4 \, 7 \, h5 \, f5 + \, 8 \, Kh3! \, etc.

'I think that this manoeuvre must have occurred even earlier' writes Nedeljkovic. This supposition is correct (cf. for example Grigoriev's study 85, White's 4th move).
Three Pawns against Two or Three Pawns

558. For a long time this interesting and complicated position remained little-known, since its solution was not published. In order to understand it better, let us first examine the following, more simple ending.

Volpert-Byelova
1949

559.

\[
\begin{array}{cccccccc}
& a & b & c & d & e & f & g & h \\
8 & & & & & & & & \\
7 & & & & & & & & \\
6 & & & & & & & & \\
5 & & & & & & & & \\
4 & & & & & & & & \\
3 & & & & & & & & \\
2 & & & & & & & & \\
1 & & & & & & & & \\
\end{array}
\]

559. 1... h5+? proved to be a fatal mistake, in view of 2 Kf4 Kf6 3 h4. Romanovsky showed that Black should have played 1... Kf6 2 Kf4 Kg6 (or even 2... Ke6), or 2 h4 Kg6 3 h5+ Kf6 4 Kf4 Ke6 5 Ke4 Kf6, when the draw is obvious.

An analogous final position must obviously result in example 558, but with the difference that Black’s Q-side pawns are much weaker, and that to obtain a queen White requires considerably much less time.

It is clear that the white king must move into the centre of the board, maintaining the opposition. Therefore: 1 Kf2! Kf7!

Of course, not 1... Kg7? 2 Ke3, while if 1... Ke7 2 Ke3, and with the seizure of the opposition on the e-file, White’s problem is in general solved, e.g. 2... Ke6 3 Ke4 Kf6 4 Kd5, or 3... Kd6 4 Kd4!

2 Kf3 Kf6 3 Kf4 Kg6! (we have reached the most difficult point of the solution; 4 Ke5 appears to win, but this is not so, as will be seen later) 4 Kg4!! (White voluntarily creates this position, which to all appearances is hopelessly drawn!) 4... Kf6 (4... Kg7? loses immediately) 5 h4! Kg6 6 h5+ Kf6 7 Kf4 Kf6 8 Ke4 Ke6 (it is still not clear how White can win; both sides require an identical 8 moves to obtain a queen) 9 Kd5 Kg5 10 Kc5 K×h5 11 K×b5 Kg4 12 Kc4!!

Here is the solution: the white king has not only cleared the way for its pawn, but has also stepped into the ‘square’ of the h6 pawn.

12... h5 13 Kd3! Kf3 (otherwise the white king goes to f1; in the event of 13... Kg3 the king is checked from b8) 14 b5, and the queen at h1 will be won by a skewer.

One question remains to be clarified: why does not 4 Ke5? win in the same way? It is because there would have followed 4... Kg5 5 Kd5 K×h4 6 Kc5 K×h3 7 K×h5, and now not 7... Kg4? as in the main variation, and not 7... Kg2? in view of 8 K×a4 with a subsequent exchange of queens (by check along the diagonal), but 7... h5! 8 Kc4 h4!, when White is faced with an unpleasant choice: either 9 b5 Kg4!, or 9 Kd3 Kg2!—in both cases with a draw. Now it is clear why the pawn had to be driven to h5, denying Black the possibility of this counter-play.

One of the many Grigoriev gems!

Louma, 1939

560.

\[
\begin{array}{cccccccc}
& a & b & c & d & e & f & g & h \\
8 & & & & & & & & \\
7 & & & & & & & & \\
6 & & & & & & & & \\
5 & & & & & & & & \\
4 & & & & & & & & \\
3 & & & & & & & & \\
2 & & & & & & & & \\
1 & & & & & & & & \\
\end{array}
\]

560. As in example 558, the main file is the e-file (Black has weak pawns at b7 and h7).
Endings without Passed Pawns

The position on the Q-side is even more favourable for White.

1 Kf2! (beginning a by-pass and approach to the h7 pawn; at the same time ... Kc6-b5 can now be met by Ke3-d4) 1 ... Ke6 (1 ... Ke7 2 Ke3; 1 ... Kd6 2 Kf3! Kd5 3 Kf4 Ke5 4 Ke5 or 3 ... Kd6 4 Kg5) 2 Kg3! Kf7! 3 Kh4! Kg6 (3 ... Kf6 4 Kh5) 4 Kg4 h6 5 h3! Kf6 6 Kf4 Ke6 7 Ke4, winning.

But Black could have drawn by the subtle move suggested by Bondarevsky, 4 ... Ke6!, e.g. 5 Kf5 Kd5 6 Kg5 Ke4! 7 f4 h4, or 5 a5 h4 6 Kf4 Kd5 7 Kg4 Ke5! (7 ... Ke4? 8 a6 h3 9 Kg3!, winning) 8 a6 (8 f3 a6) 8 ... Ke4 9 f3+ Ke3 10 f4 h3. Draw.

After the incorrect 4 ... a5 there followed: 5 Kf5 Kd6 6 Kg5 Ke5 7 f3! Ke6 (7 ... h4 8 Kg4) 8 f4 h4 9 Kxh4 Kf6 10 Kg4 Kg6 11 Kf3 Kf6 12 Ke4 Ke6 13 Kd4!, and White won (the pawn is just inside the winning zone—209).

7.23 Scattered pawns

By this we imply that there are pawns both in the centre, and on the wings. Among them there may be two connected pawns, but most often all the pawns are isolated. With both sides having weaknesses, the strategic difficulty is usually presented by the correct choice of direction of the so-called 'main blow', since attacking one pawn means giving up another to the opponent.

Grigoriev, 1920

562. This is one of Grigoriev's early and little-known studies (the solution to which was not published).

Here White has the more active king and a slightly superior pawn formation.

In the 1st edition of this book, Maizelis gave his own solution to Grigoriev's study,
but it proved to be inaccurate. The correct path was indicated by Knyazev (Moscow).

1 b5!

Maizelis begins with 1 Ke4, but after 1 ... b6! 2 Kd4 b5 it is unlikely that White can realize his slight advantage, e.g. 3 Ke4 Kd7 4 Ke5 Ke7 5 f5 e×f5 6 K×f5 Kf7 7 Kg5 g6, with a draw.

Probably the best that White has is 2 b5, but after 2 ... Ke5 3 Ke5 K×b5 4 K×e6 Ka4 5 Kf7 b5 6 K×g7 b4 7 f5 b3 8 f6 b2 9 f7 b1 = Q 10 f8 = Q play transposes into a theoretically unclear queen ending.

1 ... b6 2 Ke4! Ke7 3 Kd3! Kf6.

With his great freedom for manoeuvring, White can always reach the important e5 square. If, for example, 3 ... Kd7, then 4 Ke4 Kd6 5 Kd4 Kd7 6 Ke5 Ke7, and now, as shown by Maizelis, after 7 f5 e×f5 8 K×f5 Kf7 9 Kg5 Kf6! (9 ... g6 10 Kh4 Kf6 11 Kh7! Kg5 12 Kf7) 10 Kf4! White breaks decisively into his opponent's position: 10 ... Ke8 11 Ke4 Kd8 12 Kf5! Ke7 13 Kg6 Kf8 14 Kh7 Kf7 15 g5, winning.

4 Ke4 g6.

4 ... Kf7 5 Ke5 Ke7 or 4 ... Ke7 5 Ke5 Kd7 leads after f4-f5 to situations already considered.

5 Kd4 Ke7 6 Ke5 Kf7 7 Kd6 Kf6 8 Ke6 g5 9 f×g5+ K×g5 10 K×b6 e5 11 Ke5 e4 12 Kd4 Kf4 13 b6, and White wins.

563. White wins the d4 pawn, but loses his b3 pawn; the d3 and b6 pawns are also exchanged (after promoting). Victory is secured by the h2 pawn.

1 Ke2 Ke7 2 Kf3 Kd6 3 b4! Kc6 (3 ... Ke5) 4 Ke4 Kb5 5 K×d4 K×b4 6 Ke5 b5 7 d4 Kc3 8 d5 b4 9 d6 b3 10 d7 b2 11 d8 = Q b1 = Q 12 Qd4+ Kc2 13 Qe4(b6)+ Kc1 14 Q×b1 + K×b1 15 Kf6! Kc2 16 Kg7? h5 17 Kg6 b4 18 Kg5 h3 19 Kg4 Kd3 20 K×h3 Ke4 21 Kg4, and wins.

Wahluch-Mitchell
Southport, 1905

564. Black has a passed pawn, but it will be either exchanged for the c-pawn or blocked: 1 ... Kd6.

If 1 ... Ke5, then 2 Ke3 g5 3 h3 Kf5 4 a3!! Ke5 5 a4 Kf5 6 c5! b×c5 7 a5 and wins. But 4 a2? leads only to a draw in view of 4 ... Ke5 5 c5 b×c5 6 a5 Kd5 7 a6 Kc6 8 K×e4 Kb6 9 Kd5 K×a6 10 K×c5 Kb7.

2 Ke3 Ke5 (2 ... Ke5 3 a4! Kf5 4 c5!!) 3 K×e4 K×e4 4 h4! b5 5 Ke4 b4 6 Kd6 (6 Kf6? Kc3 7 K×g6 Kd2 8 h5 K×a2 9 h6 b3—draw) 6 ... Kd5 7 Kd6 Kc4 8 Kc4, and wins (550).

565. 1 Ke5 K×f3 (1 ... K×h4 2 K×e6 K×h5 3 Kd7) 2 K×e6 Kg4 3 h6!! (3 Kd6? K×h5 4 Kc6 K×h4—draw) 3 ... g×h6 4
Kd6, and neither 4 ... K×h4 nor 4 ... Kf5 saves the draw.

566. 1 Kf2 Ke7 2 Ke3 Kf6 (after 2 ... Ke6
3 K×d3 Kd5 4 Ke3 Kc4 the win is achieved as in example 241) 3 Kd4! (3 K×d3? Kg6) 3
... Kg6 4 Ke5 K×h6 5 Kf6 Kh5 6 K×f5 Kh6 7
Kf6 Kh5 8 f5 Kg4 9 Ke6, and wins.

567. 1 Ke8!

Now there are three possibilities:

(a) 1 ... Ke6 2 Kd8 Kd6 3 Ke8 Kc6 4 Kb8
Kb5 5 K×b7 K×a5 6 Kc6 Ka6 7 Kd5 Kb6 8
K×e4, and the king penetrates to f6.

(b) 1 ... Ke6 2 Ke7! Ke5 3 Ke6 Kc4 4 Ke5

Kb5 5 Kf6! K×a5 6 Kg7 b5 7 K×h7 b4 8 Kg7!
b3 9 h7 b2 10 h8 = Q b1 = Q 11 Qa8 +, and
wins.

(c) 1 ... Ke5 2 Kd7! Kf4 3 Kd6! Kg4 4 Ke5
Kg5 5 K×e4 K×h6 6 Kf5 Kg7 7 Ke6! h6 8 f3!
h5 9 Kf5 Kb6 10 f4!, and wins.

Tattersall, 1910

568. 1 e4! f3 2 c5!! d×c5 3 Ke3, and wins.
Black also fails to save the game after 1 ...
f×e3 2 K×e3 Kf6! 3 Kd4! Ke7 4 Ke3, when
White wins by a by-pass (146), or 1 ... Kf6 2
Ke2 Kg6 3 Kf3 Kg5 4 c5! etc.

In the position moved to the left, the by-
pass is not possible, and therefore it is a draw
(144).
8. Endings with Several Pawns (material advantage)

In endings with several pawns, with only very rare exceptions an advantage of one pawn is sufficient for a win. The realization of such an advantage does not normally present any difficulty. If the opponent's king succeeds in stopping this pawn, then, exploiting the fact that it has been diverted, the stronger side breaks through with his king to the enemy pawns on the opposite wing and gains there a decisive material advantage.

Before setting up a passed pawn, it is normally useful to improve the position of one's own king, so as to advance the pawn most effectively towards the queening square, without allowing the opponent any chances.

As an example of the realization of a passed pawn, let us consider the following position.

569. 1 Kf1 (the king must be brought into play) 1... Ke7 2 Ke2 Kd6 3 b4 (or 3 Kd3 Kc5 4 Kc3 and 5 b4) 3... Kd5 4 Kd3 f5 5 f4 g6 6 g3 a6 7 a4 Kc6.

The king cannot be maintained in its active position—Black has run out of moves, and he has to retreat.

8 Kd4 Kd6 9 b5 a x b5 10 a x b5 Kc7 11 Ke5.

The simplest. White gives up his passed pawn, but gains a decisive material advantage on the opposite wing. The alternative was 11 Kc5 Kb7 12 b6 Kba 13 Kb6 Kc6 14 b7+ Kb8 15 Kb6 h6 (15... h5 16 Kd6) 16 h4 g5 (a desperate attempt to play for stalemate) 17 h x g5 h x g5 18 f x g5 f4 19 g6 f3 20 g7 f2 21 g8=Q mate.

11... Kb6 12 Kf6 K x b5 13 Kg7 Kc4 14 K x h7 Kd4 15 K x g6, and White wins.

The reader should note how easily White carried out his winning plan. Of course, to a certain extent this example was an ideal one, but it can safely be said that in the overwhelming majority of pawn endings a material advantage can be realized without difficulty. But even so, it should not be thought that an extra pawn in a pawn ending guarantees victory, or that the win is achieved automatically.

As we have established, the winning plan consists of three stages: the creation of a passed pawn, supporting it with the king, and the breakthrough of the king to the enemy pawns. At each of these stages problems may arise, associated with this or that feature of the position. It may, for example, prove difficult to create a passed pawn due to some defect in the pawn formation (backward pawn, doubled pawns etc.), or it may be difficult for the king to support it, since it is tied up with other duties (stopping an enemy passed pawn, defending invasion squares etc.). Finally, it may happen that it is alto-
gether difficult for the king to penetrate into the enemy position. In the following examples we will show how these difficulties are overcome.

570. After 1 Kf2 Kf4 Black tries to prevent the creation of a passed pawn, but White easily overcomes this barrier by a temporary pawn sacrifice: 2 g3+! h×g3+ 3 Kg2 Ke5 4 K×g3 Kf5 (setting up a second line of defence, but it too can be overcome) 5 f4! g×f4+ 6 Kf3.

By his pawn sacrifice White has set up an outside passed pawn, and now, after the enforced exchange of the K-side pawns, what proves decisive is the fact that his king is much closer than the opponent's king to the Q-side pawns.

6... Kg5 7 h4+ (or 7 Ke4 Kh4 8 K×f4, and White wins).

In the following example White successfully combines an undermining of the enemy pawns with the threat of a breakthrough.

571. White first carries out the undermining: 1 e4+! d×e4+ 2 Ke3 Kf6 (if 2... Kf6, the breakthrough 3 d5 c×d5 4 c6 is decisive) 3 K×e4 Ke7 4 Ke5.

4 d5 also wins, but the text move is simpler.

4... Kd7 5 Kf6 Kd8 6 Ke6 Kc8 (6... Ke8 7 d5 c×d5 8 c6) 7 Ke7 Kb8 8 Kd7 Ka8 9 d5 c×d5 10 c6 b×c6 11 Kc7, and White gives mate in three moves.

Note that if this position is moved one file to the left, the assessment changes—Black easily draws by taking his king into the a8 corner.

Sometimes the opponent's counter-play may force the stronger side (before setting up a passed pawn) to resort to very subtle manoeuvring, to suppress this counter-play.

Averbakh-Bachtiar
Djakarta, 1979

572. How is White to realize his advantage? If, for example, 1 b4 a×b4+ 2 Kb3, Black launches an attack on the K-side pawns: 2... Kd4 3 K×b4 Ke3 4 Kd5 Kf3 5 K×b6 K×g3 6 a5 K×f4 7 a6 g3 8 a7 g2, and
the queens appear simultaneously. 4 c5 (instead of 4 Kb5) 4 ... bxc5+ 5 Kxc5 gains an important tempo, but after 5 ... Kf3 6 a5 Kxg3 7 a6 Kh2 8 a7 g3 9 a8=Q g2 10 Qa2 Kh1 11 Qd5 Kh2 no win is apparent.

However, the game lasted only another three moves: 1 Kd3! Ke6 (totally bad is 1 ... Kb4 2 Kc2 Ke5 3 Ke3 and the inevitable b3–b4) 2 Kd2! Kd6 3 Ke2! and Black resigned, since after 3 ... Kc6 4 Kd3 Ke5 5 Ke3 Kd6 6 b4 a x b4 + 7 Kx b4 Kc6 there follows 8 c5 bxc5 + 9 Kc4 etc.

It is easy to establish that we have here a case of corresponding squares: to c3 the corresponding square is c5, d4–d6, d3–c6, d2–d6 (again), but to c2 the black king has no corresponding square—the familiar 'triangulation'. And if the black king retreats to the 7th rank. White plays his king to c3 and follows up with b3–b4.

Analysis has shown that White also had another, albeit more complicated path, nevertheless beginning with 1 b4: 1 ... a x b4 + 2 Kb3 Kd4 3 K x b4 Ke3, and now not 4 c5, but 4 a5! b x a5 + 5 K x a5 Kf3 6 c5 K x g3 7 c6 Kh2 8 c7 g3 9 c8=Q g2 10 Qc2! Kh1 11 Q x f5! g1 = Q 12 Qh3 + Qh2 13 Q x h2 + K x h2 14 f5 etc.

573. If there were no pawns on the Q-side, White would easily be able to carry out the undermining move g2–g3 and win as in position 223. But here Black has counter-play associated with ... a4 and the threat of an attack by his king on the Q-side pawns. This complicates White's task, but not to the extent that he is unable to win.

1 ... Ke5 2 a4! (on 2 Kf2 there could have followed 2 ... a4 3 g4 f x g3 + 4 K x g3 Kd4 with dangerous counter-play) 2 ... Kd4 3 Kd2 Ke5 4 Kd3! Kf5 5 Ke2 Ke5 6 Kf2 Kd4 7 g4 f x g3 + 8 K x g3 Ke5 9 Kg2! Kd4 (if 9 ... Kf5 10 Kf1!!) 10 f4 Resigns.

Taimanov shows that 1 ... a4 would have been more tenacious, when White wins by an instructive variation: 2 b x a4! Ke5 (2 ... Kf5 3 Kd3 Ke5 4 Ke3 Kd6 5 Kb3 Kc6 6 a5 b x a5 7 Ka4 Kb6 8 a3! Ka6 9 K b3 K b6 10 Kc3 Kc6 11 Kd3 Kd6 12 Ke4, and the battle for the invasion squares is won) 3 Kd3 Kf5 4 Ke2 Ke5 5 Kc3! (in moving to the Q-side, White simultaneously defends the invasion squares) 5 ... Kf5 6 Kb3 Ke5 7 a5! b x a5 8 Ka4 Kd4 9 Kb5 a4 10 a3! Kc3 11 K x c5 Kb3 12 Kd5 K x a3 13 c5 Kb4 14 c6 a3 15 c7 a2 16 c8 = Q a1 = Q 17 Qc5 + Kb3 Qc4 + , exchanging queens with an easy win.

Taimanov-Barden
London, 1954

Moiseyev-Elbekov
Tbilisi, 1951
Black’s plan is clear: to place his king at d7 and play ... c5, creating a passed pawn. White can counter this only by an attack on the h7 pawn: 1 Kf4.

1 Kf3 is more cunning, and if 1 ... Kf6, then 2 Kf4 Ke7 3 Kg5 Kd7 4 Kh6 c5 5 d × c6+ K × c6 6 K × h7 d5 7 h5 d4 8 h6 d3 9 Kg6 d2 10 h7 d1 = Q 11 h8 = Q with drawing chances. But Black could play the more accurate 1 ... Kf7!, transposing into the game continuation.

1 ... Kf6 2 Ke4 Ke7 3 Kf5 Kd7 4 h5 c5 5 d × c6+ K × c6.

The assessment of this position provoked a discussion on the pages of the magazine Shakhmaty v SSSR. The game continued as follows: 6 h6 Kd7 7 Kf6 d5 8 Kg7 d4 9 K × h7 d3 10 Kg8 d2 11 h7 d1 = Q 12 h8 = Q Qg4+ 13 Kf7 Qe6+, and White resigned, since the exchange of queens is inevitable.

Instead of 6 h6, A. Zamikhovsky recommended 6 Ke6 h6 7 Kf5 as being more tenacious, but after 7 ... Kd5 8 Kg6 Kc4 this leads to a favourable queen ending for Black. (Note that White also fails to save the game by 6 Ke4 h6 7 Kd4 Kc7 8 Ke4 Kd8! 9 Kf5 Kc7 10 Ke4 Ke6 11 Kd4 d5 12 Ke5 Ke5 etc.).

I. Asaritis remarked that it wasn’t 6 h6 that was a mistake, but only 8 Kg7, and that instead of this White should have continued 8 Ke5! Kc6 9 Kf4! Kd6 10 Kf5 with a draw, e.g. 10 ... d4 11 Ke4 d3 12 K × d3 Kd5 13 Kc3 Ke4 14 Kc2! etc.

But analysis shows that Black’s 6th move was also wrong. Correct was 6 ... Kd5 7 Kf6 Kc4 8 Kg7 d5 9 K × h7 d4 10 Kg6 d3 11 h7 d2 12 h8 = Q d1 = Q, with the same favourable queen ending.

As shown by L. Knyazev (1983), a precise assessment of this ending depends on whether Black can overcome his opponent’s defences after 13 Qb2!, but a detailed analysis here would take us far off the main path.

But what if there were no pawns on the h-file—the source of White’s counter-play?

575. Even here Black encounters certain difficulties over realizing his advantage. The following is a possible continuation: 1 Kf3!

Kf7 2 Kg3!

White makes the only moves, all others losing quickly, e.g. 2 Ke4 Kg6 3 Kf4 Kf6 4 Ke4 Kg5, and the d5 pawn is lost.

2 ... Ke7 3 Kg4 Kd7 4 Kf5 c5 5 d × c6+ K × c6 6 Ke6 Kf7 7 Kf5 Kd6!

As in example 14, Black wins by triangulation. 7 ... Kd7 achieves nothing due to 8 Kf6.

8 Kf6 Kb7! 9 Kf5 Kc7! 10 Kf6 Kd7 11 Kf5 Ke7 etc. (14).

Randviir-Keres
Parnu, 1947

Black’s material advantage comprises his backward h7 pawn. But it nevertheless
decides the game, since the threat of a breakthrough (... h5) restricts the manœuvreability of the white king, and it also provides an extremely important reserve tempo for zugzwang positions.

1 ... Kb5!

A necessary finesse. After 1 ... Kb6? 2 Kc4 a5 3 a4 Black has to use up his reserve tempo, and after 3 ... h6 4 Kd3 Kc7 5 Kc3 the 'untouchable' pawns lead to a draw.

2 a4 + (2 Kd3 c4 + 3 Kd4 c3 4 K × c3 Kc5)

2 ... Kb6 3 Ke4 a5.

The only move; after 3 ... h6 4 a5 +! only White has winning chances. But now he has to exchange his d-pawn for the c-pawn, since he loses quickly after 4 Kd3(c3) Kc7 5 Ke3(d3) Kd6 6 Kc4 h6! This is where Black requires his reserve tempo.

4 d6 Kc6 5 d7 K × d7 6 K × c5 Ke7.

Since the white king cannot attack the a-pawn due to the threat of ... h5, Black goes to the aid of his h-pawn.

7 Kd5 Kf7 8 Ke4.

On 8 Kd4 there could have followed 8 ... h5 9 g × h5 Kg7 10 Kd5 g4 11 Ke4 Kh6, when Black wins.

8 ... Kf8!!

Unexpectedly White finds himself in zugzwang. On 9 Kd5 or 9 Kd4 Black can play 9 ... h5. Therefore White does not risk leaving the e-file, but now, after parrying the threat of a breakthrough, he loses due to the opponent's reserve tempo.

9 Ke3 Ke7! 10 Kc4 Kd6 11 Kd4 h6! 12 Ke4.

There is nothing better. If 12 Ke4 Kc5 13 Kb5, then 13 ... h5 14 g × h5 K × f5 15 K × a5 g4, and Black wins.

12 ... Kc5! 13 Ke3 Kd5.

This is much simpler than working out the ending arising after 13 ... Kc4 14 Kd4 K × a4 15 Kd5 Kb3 16 Ke6 a4 17 K × f6 a3 18 Ke7 a2 19 f6 a1 = Q 20 f7, although by 20 ... Qa3 +! 21 Ke8 Qa4 + 22 Ke7 Q × g4 23 f8 = Q Qb4 + Black again wins.

14 Kd3 Ke5 15 Kc3 h5 16 g × h5 K × f5 17 Kf3 Ke6.

Of course, Black could also have played 17 ... g4 + 18 Kg3 Kg5 19 h6 K × h6 20 K × g4 Kg6. Since his pawn has not crossed the c2-h7 diagonal (Bähr's rule), he must win.

18 Kg4 Kf7 19 Kf5 Kg7 White resigns.

Persitz-Pfally
Southend, 1955

577. The attempt by White to support his passed pawn does not succeed: 1 c3 a5 + 2 K × a5 K × c5, and Black has the chances. White's plan is to exchange his c5 pawn for the a-pawn, and then break through with his king to the g5 pawn.

1 Ka5 K × c5.

1 ... Kb7 2 c3! (2 c6 +? K × c6 3 K × a6 d4 with a draw) 2 ... Kc7 3 K × a6 Kc6! was more tenacious, but after 4 Ka7 Kc7 5 c6! K × c6 6 Ka6! White nevertheless attains the position from the game.

2 K × a6 Kc6 (if 2 ... d4 3 Kb7, while on 2 ... Kc4 there follows 3 Kb6 Kc3 4 Kc5) 3 c3!

In this way White puts his opponent in zugzwang, and forces him to allow the king into his position.

3 ... Kc5 4 Kb7 d4.

Or 4 ... Kd6 5 Kb6! (5 Kc8? Kc6 with a draw) 5 ... Kc5 6 Kc5 Ke6 7 Kc6 Ke5 8 Kd7, and White wins.

5 c × d4 + K × d4 6 Kc6 Ke5 7 Kd7 Kf6 (7 ... Kf4 8 Ke6 Kg3 9 Kf5 etc.) 8 Kd6!, and White won.
Endings with Several Pawns (material advantage)

If a passed pawn cannot be created by manoeuvring, only one possibility remains—a breakthrough (the sacrifice of one, or even several pawns, with the aim of carving a way to the queening square for another pawn).

Minckwitz, 1879

578

579. On the Q-side the two black pawns are restraining three enemy pawns, and the white king cannot go there, since this loses the g-pawn.

White nevertheless wins, by employing a typical device—the breakthrough. By the sacrifice of two pawns he creates an outside passed pawn, which the enemy king is unable to stop.

1 e4! b×c3 2 Ke3 Kg5 3 a4! K×g4 4 b4! a×b4 5 Kd3!

A necessary finesse. After 5 a5 b3 6 Kd3 b2 7 Ke2 Kf3! 8 a6 Ke2 9 a7 b1 = Q + 10 K×b1 Kd2 Black saves the draw.

5 ... h5 6 a5, and White won.

In the following example White wins by a breakthrough on both wings.

Capablanca-Condé
Hastings, 1919

580

578. If the black king were at d6, the result would be a draw, but the remoteness of the king from the pawns creates the possibility of a breakthrough: 1 g4 f×g4 2 f5 g×f5 3 g6, and White wins.

The following example is more complicated.

Berger-Bauer
correspondence, 1889–1891

579

580. The pawn formation on the K-side permits the possibility of a breakthrough, so that the black king cannot move far away from that wing. This allows White to play b3—b4 at a convenient moment, creating a passed pawn.

1 f4 Kd6 2 g4 Ke7 3 Kf2 Kd6 4 Kf3 Ke7 5 Ke4 Kd6 6 b4 Kd7.

Having deployed his king and his pawns in the best possible way, Capablanca turns to decisive action.
Endings with Several Pawns (material advantage)

7 b4! a×b4 8 a5 Kc7 9 g5 f×g5 10 f×g5 h×g5 b3 12 Kd3 Kd7 13 g6 f×g6 14 f×g6
Resigns.

Szabo-Pirc
Hastings 1938/39

581. The black king is extremely active, and so the usual plan of creating a passed pawn does not succeed here: 1 e3 f×e3 2 f×e3 Kd5 3 Kf3 a4 4 Ke2 Kc4 5 Kd2 K×b5 6 Kc3 Kc5 8 e4 c6, and after losing a pawn White no longer has any winning chances. The only thing that can stop the black king is the threat of a pawn breakthrough, but such that it is the knight’s pawn or even the rook’s pawn on the K-side that becomes the passed pawn.

1 f3!

Splendidly played! If now 1 ... Kd4 2 Kf2 a4, first comes the standard procedure of undermining: 3 e3+! f×e3+ 4 Ke2 Kc4 5 K×e3 Kb3 6 f4 g×f4 7 Kf3 K×b2, and then the breakthrough: 8 g5!, and White wins.

1 ... Kd5 2 Kf2 Kd4 3 Ke1 Kc4.

If 3 ... Ke3, then 4 b3 Kd4 5 Kd2 Kc5 6 e3 f×e3+ 7 K×e3 K×b5 8 f4 g×f4+ 9 Kf3 and 10 g5.

4 Kd2!

This cool move was not played in the game, which took a sharp turn: 4 e3 f×e3 5 Ke2 Kd4 6 b3 Kc3 7 f4 g×f4 8 g5 Kd4! 9 g×h6 f3+ 10 K×f3 Kd3 11 h7 e2 12 h8=Q e1=Q 13 Qd8+ Kc3 14 Q×c7+ K×b3 15 b6 Qd1+ 16 Kg3. In this queen ending White has every chance of winning, but the struggle could still have continued for a long time (White’s king would have had to hide behind the b6 pawn). The game in fact concluded surprisingly quickly: 16 ... Q×h5 17 b7 Qg5+ 18 Kf3 Qf5+ 19 Ke3 Qg5+ 20 Kd4, and Black resigned since the king finds a shelter at a7.

The analysis of 4 Kd2 is by Euwe.

4 ... a4 (there is nothing better; if 4 ... K×b5 5 e3, or 4 ... Kd4 5 b3) 5 e3 f×e3+ 6 K×e3 K×b5.

6 ... Kb3 leads to variations already considered: 7 f4 g×f4+ 8 Kf3 K×b2 9 g5 a3 10 g×h6 a2 11 h7 a1=Q 12 h8=Q+ etc.

7 f4 g×f4+ 8 K×f4 c5 9 Kc3 c4 10 Kd2, and Black can resign: against the g4–g5 breakthrough there is no defence.

Bronstein, 1967

582. White’s king is a long way from the K-side, and in the first instance he must defend against the threat of a breakthrough: 1 Kb2 f5 2 Ke3 h5 3 Kd3! h4 4 f4!

The decisive move! By clarifying the situation on the K-side, White concludes the game.
in his favour. If now 4 ... g×f4 5 c×f4, and Black’s counter-play is neutralized.

4 ... g4 5 Ke2 g3.

There is nothing better. If 5 ... Kd7, then 6 h×g4! f×g4 7 f5, and the black king cannot cope with the enemy passed pawns.

6 e4! f×e4 7 f5, and one of the white pawns queens.

With doubled pawns the creation of a passed pawn is often a difficult, and sometimes altogether hopeless matter. Other factors come to the rescue, as we will now see.

Spiess-Bürger
Berlin, 1905

584. The defects in White’s pawn formation (two sets of doubled pawns) are strikingly apparent. Nevertheless, thanks to the existence of several reserve tempi, he succeeds in winning after 1 b5! f5 2 f4 + Kg6.

If 2 ... e×f4 + 3 Kf3 Kg6 4 K×f4 g5+, then 5 Kg3 followed by 6 h3 and 7 f4, while 5 ... Ke5 can be met by 6 f4 + ! Ke4 7 f×g5 Ke3 8 g6 f4 + 9 Kg2! Ke2 10 g7 f3 + 11 Kg3 f2 12 g8 = Q f1 = Q 13 Qc4 +., exchanging queens.

3 f×e5 + K×e5 4 Kf3 g5 5 Ke3 Kh5 (5 ... g4 6 f3!) 6 h3 Ke5 7 f4, and White won.

Other continuations would not have saved Black, e.g. 1 ... Kh5 2 f4! e4 (2 ... e×f4 + 3 K×f4 Kh4 4 Ke4 Kh3 5 Kd5) 3 f5! g5 4 h3 f6 5 b3 Kh6 6 Kg4 Kg7 7 h4 etc., or 1 ... Kf5 2 h4 g5 3 h5 f6 4 b3 Ke6 5 Kg4.

1 ... f6 is evidently the most tenacious, but here too White is helped by his reserve tempo: 2 b3! Kf5 3 h4, or 2 ... Kh5 3 f4, transposing into continuations already considered.

585. Here the extra pawn is of importance only as a reserve tempo ... c5. Apart from his more active king, Black has another advantage: the possibility of ... h4, exchanging pawns and creating invasion squares.

1 Kh2 Kf6 2 Kg2 Kg6 3 Kh2 h4! 4 Kh3.
On 4 f4 Black wins by 4 ... g×f4 5 g×h4
attached question marks, pointing out that playing the king to g6 should not have succeeded, since White could have replied 3 Kg2!, and if 3 ... h4 4 f4 with an obvious draw.

Bonch-Osmolovsky and Ter-Pogosov made a thorough study of this ending, and determined a number of positions in which ... h4 is possible. Here, for example, is one of them.

586. 1 ... h4! 2 f4!
This counter-blow is White’s main chance, otherwise play reduces to the continuation already seen in the game.
2 ... g x f4 3 g x h4 Kf5!
An important finesse. It is useful to lure the white king to h3 (4 ... Kg4 is threatened).
4 Kh3 Ke4! (the goal is achieved via a queen ending) 5 h5.
Even worse is 5 Kg2 Kd3! 6 Kf3 (6 h6 Ke2) 6 ... K x c3 7 b5 Kb3 8 h6 c3 9 h7 c2 10 h8=Q c1=Q, when Black wins easily.
5 ... Ke3 6 h6 f3 7 h7 f2 8 h8=Q f1=Q + 9 Kg4 Qf3 + 10 Kg5 Kd2, with a won queen ending.

Note that in the initial position the black king could have been at e6, when the play would have been exactly the same.

The ... h4 breakthrough also proves possible with the white king at f2: 1 ... h4! 2 f4 h3 (of course, not 2 ... g x f4? 3 g x h4, when

Kh5 6 Kh3 c5! As we see, the extra tempo comes in very useful!
4 ... h x g3 5 K x g3 (5 Kg2 merely prolongs matters: 5 ... Kf6! 6 Kh3 Ke5!, and White has to take on g3) 5 ... Kf5 6 Kf2.

In the event of 6 Kg2 Kf4 7 Kf2 Black can again exploit advantageously his reserve tempo: 7 ... c5! 8 Ke2 Kg3 9 Ke3 Kh3! This move is the whole point: d3 is inaccessible to the white king, and Black succeeds in carrying out a decisive by-passing manoeuvre: 10 Kf2(d2) Kh2! 11 Ke3 Kg1 12 Ke2 Kg2 13 Ke3 Kf1, and wins.
6 ... Kf4 7 Ke2 Kg3 8 Ke3 c5! 9 Ke2 (9 Ke4 Kf2) 9 ... Kg2, and White resigned in view of 10 Ke3 Kf1 11 Ke4 Ke2 etc.

In 1922 Berger expressed doubts as to whether White had played the best. He indicated the possibility of 3 Kh3 (preventing 3 ... h4), but refrained from drawing any conclusions. In Basic Chess Endings (1941) Fine gave this ending with Black to move, placed under the diagram ‘Black wins’, and gave the continuation 1 ... Kf6 2 Kh3 Kg6 3 Kh2 h4 4 Kh3 h x g3 5 K x g3 Kf5 etc., as in the above game.

In 1950 the continuation given in Fine's book was criticized in the magazine Shakhmaty v SSSR by Bonch-Osmolovsky and Ter-Pogosov. To 2 ... Kg6 and 3 Kh2 they
Endings with Several Pawns (material advantage)

only White can win) 3 f×g5 + K×g5 4 Kf3 e5, and Black wins.
The following position is also of interest.

587

587. 1 ... h4! 2 g×h4.

After 2 f4 + g×f4 3 g×h4 Ke4 a position considered in the analysis of example 586 is reached.

2 ... g×h4 3 Kg4 e5!
The reserve tempo again comes in useful. White is in zugzwang.

4 f4 + Ke4 5 f5 h3 6 f6 h2 7 f7 h1 = Q 8 f8 = Q Qg2 + 9 Kh5 Kd3, and Black must win.

These two subsidiary positions allow us to find immediately the correct path in position 585 with Black to move: 1 ... h4! 2 Kh2(12) Kf6!, and we reach position 586, or 2 Kh3 Ke5, and we have obtained position 587.

However, as we know, in position 585 it was White to move. In this case Black's task is rather more difficult, and some preliminary manoeuvring with the king is required.

1 Kh2 Kf6!

Nothing is achieved by 1 ... h4 2 Kg2!, and any king move by Black is met by 3 f4! with a draw.

2 Kh3 Ke6! 3 Kg2.

Here it is useful to consider another active possibility for White — 3 g4, e.g. 3 ... h4 4 f4! g×f4 5 K×h4. But it too fails to save him: 5 ... Ke5 6 g5 (6 Kh3 Ke4 7 Kg2 Kd3! 8 Kf3 K×c3
9 g5 Kb2 10 g6 c3 etc.) 6 ... Ke4 7 g6 f3 8 g7 f2 9 g8 = Q f1 = Q. Although it is White to move, he appears to have no way of saving even this relatively favourable ending, e.g. 10 Qe8 + Kd3 11 Q×c6 Qf2 + 12 Kg4 Qc2, winning the c-pawn, or 10 Qg6 + Ke3 11 Qe6 + (11 Q×c6 Qf4 + and 12 ... Qf3 +, exchanging queens) 11 ... Kd3 12 Qd6 + Kc2 13 Q×c6 (13 Qb4 does not help, since after a couple of checks the black queen reaches b5) 13 ... Qe1 + 14 Kg4 Q×c3 15 Q×a4 + Qb3, and Black queens his c-pawn.

3 ... Ke5!

It is not hard to see that White is in zugzwang. If 4 Kh3 or 4 Kh2, then 4 ... h4 with continuations already considered, while if 4 Kf2, then again 4 ... h4 5 f4 + Kf5 6 f×g5 (in the event of 6 Kf3 g4 + 7 Kf2 h×g3 + 8 K×g3 the reserve tempo 8 ... c5! is again decisive) 6 ... h3!, and Black wins.

Only now can we say that this complicated ending has fully revealed its secrets.

Fine-Reshevsky
Nottingham, 1936

588

588. Black's king is threatening to go to d5 to attack the advanced white pawn, and the attempt to meet it with the white king does
not succeed, since in this case it does not help its own pawns.

The correct plan, combining the advance of the rook's pawn with manoeuvring by the king, was shown by Fine. Here is his analysis:

1 h4 Ke7 2 Kg2 Kd7 3 h5! Ke6.

Or 3 ... g×h5 4 Kh3 Kc6 5 Kh4 Kd5 6 f4 Ke4 7 K×h5 Ke3 8 Kg5 K×e2 9 g4 Kf3 10 f5 and White wins. No better is 3 ... g5 4 g4 Kc6 5 f4! Kd5 6 Kf3 Kd4 7 h6 Kd5 8 e3.

4 h×g6 f×g6 (after 4 ... h×g6 5 f4 Kd5 6 Kf3 White's king reaches f6) 5 f4 Kd5 6 Kf3 Kd4 (by activating his king, Black complicates his opponent's task) 7 g4 g5.

If 7 ... Kd5 8 Ke3 Ke5, then 9 Kd3 followed by 10 e4 and 11 f5, while on 7 ... h6 there follows 8 e3+ Kd5 9 Ke2 Ke4 10 Kf2 g5 11 Kg3 etc.

8 f×g5!

Also possible is 8 e3+ Kd5 9 e4+ (but not 9 Kf2 Ke4 10 Kg3 h6! 11 f5 K×e5 12 f×e6 K×e6 13 Kf3 Ke5! with a draw) 9 ... Kd4 10 f×g5 K×e5 11 Ke3 Kd6 12 Kf4, and White wins.

8 ... K×e5 9 Ke3 Kd5 10 Kf4 Kd4 11 g6!

h×g6 12 Kg5 Ke3 13 K×g6, and White wins.

We will now consider cases where one side already has a passed pawn or has the possibility of creating one, but where problems arise in supporting this pawn with his king.

Alekhine-Yates
Hamburg, 1910

589. The attack by White on the opponent's pawns does not succeed, since Black in turn can capture the e5 and e3 pawns. The direct support of the passed pawn by 1 Kd4 is also unpromising due to 1 ... Ke6, when White is in zugzwang.

The win is achieved by the following instructive manoeuvre: 1 Kd3! Kd7 2 e4! f4 3 Ke2 Ke6 4 Kf2! K×e5 5 Kf3 Resigns.

Botvinnik-Taimanov
Moscow, 1967

590. Black has an extra doubled pawn, but the attempt to defend it does not succeed: 1 ... Kf6 2 Kg3 Kf5 3 e4+! K×e4 4 K×g4 e5 5 Kg5, and the pawns queen simultaneously.

But Black has a strong move, which reduces the play to 'untouchable pawns': 1 ... g5! Now it is completely bad for White to play 2 Kg3 K×g6 3 K×g4 h×h4 4 K×h4 Kf5 5 Kg3 Ke4 6 Kf2 Kd3 7 Kf3 e5. His best reply is 2 h5, but then comes 2 ... Kg7 3 Kg3 Kh6.

The game continuation was 3 ... Kh7 4 K×g4 Kh6, similar to the previous ending. But here this subtlety is not essential, since even so Black does not have the tempi to create a zugzwang position, and all the same play reduces to the game continuation.

4 K×g4 e6! 5 a4 a5 6 e4 e5 7 Kf5 (the only
chance of complicating things) 7... K × h5 8
K × e5 g4 9 Kf4.

In the event of 9 Kd4 g3 10 Ke3 Kg4 11 e5
(11 Ke2 Kh3) 11... Kh3 12 e6 g2 Black again
wins.

9... Kh4 10 e5 g3 11 e6 g2 12 e7 g1 = Q 13
e8 = Q Qf1 +, and White loses his queen.

Keres-Alekhine
Dresden, 1936

591. The protected passed pawn restricts
the black king, but by 1... Kg4 2 d6 g2 3 Kf2
Kh3 4 d7 e3 + ! Alekhine attained a won
queen ending: 5 Kf3 g1 = Q 6 d8 = Q Qf2 + 7
Ke4 c2, and Black won.

Grigoriov showed another, no less in-
teresting way, by which Black undermines and
eliminates the opponent's protected passed
pawn: 1... Ke5 2 Ke2 Kd6 3 Ke3 Kc7 4 Ke2
Kb7 5 Ke3 a5 6 b × a6 + K × a6 7 Ke2 Kb7 8
Ke3 Kc7 9 Ke2 Kd6 10 Ke3 b5 11 c × b5 K × d5
e tc.

592. Black's problem here is to prepare the
transition into a favourable queen ending.
1... Kd5 2 Kf3 a5! 3 Kg2 a4 4 Kf3 Ke5 5
Ke2 f5 6 g × f5 K × f5 7 Kf3 g4 + 8 Kg2 f3 +
9 Kf2 (9 Kg3 Ke5 10 Kf2 Kf4) 9... Kf4 10
c6 g3 + 11 Kf1 Ke5 12 Kf2 + 13 Kg1 f2 + 14
K × g2 Ke2 15 c8 = Q f1 = Q + 16 Kg3 Qf3 +

17 Kh4 Qf4 + 18 Kh3 Qe3 + ! 19 Kh2 (19 Kg4
Qg5 + ) 19... Qe5 + ! (this subtle queen ma-
noeuvre is the whole point: Black wins the h-
pawn with check) 20 Kg2 Qg5 + ! 21 Kh3
Q × h5 + 22 Kg3 Qg5 + 23 Kh3 Kd2! 24 Qc5
Kd3 25 Kh2 Qh4 + 26 Kg2 Qc4! 27 Qd6 +
Ke2 28 Q × h6 Kb2 29 Qe3 Qc2 + White
resigns.

We will now consider a number of ex-
amples where the stronger side is faced with
the problem of how to penetrate into the
enemy position.
593. After 1 a5 (1 b5 is also possible) 1 ... Kd7 2 b5 Ke8 3 a6 Kb8, in view of the threat of a6–a7 the black king has only the squares b8 and a8. It remains for White to penetrate into the enemy position, but for the moment this cannot be done due to ... d4.

To win, White must reach d6 when the black king is at a8—hence he has to give his opponent the move. This is achieved by triangulation, on practically any squares. For example: 4 Kb4 Ka8 5 Ka5 Kb8 6 Ka4 Ka8 7 Kb4 Kb8 8 Ke5 Ka8 9 Kd6! Kb8 (9 ... d4 10 Kc7 16 K × e6 d4 11 Kf7 d3 12 e6, or 4 Kd4 Ka8 5 Ke3 Kb8 6 Kd3 Ka8 7 Kd4 Kb8 8 Ke5, and White wins.

In the following example it is Black’s own passed pawn that prevents him from penetrating into the opponent’s position.

Bähr, 1936

594

594. In order to penetrate with his king, Black will be forced to sacrifice his d-pawn, and the whole question is whether or not White can defend all the invasion squares. But first White must be forced to use up his reserve tempi, which is very important for the creation of a zugzwang position.

1 ... a6! (securing the Q-side) 2 Kd4 Kd7 3 Kc5 Ke7 4 Kd4 Kf7.

Threatening to open lines by 5 ... g5. Therefore White’s reply is forced, to be able to meet 5 ... g5 with 6 h5, blocking the king’s path.

5 h4 Ke7 6 Kc3 Kd7 7 Kd4 Kc6 8 Kd3 Kb6 9 Ke3!

In a game played by Bähr his opponent went 9 Kd4, allowing 9 ... a5 10 a3 a × b4 11 a × b4 etc. But now on 9 ... a5? comes 10 g4!, when Black cannot play 10 ... f × g4 11 h5! g × h5 12 f5! e × f5 13 e6 Kc7 14 b × a5, when a white pawn is the first to queen, and meanwhile there is the threat of 11 h5 g × h5 12 g5. In defending against this threat, Black is forced by 10 ... h5 to waste his reserve tempo, after which there can be no question of a win.

9 ... Kb7(c7) 10 Ke2 Kc6 11 Ke3 a5!

Black’s king is one move closer to the e-file, and the breakthrough no longer works: 12 g4? f × g4! 13 h5 g × h5 14 b × a5 g3 15 f5 e × f5 16 e6 d4 + 17 Kf3 d3 18 e7 Kd7 19 a6 d2 20 Ke2 g2 21 a7 d1 = Q + 22 K × d1 g1 = Q + and 23 ... Q × a7.

12 a3 (12 b × a5 b4 13 g4 h5) 12 ... a × b4 13 a × b4 Kb7(c7) 14 Kd4 Kb6 15 Kc3 d4 +! 16 Kd3 Kb7 17 K × d4 Kc6 18 Kc3 Kd5 19 Kd3 h5, and Black wins.

Sometimes, in order to penetrate into the opponent’s position, one has to resort to very subtle manoeuvring, employing the theory of corresponding squares.

Neustadtl, 1907

595 +
In the first instance White must control the invasion point d4. This problem is easily solved by determining the corresponding squares. It is clear that the square corresponding to e3 is d5, to f3—c6, and to e4—c5. If after 1 Kf3 Ke6 White plays 2 Kf2 (or 2 Ke2), Black replies 2...Ke5, but after 3 Ke2 he has no good reply, and this means that he cannot maintain the correspondence.

1 Kf3 Ke6 2 Kf2(e2)! Ke5 3 Ke2! Ke6 4 Kf3! Kd5 5 Ke3! Ke5 6 Ke4.

Now Black loses immediately after 6...Kc6 7 Kd4 Kb5 8 h4, and therefore he has to advance his h-pawn.

6...h4 7 Kf3! Kd5 8 Ke3! Ke5 9 Ke4 h3.

9...Kc6 10 Kd4 Kb5 11 h3 is bad for Black, while after 9...Kb5 10 Kd4 he is all the same forced to move his pawn.

10 Ke3.

Having forced the advance of the pawn, White makes for it with his king. But in doing so he must watch carefully that the opponent’s king does not become too dangerous.

10...Kc6 11 Kf2!

An essential finesse. If 11 Kf3, then 11...Kd5, and after 12 Kg3 Ke4 Black has sufficient counter-play.

11...Kd5 12 Kf3! Ke5 13 Kg3! (now this move is possible) 13...Kd5 14 K×h3 Ke4 15 Kg4 (596).

596. In this position, where White is now two pawns up, he either breaks through with his pawns or invades the opponent’s position.

Three finishes are possible: the first two involve a pawn breakthrough, while the third is based on invading the enemy position.

(a) 15...Ke3 16 Kg3 Ke4 17 h4 Ke3 18 h5 g×h5 19 f5! e×f5 20 g6! f×g6 21 e6 f4+ 22 Kh2.

(b) 15...Kd3 16 Kf3 Ke2 17 h4 K×b2 18 h5 g×h5 19 f5 e×f5 20 g6 f×g6 21 e6 K×a3 22 e7 Kb2 23 e8=Q a3 24 Qb5+.

(c) 15...Kd5 16 Kf3 Ke6 17 Ke4 Kc5 18 h3 Ke6 19 Kd4 Kb5 20 h4, and White wins.

We have already seen several times that, for the sake of various positional gains it does no harm to part with a pawn. This procedure is also strikingly illustrated by the following study.

Chekhover, 1951

597. The immediate advance of the king towards the c-pawn does not promise success, e.g. 1 Kf2 e3 2 Ke2 g×f4 3 e×f4 Kd5 4 Ke3? Kc4 5 f5 Kb3 6 f6 c2, and the pawn queens with check. Of course, stronger is 4 f5 K×d4 5 f6 e3 6 f7 c2 7 f8=Q c1=Q, but even here there is no way that Black can lose.

White wins by 1 f5+! K×f5 2 Kf1!!

It is a mistake to play 2 Kf2 g4 3 h×g4+ K×g4 4 Ke2 c3!, when White is in zugzwang, and after 5 Kd1 (5 d5 Kf3) 5...Kf3 6 d5
K × e3 7 d6 Kf2 8 d7 e3 the game ends in a draw.

2 ... c3.

If 2 ... g4, then 3 h × g4 + K × g4 4 Kf2! c3 5 Ke2, and it is Black who ends up in zugzwang.

3 Ke1! g4 (3 ... Ke6 4 Kd1 Kd5 5 Kc1 etc.)
4 h × g4 + K × g4 5 Ke2! c2 6 Kd2 Kf3 7 d5
c1 = Q + 8 K × c1 K × e3 9 d6, and White wins.

The following study shows an exceptional situation.

Dawson, 1923

598. How can there be any question of a win here, when the black pawn is almost on the threshold of queening? And yet, by creating one mating threat after another, White succeeds in stopping it.

1 Kb2 Kb4.

2 Ka3 with inevitable mate was threatened, so Black is forced to move his king out of the danger zone.

2 c3+ Ke5 3 Ke2 f6 (a further tempo gained; if 3 ... g2, then 4 Kd3 and 5 b4 mate)
4 Kd3 f × e5 5 Ke2 and by eliminating the g3 pawn White wins easily.

In the examples considered, in spite of various difficulties, the stronger side normally succeeded in realizing his advantage. This is by no means always the case. We now wish to acquaint you with a whole series of positions in which the material advantage proves insufficient for a win. In each case we will try to establish what prevented the stronger side from winning.

Kasparov-Averbakh
Kislovodsk, 1982

599. How should White set up a passed pawn? 1 Kf3 f5 leads to an immediate draw, since the threat of 2 ... Kh5 with an attack on the h4 pawn restricts all his actions. Incidentally, if the black pawn were at h5, White would be able to win: 1 Kf3 Kf5 (1 ... f5 is too late due to 2 Ke2 Kf6 3 Kd3 Ke6 4 Ke3 Kd6 5 Kb4, winning) 2 e4 + d × e4 3 Ke3 Kg4 4 K × e4 K × h4 5 Kf3! etc.

Little is also promised by the immediate sacrifice of a pawn, e.g. 1 h5 + K × h5 2 e4 Kg6! (but not 2 ... d × e4 3 d5 Kg6 4 d6 Kf6 5 f5! h5 6 Kf4, when White wins) 3 e × d5 Kf6 4 Kg4 Ke7 5 Kh5 Kd6 6 K × h6 K × d5 7 Kg7 K × d4 with a draw.

600. Chekhov reached this position (1936) when analyzing the 32nd game of the Capablanca-Alekhine World Championship match. White is a pawn up, but how is he to organize its advance? We should first remark
that Black’s defence requires a certain accuracy. Thus 1... Kg6?, for example, is bad: 2 Kh3! Kf5 (2... b5 3 Kg3 Kf6 4 Kh4 Kg6 5 a6) 3 Kh4 Kg6 4 h5.

1... a6! is correct, depriving White of any reserve pawn moves, e.g. 2 Kh4 Kg6 3 Kh3 Kf4 4 Kg3 Kf5 5 Kf3 h4! 6 Kf2 Kg6!

A precise retreat. Chekhov gave the variation 6... Kg4 7 Kg2 h3+ 8 Kh2 Kh4, but the king move to g4 is an obvious mistake, since after 8 Kf2! (instead of 8 Kh2) 8... Kh4 9 Kf3 White wins.

7 Kg2 Kh6! (again the only reply which allows Black to maintain the balance) 8 Kf3 Kh5 9 f5.

There is nothing better. After 9 e4? dxe4+ 10 Kxe4 Kg4 it is Black who wins.

9... Kg5 10 f6 Kxf6 11 Kg4 h3 12 Kxh3 Kf5 13 Kg3 Ke4 14 Kf2 Kd3 15 Kf3 Kc4 16 Kf4 Kxb4 17 Ke5 Kxa5 18 Kxg5 h5! 19 Kc5 b4 20 d5 b3 21 d6 h2 22 d7 b1=Q 23 d8=Q + Ka4 24 Qd4+ Ka5. Draw.

601. Here the white pawns divert Black’s king to such an extent that it is quite unable to support his own pawn phalanx: 1 a4!

White has to hurry. If 1 Kg4, then 1... Ke6 2 a4 Kd5 3 Kf5 g6 4 Kf6 h5, and Black wins.

1... Ke6 2 a5 Kd5 3 a6 Kc6 4 Kg4 g6 5 Kg5 h6+! 6 Kh4! Drawn.

Attempts by Black to win here look risky, e.g.:

(a) 6... g5? 7 Kg4 Kc7 8 Kf5 e4 9 Kxe4

h5 10 Kd5! g4 11 c6 Kb6 12 Kd6 gxf3 13 c7 f2 14 c8=Q f1=Q 15 Qb7 + Ka5 16 a7.

(b) 6... h5 7 Kg5 Kc7 8 Kh4 (8 Kf6? e4 9 a7 Kh7 10 Ke7 e3 11 c6 + K×a7 12 c7 e2 13 c8=Q e1=Q+, and the only chances are with Black) 8... e4? 9 f×e4 g5+ 10 Kh3 g4+ 11 Kg2 h4 12 Kg1!, and White wins.

The saving procedure employed by White in the following example is characteristic of instances where the passed pawn is on a rook’s file.

from a practical game
East Germany, 1946

602. White has in reserve three pawn
tempi, and so he plays 1 Kc4, so as not to allow the opponent’s king out from in front of the pawn: 1 ... Ka3 2 Kc3 a4 3 h3! Ka2 4 Kc2 a3 5 h4 h6 6 h5. Draw.

van Desburg-Maroczy
Zandvoort, 1936

603

W

604

Wotava, 1963

604. After the natural 1 ... d4! White’s position becomes critical: 2 ... b5 and then 3 ... Kd5 is threatened, when the king breaks through to the c3 pawn. If 2 c×d4+, then 2 ... K×d4 3 h3 h6 4 Kf1 Ke3 5 Ke1 b5, and Black wins. Nevertheless White finds a surprising resource.

2 c4! b5!

3 b4 was threatened, whereas now 3 c5 can be met by 3 ... b4, cutting off the pawn from the support of its b-pawn. Bad is 3 c×b5 Kd5 4 Ke1 Kc5 5 Kd2 K×b5 6 Kc2 Kb4 7 h3 (7 b3 Ka3 8 h3 h6) 7 ... h6 8 Kb1 Kb3 9 Ke1 e3, when White can resign.

3 c5! b4.

It only needs White to delay slightly, and after 4 ... Kd5 his position will become hopeless. But it is here that the sublety of White’s plan, involving the advance of the c-pawn, is revealed.

4 e3! d3 5 h3!

Accuracy to the end! If first 5 b3, then 5 ... h3, and the black king gains the opportunity to invade via g4.

5 ... Kd5 6 b3.

Now Black’s king has no way of breaking into the opponent’s position. Draw.

In conclusion, here are two further examples from studies. The first position could hardly arise in a practical game, but the second is a perfectly natural one.
605. Two pawns down, White's position looks completely hopeless. Only a miracle can save him, and that is what happens: 1 h5! g×h5 2 g6 f×g6 3 e6 (this would also have been the answer to 2 ... f6) 3 ... d×e6 4 c5 d×e5 5 a6! b×a6 6 b6!! a×b6—stalemate!

606. Black's passed pawn cannot be stopped, but in fact there is no need to do so. After 1 Kb2! h4 2 Ka3 h3 3 Ka4 h2 4 a3 h1 = Q it transpires that White has stalemated himself. Draw!
9. Endings with Several Pawns
(positionational advantage)

In pawn endings the assessment of a position and the choice of plan are usually based on three factors: features of the pawn formation, the presence of reserve tempi, and the relative placing of the kings.

These factors are linked to one another, are not permanent, and in the course of play their role may change sharply. And the influence of these factors on the assessment of a position is not identical. Sometimes it can be very difficult to establish which of them in the given situation is the most important, the most significant.

In classifying the examples selected for this chapter, we have done things in the reverse order: after establishing beforehand which factor is the most distinctive, we have grouped examples on the basis of this indication. But the features of the pawn formation is too broad a concept, and we considered it expedient to expand it.

Both the assessment of a position, and the plan of play, depend to a considerable extent on whether one of the sides possesses the following features of the pawn formation:

(a) an outside passed pawn, or the possibility of creating one;
(b) the more dangerous pawns, if both sides have passed pawns;
(c) a protected passed pawn;
(d) the possibility of a breakthrough;
(e) organic weaknesses.

These five basic features of the pawn formation have in fact been taken as the basis of our classification. To them we have added examples where one side possesses reserve tempi, and also positions where one of the kings is more actively placed.

9.1 OUTSIDE PASSED PAWN

If one side has an outside passed pawn or has the possibility of creating one, all other things being equal this normally constitutes a decisive positional advantage. The winning plan is to divert the opponent's king by the advance of this pawn, and then break through with one's own king into the enemy position.

Here is a typical example.

607

607. The outside passed a-pawn is much more dangerous than the c-pawn. White exchanges these pawns, as a result of which his king ends up closer to the remaining pawns, which leads to a decisive gain of material.

1 Ke2 Ka3.

If Black stubbornly plays 1 ... Kc5, after 2 Kc3 Kb5 3 a3 Kc5 4 a4 Kd5 5 a5 Kc5 6 a6 the
Outside Passed Pawn

exchange is forced in an even more unfavourable situation for him.

2 Kc3 K × a2 3 K × c4 Kb2 4 Kd4 Kc2 5 Ke4 Kd2 6 Kf5 Ke2 7 K × g5 Kf2 8 K × f4 K × g2 9 Kg4, and White wins.

The winning method is usually based on the typical procedure of ‘transformation of advantages’—in return for the outside passed pawn the king is strongly activated.

As the following two practical examples show, in the majority of cases the realization of the advantage in such endings is of an elementary nature.

Gheorghiu-Gligoric
Hastings, 1964

608. For the moment there is no outside passed pawn, but Black quickly obtains it: 1 ... f5! 2 Ke3 f4+! 3 Kf2 (3 g × f4+ Kf5) 3 ... b5 White resigns. After 4 Kg2 b4 5 Kf2 f × g3+ 6 K × g3 h4+ 7 Kh3 Kf4 8 K × h4 K × f3 the difference in the placing of the kings decides the game in Black’s favour.

609. 1 h4.

Of course, White could have played the immediate 1 a4 b × a4 2 b × a4 Kc5 3 a5 Kb5 4 a6 K × a6 5 Kc4, but he wishes first to create weaknesses in the opponent’s position.

1 ... h6 2 a4 b4 (2 ... b × a4 3 b × a4 g5 4 h × g5 h × g5 5 a5 Kc5 6 g4 etc.) 3 a5 Kc5 4 a6 Kb6 5 Kc4 K × a6 6 Kd5.

Faibisovich-Gutman
USSR, 1972

609

W

This is simpler than 6 K × b4 Kb6 7 Kc4 Kc6 8 b4, when White has to use this pawn to divert the king.

6 ... e4 7 f × e4 f × e4 8 K × e4 Kb6 9 Kd4 Kh5 10 Kd5, and White wins.

But sometimes the path to victory can be rather complicated, and can require accurate and subtle play.

Averbakh-Tukmakov
Kislovedsk, 1982

610

B

610. The pawn structure here is such that White threatens to create an outside passed pawn by f2–f3, g3–g4 and h4–h5. For ex-
ample: 1 ... Ke6 2 f3 Kd5 3 Kd3 Kc5 4 g4! Kd6 (4 ... e4 5 Ke3!) 5 g×f5 g×f5 6 h5 Ke6 7 Kc4 Kf6 8 Kd5, and White wins.

Therefore Black played 1 ... g5, exploiting the fact that 2 h5 is not possible due to 2 ... g4. The game concluded: 2 h×g5 + K×g5 3 f3 Kf6 4 Kd3 Ke6 5 g4 e4 +!

The saving move. After 5 ... Kf6 6 g×f5 K×f5 7 Ke4 Kf4 8 Kd5 Kf5 9 b4 Kf6 10 Kd6 Kf5 11 Ke7 Kf4 12 Ke6 e4 13 f×e4 K×e4 14 Kd6 Kd4 15 Kc7 Kc3 16 K×b7 K×b4 17 K×a6 White would have won.

6 Ke3 e×f3 7 g5 f2. Drawn.

But by retaining his outside passed pawn with 2 Kd3!, White could have won this ending, e.g. 2 ... g×h4 3 g×h4 Kg6 4 Kc4 f4 5 h5 +!

Weaker is 5 Kd5 Kf5 6 h5 e4 7 h6 e3 or 7 Kd4 e3 8 f×e3 f3 9 Kd3 Kg4 10 h6 Kg3, while if 5 Kd3, then 5 ... Kf5 6 f3 e4 +! 7 f×e4 Kg4 8 e5 f3 9 e6 Kg3, when Black queens with check and draws the queen ending.

5 ... K×h5 (if 5 ... Kf5, then 6 h6) 6 Kd5 Kg4 7 Ke4!

The only move which wins. After 7 K×e5 Kf3 the game ends in a draw: 8 b4 K×f2 9 K×f4 Ke2 10 Ke4 Kd2 11 Kd4 Kc2 12 Kc4 Kb2.

7 ... Kh3.

There is nothing better. If 7 ... f3, then 8 K×e5 Kh3 9 Kf4 Kg2 10 Ke3.

8 K×e5 Kg2 9 K×f4 K×f2 10 Ke5, and White wins by reaching the opponent's pawns first.

In the following example the play is exceptionally complicated, but the theme of it is the struggle for an outside passed pawn.

611. White played 1 h3, and after 1 ... Kf6 2 Kf4 e5 + 3 Ke3 a6 4 b3 Ke6 5 e×f5 + Kxf5 6 f3 Ke6 7 g4 f5 8 g×f5 + K×f5 9 h4 Kf6 10 Ke4 Ke6 11 a3 b5 12 c×b5 a×b5 13 Kd3 Kd6 the game ended in a draw. The diagram position was thoroughly analyzed by Smyslov, Furman and Averbakh, and in the end it was shown that by 1 e5! White could have won.

White's advantage here is determined by two main factors: he has the superior pawn formation and a greater number of pawn moves, which in a struggle for zugzwang should play a decisive role.

By 1 e5 White begins a plan which is typical of such positions: he intends to drive back the black king by h2–h4, occupy f4 with his king, and by f2–f3 and g3–g4 create an outside passed pawn on the h-file.

How can Black forestall this? Let us first try maintaining the king at g5, and to do this play 1 ... h4. It was convincingly shown by Smyslov that after 2 h3 a6 3 a3 a5 4 a4 Kg6 5 Kf4 Kh5 6 b3 Black ends up in zugzwang and is forced to retreat, and 6 ... Kh6 is met by 7 g4 Kg6 8 g×f5 e×f5 9 f3, winning. If instead Black plays 6 ... h×g3 7 f×g3 Kg6, White carries out his plan—8 g4 f×g4 9 K×g4, and wins easily thanks to his outside passed pawn.

Thus Black is not able to hinder the initial stage of the opponent's plan, but by 1 ... f6 he can create a tense situation, in which every tempo is vital. White makes the second step: 2 h4 + Kg6 3 Kf4.

Now passive tactics by the opponent allow White easily to achieve his aim: 3 ... Kf7 4 f3 Kg6 5 a3 a6 6 a4! a5 7 b3 (the battle for a tempo is won) 7 ... Kf7 8 g4! f×g4 9 f×g4

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Outside Passed Pawn

fxe5 + 10 Kxe5 h×g4 11 Kf4, and the game is decided.

Black need not fear the breakthrough by f2–f3 and g3–g4, only if his king is at g6. His best plan of defence is 3...a6!

Now on 4 f3 comes 4...b5! 5 c×b5 (5 b3 a4) 5...a×b5 6 a3 c4!, and in the struggle for a tempo it is Black who emerges the winner. The breakthrough 7 g4 even loses for White after 7...f×e5+ 8 K×e5 f×g4 9 f×g4 h×g4 10 Kf4 Kh5.

But White has the subtle reply 4 a3!, and if 4...b5 5 c×b5 a×b5 6 b3! (612).

Unexpectedly White has acquired a new threat—that of creating an outside passed pawn... on the Q-side. True, the situation has become much sharper, since Black too has the possibility of creating a passed pawn.

612. 6...f×e5 +!

What is White to do now? On the natural 7 K×e5 comes 7...Kf7 8 b4 c4 9 Kd4 e5 + 10 Kc3 Ke6 with a draw.

He wins by the subtle 7 Ke3!!, found by Smyslov.

The threat of creating a passed pawn on the a-file is so strong that White can permit himself to give up a pawn. The subsequent events are extremely interesting, since Black has many ways of complicating matters.

7...Kf6 (the king strives to enter the 'square' of the a-pawn) 8 a4 b×a4.

Forced. If 8...Ke7 9 a×b5 Kd6, White makes a 180° turn—10 f3 Kc7 11 g4!, and nevertheless carries out his original plan.

9 b×a4 Ke7 10 Kd3 Kd6 11 Kc4 Ke6 12 a5 f4 13 g×f4 e×f4 14 a6 f3!

The most tenacious defence. 14...e5 15 a7 Kb7 16 K×c5 e4 17 Kd4 e3 18 f×e3 f×e3 19 K×e3 K×a7 results in the 'tragedy of one tempo'—White wins, by outstripping his opponent by precisely one move: 20 Kf4 Kd7 21 Kg5 Kc7 22 K×h5 Kd8 23 Kg6 Ke8 24 Kg7 etc.

15 a7 Kb7 16 K×c5 K×a7 (613).

613. White has to decide—which pawn should he make for, f3 or e6? The correct path is easy to miss. After 17 Kd4 Kb6 18 Ke3 Kc5 19 K×f3 Kd4 20 Kf4 e5 + 21 Kg5 Ke4 22 K×h5 Kf3 23 Kg5 K×f2 24 Kf5 it appears that White has succeeded, but the king makes a feint—24...Kf3!, and after the forced 25 K×e5 Kg4 it catches the pawn.

The only correct continuation is 17 Kd6! Kb6 18 K×e6 Kc7 19 Kf5 Kd6.

White again has a choice: should he go to the h5 pawn or the f3 pawn? It turns out that the immediate attempt to win the h5 pawn is incorrect: 20 Kg5 Ke5 21 K×h5 Kf5 22 Kh6 Kg4 23 h5 Kh3 24 Kg5 Kg2 25 h6 K×f2 26 h7 Kg2 27 h8=Q f2, and the ending is a draw.

White should play 20 Kf4 Ke6 21 K×f3 Kf5 22 Ke3! Kg4 23 f4 K×h4 24 Kf3!
The irony of fate! Black’s king has at last eliminated the h-pawn, which has caused him so much trouble, but . . . it has ended up in a trap, and is now preventing its own pawn from advancing. The rest is simple: 24 . . . 

Kb3 25 f5 h4 26 f6 Kh2 27 f7 h3 28 f8 = Q, and White wins.

Difficulties over realizing an advantage can occur when: (a) for some reason or other it is difficult, or perhaps even impossible, to create an outside passed pawn; (b) the exchange of the pawn does not lead to the activation of the king, and does not enable it to penetrate into the enemy position; (c) both kings end up in the opposite camp, and the play leads to the mutual elimination of the pawns.


Kc4 Kb6 13 a5 + Kc6 14 h3 White wins very simply.

10 b5 Kc5 11 f6 Kb6.

Black is relying on the fact that the opponent’s king will after all not be able to break into his position.

12 Kf3 Kc5 13 Kg4.

Exploiting the fact that the black king is completely tied down by the protected passed b-pawn, White strengthens his position on the K-side to the maximum, by advancing his pawn to h6.

13 . . . Kb6 14 h3 Kc5 15 h4 Kb6 16 h5 Ke5 

17 h6 Kb6 18 Kf4 Kc5 19 Ke3 Kb6 20 Kd4 

Kb7 21 e5! (now White’s problem is to create an invasion point in the opponent’s position) 21 . . . d5 22 Kc5 Kc7 23 b6 + Kb7 24 Kd6!

The culmination of White’s plan! Although Black queens two moves earlier, he cannot prevent White from acquiring a new queen, and the resulting ending proves completely hopeless for him.

24 . . . d4 25 Ke7 d3 26 K × f7 d2 27 Kg8 

d1 = Q 28 f7 Q × a4 29 f8 = Q Qb3 30 K × h7 Resigns.

The opinion of the commentators was unanimous—the ending is won for White.

But did Black exploit all his defensive resources? It appears that he did not. He could have hindered his opponent’s task by 1 . . . g5!, not allowing White to advance his K-side pawns. How would the play have gone then? Let us consider some possible continuations:

(a) 2 Ke3 Ke6? (Black cannot remain passive, and must exploit any possibility of activating his forces) 3 f4 g × f4 + 4 K × f4 d5? 5 

Ke5 (5 g5 d4!, and the only chances are with Black) 5 . . . d × e4 6 K × e4 f6 7 Kd4 Kd6 8 

Kc4 a6 9 b5 a × b5 10 a × b5 e5, and it is not apparent how Black can lose.

(b) 2 Ke4 (White tries first to fix the position on the Q-side) 2 . . . a6 (bad is 2 . . . Ke6 3 

b5 + Kb6 4 Kb4 and 5 a5 + ) 3 b5 a × b5 + 4

614. White has an undisputed advantage, comprising a Q-side pawn majority, command of greater space, and, finally, a more active king position. But for the moment it is not apparent how he can create a passed pawn, and especially how he can penetrate with his king into the enemy position.

The game continued 1 . . . Ke7 2 g5! Kb6 3 

Kc4 a6 4 Kd4 Kc6 5 Ke4 Ke7 6 f4.

White embarked on active measures only after some harmless king manoeuvring, which here we will omit.
Later Leick (1942) found that Black had been wrong to resign: after 4 . . . d4! 5 K × g4
Kd5 6 Kf4 a6 7 a3 a5! 8 Kf3 a × b4 9 a × b4
Ke5 (344) a draw is obvious.

However, it was correctly shown by Zinar
(1974) that 3 d5 was an obvious mistake,
throwing away the win, and that White
should have played 3 a3!, supporting the b4
pawn in advance. After 3 . . . g3 4 Kf3 Kd5 5
K × g3 K × d4 6 Kf4! Black is not saved by
either 6 . . . a5 7 b × a5 K × c5 8 Ke5, or 6 . . .
Kc4 7 Ke5 Kb3 8 Kd6 K × a3 9 K × c6, when
White queens first.

We can therefore agree with the opinion of
Berger (1922), that 1 . . . b5 was already the
decisive mistake.

But of course this doesn’t mean that posi-
tion 615 is lost for Black. If he sticks, for
example, to waiting tactics, he is in no danger
of losing: 1 . . . Kf6 2 a4 a6 3 d5 c × d5 + 4
K × d5 g4 5 Ke4 g3 6 Kf3 Ke5 7 K × g3 Kd4 8
c5 b × c5 9 b × c5 K × c5 10 Kf2 Kb4 11 Ke1
K × a4 12 Kd1 Kb3 13 Ke1 with a draw.

But if Black intends to play actively, 1 . . .
a5 is possible, when 2 a3 is bad because of 2 . . .
a4! After the correct 2 c5 a × b4 3 e × b6
Kd7 4 Kf5 Ke8 5 K × g5 Kb7 6 Kf6 K × b6 7
Ke6 Kb5 8 Kd6 the game ends in a draw.

Thus if the passed pawn is not too far from
the opponent’s king, and the latter is suffi-
ciently active, the advantage may prove insuf-
cient for a win.
616. In its time this position was the topic of a lively discussion in the chess press. It began in 1956 when Levenfish gave it as an example illustrating the strength of the outside passed pawn arising after 1 ... b6, 2 ... a6 and 3 ... b5. This was challenged by Romanovsky, who pointed out that the main factor in this position was in fact the active position of the white king, due to which Black's advantage was neutralized. After 1 ... b6 2 h5 g × h5 3 g × h5 a6 4 f5 + Ke6 (4 ... Kd5 5 f6 Ke6 6 f7) 5 Kd5 K × f5 6 Kc6 b5 7 a × b5 a × b5 8 K × b5 Kg5 9 Ke4 K × h5 10 Kd3 the king reaches f1 in time.

Instead of 1 ... b6, Maizelis (1956) suggested 1 ... h5! with the following continuation: 2 f5 + g × f5 + 3 g × f5 + Kf6 4 a5! (4 Kd5? K × f5 5 Kd6 a4! or 5 a5 Kg4 6 Kd6 b5!). But even in this case, as he writes, '4 ... b6 or 4 ... b5 is futile in view of 5 a6!, and White is saved solely by the attack on the a7 pawn'.

This last variation is worth examining. Indeed, after 4 ... b5 5 a6! b4 6 Kd4 K × f5 7 Ke4 Kg4 8 K × b4 K × h4 9 Kc5 White manages to capture on a7 just in time: 9 ... Kg4 10 Ke6 h4 11 Kb7 h3 12 K × a7 h2 13 Kb8 h1 = Q 14 a7 with a draw. But, as was shown by Botvinnik, Black can play more strongly: 7 ... Ke5! 8 K × b4 Kd4!, forcing White to make a choice. Bad, for example, is 9 Kb3 Kc5 10 Ka4 Kb6 11 Kb4 K × a6, and according to Bahr's rule White loses, since the black pawn has not moved from its initial square. Botvinnik thought that even after the best move 9 Kb5! Kd5 10 Kb4! Kc6 11 Kc4 Kb6 Black would win, but in fact White is saved by a subtle king manoeuvre: 12 Kd5! K × a6 13 Kc6!, when we reach a drawn position which could have arisen in example 214. Without advancing his pawn Black has no way of getting his king off the rook's file, but then, according to Bahr's rule, it moves out of the winning zone. For example: 13 ... Ka5 14 Kc5! a6 15 Kc4 Kb6 (15 ... Ka4 16 Kc5) 16 Kb4. Draw.

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617. Here the typical plan—exchanging the outside passed pawn with the aim of invading the opponent's position with the king—does not succeed for Black, since the white king in turn is able to attack the pawns. For example: 1 ... h4 + 2 Kh3 Kf4 3 K × h4 K × f3 4 Kg5 Ke3 (4 ... e4 5 d × e4 K × e4 6 Kf6! comes to the same thing) 5 Kf5 K × d6 K × c5 4 Kd6 K × b4 8 K × c6 K × a5 9 Kc5. Draw.

But due to the threat of a breakthrough, Black cannot win the necessary tempo even by manoeuvring. If 2 ... Kh5, then 3 f4! (bad is 3 Kg2 Kg6 4 Kh2 Kf5 5 Kh3 Kg5!, when Black has gained the tempo needed) 3 e × f4 d × f4 5 d5! c × d5 6 b5 d4! 7 b × a6 d3 8 a7 d2 9 a8 = Q d1 = Q 10 Qe8 + Kg5 11 Qe5+, with a draw by perpetual check.

Thus against correct defence, Black here is unable to realize his positional advantage.

In the struggle against an outside passed pawn, the threat of a breakthrough is the most effective method of defence. The following example is highly instructive.

618. In this game, which constituted a ‘comedy of errors’, after 1 ... h5?? 2 g5?? (2 h5! followed by 3 g5!) would have won) 2...
More Dangerous Passed Pawns

Nakagawa-Day
Buenos Aires, 1978

618

Kf5, when he can resign. His only chance is to try to set up a passed pawn—1 d4! h4.
If 1... cxd4 2 Kxd4 h4, then 3 Ke3, and thanks to the threat of 4 b5 White is no longer in danger of losing.

2 dxc5 Kf7 3 Kd6!
Everything else loses, e.g. 3 c6 Ke7 or 3 f4 h3 4 c6 Kc8 5 Kd6 Kd8.

3... Ke8 4 Kc7! h3 5 c6 h2 6 Kb8 h1 = Q 7 c7 Qh2 8 Kb7 Qh7 9 Kb8, with a draw by repetition.

9.2 MORE DANGEROUS PASSED PAWNS

To this group we have assigned positions where both sides either have passed pawns, or can create them, which naturally leads to sharp situations. The question of which pawns are the more dangerous depends on the specific situation: how far advanced the pawns are, what part the kings are playing in the struggle with them, and so on.

Positions of this type lend themselves least well to generalization. Here it often happens that the result depends on a single tempo.

Taimanov-Botvinnik
Moscow, 1953

Averbakh, 1983

619

619. How is White to cope with the outside passed pawn? Bad is 1 Ke4 h4 2 Ke3

620

620. If it were White’s move, he would draw most easily by 1 a3, e.g. 1... bxa3 (1...
... b3 is bad due to 2 Kb2 and 3 a4! 2 Ka2 d4 3 K x a3 d3 4 Kb2 d2 5 Kc2 e3 6 e6 Kg7 7 f4, and White's connected pawns are in no way inferior. However, also possible is 1 Kb2 d4 2 Kc2 d3+ 3 Kd2, when the pawns are stopped.

But in this position the right of the first move proves decisive, since Black is able to advance both his pawns to the 3rd rank, creating a zugzwang position.

1 ... d4! 2 e6 Kg7 3 f4 Kf6 4 f5 d3 5 Kb2 b5 White resigns. Indeed, after 6 Kc1 b3! 7 a x b3 c x b3 the white king is forced to allow one of the pawns to queen (7 a+ c3 8 a5 b2+ 9 Khi d2 does not help).

The situation has become incredibly sharp. Whose passed pawns will prove the more dangerous?

10 b4 g4 11 b5 b4 12 b6 a2 13 Kb2 g3 14 d6+ K x d6 15 b7 Kc7 16 e7, and White outstrips his opponent by precisely one tempo, and wins.

Of particular interest is the question of the comparative strength of central and wing pawns. It is normally considered, for example, that if one of the players creates passed pawns simultaneously on both wings, these are stronger than a pair of connected pawns in the centre.

Usually this is indeed the case.

Pillsbury-Gunsberg
Hastings, 1895

621. White's e6 pawn has broken away from his main forces, but by 1 e4! he establishes contact with it: 1 ... d x e4 2 d5 + Kd6 3 Ke3 b4 4 K x e4 a4 5 Kd4 h5.

A desperate attempt to set in motion his pawns on the opposite wing, which is quickly refuted.

6 g x h5 a3 7 Kc4 f5 8 h6 Resigns.

Analysis showed that 5 ... Ke7! would have been more tenacious, preparing the breakthrough under more favourable circumstances, e.g. 6 Kc4 b3! 7 a x b3 a3 8 Kc3 f5! 9 g x f5 h5.

622. A difficult position to assess. White has the possibility of creating a passed pawn on the Q-side, but the king can neither support it, nor break through to the enemy pawns. White nevertheless shows that his pawns can become much more dangerous than the opponent's: 1 Ke3! Kb8.

Where White's king is aiming for is as yet not clear, and its opponent goes to attack the Q-side pawns. 1 ... e3 was bad due to 2 Kd3!

2 Kd2 Ke7 3 Ke2 Kd6 4 Kf1!

At last White's plan becomes clear: having

Shikova-Krumova
Bulgaria, 1972
taken up a frontal position with her king in front of the black pawns, she creates a passed pawn on the h-file, and uses her passed pawn on the Q-side to divert the opponent’s king. White’s calculations are precise. The immediate 4 g3 would have led to the opposite result: 4 ... f×g3 5 f×g3 h×g3 6 h4 f4 7 h5 f3+ 8 Ke3 g2, and the ‘self-propelled’ pawns queen on their own—9 Kf2 e3+ 10 Kg1 e2 etc.

4 ... Kc5 5 a6 b×a6 6 b×a6 Kb6 7 g3! f×g3 8 f×g3 h×g3 9 Kg2! (after 9 h4?? e3 it is Black who wins) 9 ... K×a6 10 h4. Black resigned, since after 10 ... f4 11 h5 e3 12 Kf3 Kb5 13 h6 Kc4 14 h7 Kd3 15 h8=Q she is one tempo too late.

But if the connected pawns are close to the queening square and can be supported by the king, the situation is significantly changed and the result will depend on the specific circumstances.

More Dangerous Passed Pawns

a×b4? c×b4 9 Kh6, as occurred in the game, was weaker: 9 ... a3 10 e7 K×e7 11 Kg7 (after 11 b×a3 b×a3 12 Kg7 a2 the black pawn queens with check) 11 ... a×b2 12 f6+ Kd7 13 f7 b1=Q 14 f8=Q Qg1+ 15 Kh6 Qe3+! and White resigned, since the exchange of queens is inevitable. 15 Kh7 Qh2+ 16 Kg7 (16 Kg6 Qd6+?) 16 ... Qe5+ etc. would have been no better.

8 ... c4 9 e7 K×e7 10 Kg7 e3 11 b×c3 b×a3 12 f6+ Ke6 13 f7 a2 14 f8=Q a1=Q 15 Qf6+, and White gains a draw.

The play in such positions is of an exceptionally concrete nature, and demands accuracy. The slightest delay or weakening of the position may prove fatal.

Pillsbury-Tarrasch
Nuremberg, 1896

623. The black pawns are not yet far advanced, and therefore White’s king is able to support his pawns: 1 e6! Kf6 2 f5 a5 3 Ke2 a4 4 Kf3 h5 5 Kg3 b5 6 Kh4 c5 7 K×h5 b4 8 Kh6.

White’s plan is to sacrifice his e-pawn and queen the f-pawn. From the viewpoint of the resulting queen ending, the preliminary 8

Quinteros-Andersson
Mar del Plata, 1981

624. White’s connected passed pawns are far advanced, but it is not apparent how he can support them with his king. Black’s chances appear to be better, but the game in fact ended in a win for White: 1 ... b5 2 Kg3 b4 3 Kf4 g5+ 4 h×g5 h×g5+ 5 K×g5 a5 6 d6 Kf7 7 Kf5 a4 8 e6+ Ke8 9 Kf6 b3 10 a×b3 a×b3 11 d7+ Kd8 12 Kf7. Resigns.

Later Tarrasch showed that 2 ... a5! would have been stronger, when after 3 Kf4 Kf7 4 h5 g×h5 5 Kf5 h4 6 d6 h3 7 e6+ Ke8 8 Kf6 h2 9 d7+ Kd8 10 Kf7 h1=Q 11 e7+ K×d7

225
12 e8 = Q + Kd6 13 Qd8 + Kc5 Black has every chance of winning the queen ending.

But in 1941 Fine showed that after 1 . . . b5 White can force a draw by 2 h5! The point is that after 2 . . . g × h5 White advances his king to f7 not via f5, but via h5, eliminating the dangerous black pawn on the way. Here is a possible continuation: 3 Kg3 a5 4 Kh4 b4 5 d6 Kf7 6 d7 Ke7 7 e6 a4 8 K × h5 b3 9 a × b3 a × b3 10 Kg6 b2 11 d8 = Q + K × d8 12 Kf7. Draw.

In the following example the connected pawns prove stronger than the isolated pawns on either wing.

Polikarpov-Yufyurof
USSR, 1975

625

If 3 g5, then 3 . . . d5 4 c × d5 c × d5 5 h4 d4 + 6 Ke2 Kf4 etc.

3 . . . d5 4 c5.

No better is 4 c × d5 c × d5 5 a4 d4 + 6 Ke2 d3 + 7 Ke3 Kd5 8 g5 Kc4 9 h4 Kc3, and the c-pawn queens.

4 . . . d4 + 5 Ke2.

If the king is actively supporting its passed pawns, and both sides queen a pawn, in certain cases this can lead to the creation of a mating attack. Here, for example, there could have followed: 5 Kd2 e3 + 6 Kd3 Kf4! 7 b5 Kf3 8 b × c6 e2 9 c7 e1 = Q 10 c8 = Q Qc3 mate, or 5 Kf2 d3 6 Ke3 Kd5 7 g5 Kc4 8 b5 Kc3! 9 b × c6 d2 10 c7 d1 = Q 11 c8 = Q Qf3 mate.

5 . . . d3 + 6 Ke3 Kd5 7 g5 Kc4 8 h4 Kc3 White resigns.

In endings of this type one normally has to reckon with the possibility of a queen ending arising. The correct assessment of such endings is extremely difficult. They require, firstly, a deep penetration into the special features of the position, and secondly, mathematically precise calculation.

The following Grigoriev study strikingly illustrates the diversity of procedures which have to be employed in playing these complicated endings.

Grigoriev, 1938

625. Black has the more active king, and it is this factor that enables him to carry out a typical plan—of creating two connected passed pawns in the centre. The fact that they can be actively supported by the king decides the game in Black’s favour.

1 . . . c6! 2 d × c6.

White does not manage to divert the opponent by the advance of his K-side pawns, e.g. 2 g5 c × d5 3 c × d5 K × d5 4 h4 Ke5 5 h5 d5 6 g6 d4 +, and Black wins.

2 . . . h × c6 3 b4.

An attempt to develop play on both wings.

626
626. The first impression is that the black pawns are more dangerous, but the stipulation—White to play and win—signifies that in fact the opposite is true.

First White advances his f-pawn: 1 f4 Kb4 2 g3!

Now the second pawn joins in. Incidentally, it would be a mistake to play 2 g4? Kc5, when Black is assured of a draw.

2 ... h x g3 3 h4.

Black’s king is outside the ‘square’ of this pawn, but he can advance his d-pawn.

3 ... d5 4 f5 Kc5 5 h5 d4 6 f6 Kd6 7 h6 d3 8 f7 Kc7 9 h7 d2, and now the familiar finish 10 f8 = Q + K x f8 11 h8 = Q + , and White wins.

Of course, instead of 2 ... h x g3, more tenacious is 2 ... Kc5! 3 g x h4 Kd5, trying at all costs to stop the pawns with the king.

4 h5 Kc6 5 h6! Kf7 6 f5 b5.

Thus the white pawns have been stopped, and Black reveals his trumps ... It is now that some exceptionally subtle play begins.

7 a3!

The only move to win, whereas the immediate approach of the king does not succeed: 7 Kf2 b4! 8 Kc3 e5 9 Ke4 c4 10 Kd4 c3 11 Kd3, d5, and White is unable to break up the enemy pawn chain by 12 a3? due to 12 ... b x a3 13 K x c3 d4 + , when it is Black who wins.

7 ... e5 8 Kf2 d5 9 Ke3!

The only way. On 9 Kf3? there would have followed 9 ... c4 10 Kc3 c3 11 Kd3 d4, when White can no longer win.

9 ... Kf6 10 h4 Kf7 11 h7! (by this pawn sacrifice White further strengthens his position) 11 ... Kg7 12 f6 + K x h7 13 h5 Kh6.

In the event of 13 ... b4 14 a x b4 c x b4 15 Kd4, or 13 ... c4 14 Kd4, or 13 ... d4 + 14 Ke4!, the king move to h6 is forced, which leads to variations considered below.

14 Kf4!

This decisive manoeuvre is possible only if the black king is at h6. Now three variations are possible, all leading to a won ending:

(a) 14 ... b4 15 a x b4 c x b4 16 Ke5(f5) b3

17 f7 Kg7 18 Ke6 b2 19 h6 + Kg6 20 f8 = Q b1 = Q 21 Qg7 + Kh5 22 h7 Qb6 + 23 Kd7 Qb5 + 24 Ke8! Qc6(e5) + .

Grigoriev shows that other continuations are weaker: 24 ... Qe8 + 25 Kc7, 24 ... Qe4 + 25 Kb8 Qb5 + (25 ... Qb4 + 26 Ka8) 26 Qb7 Qe8 + 27 Qc8 Qb5 + (e5) 28 Ka8. If instead 24 ... Q x a6 +, then 25 Kb8, as in the main variation: 25 ... Qb6 + 26 Qb7 Qd6 + 27 Ka8 etc.

25 Kb8 Qb6 + 26 Qb7 Qd6 + (if 26 ... Qf6 or 26 ... Qd4, then 27 Qc8 and 28 Ka8) 27 Ka8, and the check at d8 or f8 is met by 28 Qb8, and 27 ... Qf6(e5) by 28 Qc8.

(b) 14 ... c4 (the most tenacious) 15 Kf5 c3 16 f7 Kg7 17 Ke6 c2 18 h6 + Kg6 19 f8 = Q c1 = Q 20 Qf6 + 1 Kd2 21 h7 Qc6 + (stronger than 21 ... Qc8 + 22 Kd6! Q x a6 + 23 Ke5 or 22 ... Qb8 + 23 K x d5 Qa8 + 24 Kd4) 22 Kf7 Qd7 + .

Or 22 ... Qc7 + 23 Qe7 Qf4 + 24 Kg7 Qg3 + 25 Kf8 Qc3! (this reply, overlooked by Grigoriev, was pointed out by Bondarevsky) 26 Kf7! Qh8 (there is nothing better; if 26 ... Qf3 + , then 27 Qf6) 27 Qc2 + Kg5 28 Qg2 + Kf4 29 Qg7, and White wins.

(c) 14 ... d4 (the most harmless) 15 Kf5 d3 16 f7 Kg7 17 Ke6 d2 18 h6 + Kg6 19 f8 = Q d1 = Q 20 Qg7 + Kh5 21 h7 Qb3 + 22 Kd6 Qd3 + 23 Kc7, and there are no more checks.

Here are two further instructive examples, demonstrating the features of such endings.

627. White played 1 h5, creating the threat of a breakthrough.
What should Black do now? If 1...a×b5, then 2 c5! d×c5 3 d×c5 b×c5 (3...b×a4 4 c×b6 a3 5 b7 a2 6 b8=Q a1=Q 7 Qf8 mate) 4 a5 b4 5 a6 b3 6 Kd3, and White wins. No better is 1...c×b5 2 a5! b×a5 3 c5 d×c5 4 d×c5, since one of White’s passed pawns quickly queens.

But Black can solve all his problems by the unexpected counter-blow 1...d5+!, with these possibilities:

(a) 2 c×d5 c×d5 + 3 K×d5 a×b5 4 Kc6 (totally bad is 4 a×b5 g4 5 h×g4 h×g4 6 Kc4 g3 7 Kf3 K×f5 8 K×g3 Ke4) 4...g4 5 h×g4 h4! (this is the whole point—the h-pawn queens with check) 6 d5 h3 7 d6 h2 8 d7 h1=Q +.

(b) 2 Kd3 d×c4 + 3 K×c4 a×b5 + 4 a×b5 c×b5 + 5 K×b5 g4 6 h×g4 h×g4 7 d5 g3 8 d6 g2 9 d7 Ke7.

(c) 2 Ke3 c×b5 (2...a×b5? 3 c5! b×c5 4 a5) 3 c5 b×c4 4 d×c5 K×f5, and Black wins.

After 1 b5 the game was sent for adjudication, and Black was awarded a win.

628. Black’s pawns on the Q-side look very threatening, but without the help of their king they cannot promote. White’s problem is to tie down the opponent’s king by the threat of advancing his K-side pawns, but in such a way that they do not become easy booty. The following set-up would be ideal for White: pawns at f4, g5 and h5. Then the threat of h5–h6 would tie the black king to this part of the board.

Naturally, in the first instance Black prevents f3–f4: 1...e5! 2 Kf1.

2 f4 e×f4 3 g×f4 is hopeless, since the black king deals with these pawns without difficulty. It is also bad to advance the pawns to g5 and h5, since they can be attacked from f5.

2...b5 3 Kc2 c5? (this instructive mistake allows White to equalize; Black overlooks the opponent’s threat to repair his pawn structure) 4 Ke3!, and a draw was agreed, since the threat of 5 f4 cannot be parried.

Correct was 3...a5! 4 Ke3 b4, when 5 f4 is decisively met by 5...e×f4 6 K×f4 a4, since the white king is outside the a-pawn’s ‘square’; 7 Ke4 b3 8 a×b3 a3 etc. If instead White plays 4 Kd3, the play takes a standard course: 4...a4 5 Kc3 c5 6 g4 Ke7 7 Kd3 Ke6 8 Kc3 Kd5 9 a3 Ke6 10 Kd3 Kd6 11 Kc3 Kd5 12 Kd3 b4 13 a×b4 (13 Kc2 b×a3 14 Kc3 c4 15 Kc2 Kd4 16 h5 c3, winning) 13...c×b4 14 Kc2 Kc4 15 Kb2 a3 + 16 Ka2 Kc3, and Black wins.

We have examined a continuation where in the main White manoeuvres with his king. If, at any point he moves one of his pawns on the
K-side, the black king immediately makes for there. For example: 10 h5 Kf6! 11 f4 (11 Kb2 Kg5 12 Kc3 e4 13 fxe4 Kxg4 14 e5 Kxh5 15 f4 Kg6, then 16 ... Kf5 and 17 ... g5, eliminating the K-side pawns) 11 ... e x f4 12 f3 Ke5 13 Kd3 b4 14 a x b4 c x b4 15 Kc4 a3 16 Kb3 Kd4 17 Ka2 Ke3, and Black wins as in the previous variation (pointed out by Botvinnik).

9.3 PROTECTED PASSED PAWN

The possession of a protected passed pawn, or the possibility of creating one, is an important, and in many cases decisive positional advantage. If the king does not have to worry about this pawn, does not have to defend it, it can be directed into the enemy position to attack the opponent's pawns. In short, a protected passed pawn allows the king to become more active. And on the contrary, a protected passed pawn will restrict the opponent's king, which will be unable to move far away from this pawn.

The following example can well be called typical.

629. The greater manoeuvrability of White's king—a consequence of the fact that his e-pawn is protected and passed—enables him to win.

1 Ke3 Kg5 2 Kf3!

White's problem is to put his opponent in zugzwang. For this reason he should not be in a hurry to advance his pawns on the opposite wing: the tempi may later come in useful.

2 ... a5.

2 ... f4 loses immediately, since White can employ a procedure which in such positions is standard—the exchange of pawns with the aim of penetrating into the enemy position on the critical squares of the opponent's pawn(s): 3 e6! Kf6 4 K x f4 K x e6 5 Kg5, and White wins the d-pawn.

3 Kg3 (3 a4 b6 4 Kg3 is also possible) 3 ... f4 + 4 Kh3!

This is the whole point. Now the black king cannot move to h5, since after 5 e6 Kg6 6 Kg4 White's king breaks into the opponent's position. And the advance of Black's pawns on the opposite wing can no longer change the zugzwang situation which has arisen.

4 ... b5 5 a3 b4 6 a4 Kf5 7 Kh4!, and White wins.

There is one further important advantage of a protected passed pawn: an enemy outside passed pawn is not usually dangerous, since, with no longer having to worry about its own pawn, the king can head off to deal with it.

Averbakh-Bebchuk
Moscow, 1964

(variation from the game)
630. The plan is simple: White attacks the outside passed pawn with his king.
1. Ka4 Ke6 2 Kb4 Kd6 (no better is 2 . . . b6 3 
Kc4 Kd6 4 Kb5 Kc7 5 h5) 3 Kd5 Kc7 4 Ke5
Kd8 5 Kb6 Ke8 6 h5, and White wins.

It should be noted that White won only because he had a reserve tempo. Without it all his winning attempts would have been in vain.

In situations where the outside passed pawn cannot be eliminated ‘for free’, an alternative method is sometimes possible: the protected passed pawn is exchanged for it, but in doing so the king penetrates into the enemy position. With this plan it is very important that there should be pawn weaknesses in the opponent’s position.

The following two examples show various aspects of the implementation of this plan.

Em. Lasker, 1921

631. The immediate creation of a protected passed pawn does not win here: 1
b5+? cxb5+ 2 Kb4 Kd7 3 Kxb5 a6+, and Black has set up a fortress into which the white king cannot penetrate.

Correct is 1 Kb3! Kd7 (1 . . . Kb5 2 a6!
Kxb6 3 Ka4 comes to the same thing) 2
a6+! Kxa6.

The attempt to set up a barrier with pawns does not succeed: 2 . . . Kc7 3 Ka4 Kd7 4 b5
Kc7 5 bxc6 Kc8 6 c7! Kxc7 7 Kb5 Kd7 8
c6+ Kd6 9 c7 Kxc7 10 Kc5, and White
wins.

3 Ka4 Kb7 4 b5 a6.

Things are not improved by 4 . . . cxb5+ 5
Kxb5 Kc7 6 Ka6 Kb6 7 c6 Ka8! (7 . . . Kc7 8
Kxa7 Kxc6 9 Ka6?) 8 Kb5! (8 c7?? leads to
 stalemate) 8 . . . Kb8 9 Kc5.

5 bxc6+ Kxc6 6 Ka5 Kb7 7 c6+ Kxc6 8
Kxa6 Kc7 9 Kb5, and White wins.

Fine, 1941
Malzells, 1956

632. This position is given in Fine’s book as an illustration of the strength of the protected passed pawn. In his opinion, the win does not present any difficulty: 1 Kb4 Kb6 2
Ka4 a6 3 Kb4 Kc6 4 Kc4 Kb6 5 Kd5! Kb7 (5
. . . a5 6 Kd6! etc.) 6 Kb6 Kc8 7 Kc6, and after eliminating the a-pawn White wins easily.

Fine’s analysis was later criticized by a number of theorists, who showed that Black’s defence was by no means the best. Thus, instead of 2 . . . a6, more tenacious is 2 . . .
Kc6 3 Kc5 Kc7 4 Ka6 Kb8 (or 2 . . . a5 3 Ka3
Kc5 4 Kb3 Kb5), when the a-pawn can be
won only by giving up the e-pawn. The
correct plan is in fact to exchange these pawns and then invade with the king into the enemy position! But first, weaknesses must be created there. Therefore White plays 5 b5!.
Protected Passed Pawn

g×h5 6 e6 Kc7 7 K×a7 Kd6 8 Kb6 K×e6 9 Kc6! Kf7?

This suggestion of Euwe and Hooper (1958) is stronger than Maizelis’s 9...Kf7 10 Kd6 Kg6 11 Ke6 h4 12 g×h4 Kg7! 13 K×f5 Kf7 14 Ke5 Ke7 15 h5! Kf7 16 Kd6 Kf6 17 h6! (230).

After 9...Kf7, before he can reach position 230, White has to avoid a number of pitfalls!

10 Kd7!

But why not 10 Kd5? Because then comes 10...Ke7!, when White can no longer win. Black must try to ensure that, when the f5 pawn is captured, his king is already at f7. Then...h4 followed by...Ke7 is possible, and the by-pass plan employed in example 230 does not succeed: White is short of a tempo. The features of such positions are explained in detail in the analysis of examples 230–232.

It is not hard to see that here we have a case of corresponding squares: d5/e7(g7), c5/f7 and d6/f6.

10...Kf8 11 Kd6!!

The natural 11 Ke6? Ke8! 12 K×f5 leads to a draw after 12...h4 13 g×h4 Ke7!, while 12 Kf6 can be met by 12...h4! 13 g×h4 Kf8 with the same result.

11...Kg7 (11...Kf7 12 Ke5!) 12 Ke7! Kg6.

No better is 12...Kg8 13 Ke6! Kf8 14 Kf6! h4 15 g×h4 Kg8 16 K×f5 Kf7, when we reach a position from Maizelis’s variation.

13 Ke6.

At last White wins the pawn without allowing the opponent's king to go to f7. But the struggle is not yet over: there is one last problem for White to solve.

13...h6!

This defence was pointed out by E. Richter, who thought that in this way Black could draw. But this is not so (227).

14 Ke5 Kg7 15 K×f5 Kf7 16 Ke5 Ke7 17 f5 Kf7 18 f6 Ke8! 19 Kf4! Kf8 20 Ke4 Kf8 21 Ke5! Kf8 22 Ke6 Kf8 23 f7+ Kf8 24 Kf6 h4 25 g×h4 h5 26 Kg6, and White wins.

It only remains to add that, if on 1 Kb4 Black replies 1...h5, not allowing the breaking-up of his pawn chain, there can follow 2 Ka5 Kb7 3 Kb5 Kc7 4 Ka6 Kb8 5 e6 Kc7 6 K×a7 Kd6 7 Kb6 K×e6 8 Ke6, and the opposition of the kings allows White soon to win the g6 pawn, e.g. 8...Ke7 9 Kc7 Ke6 10 Kd8 Kf7 (10...Kd5 11 Ke7 Ke4 12 Kf6 Kf3 13 K×g6 K×g3 14 Kg5!) 11 Kd7 Kf6 12 Ke8 Kg7 13 Ke7 Kh7 14 Kf7 Kh6 15 Kg8, and White wins.

In the following position, by creating a protected passed pawn, White managed to win the opponent’s outside passed pawn. But had Black defended correctly, an undermining manoeuvre would have allowed him to neutralize White’s threats.

Tarrasch-Schiffers
Nuremberg, 1896

633. Black irreparably weakened his pawns by 1...h6?, and after 2 h4! the threat of 3 h5 forced him to play 2...h5, depriving him of the possible undermining move...g5.

The game continued 3 Kd5 Kd7 4 K×e4 K×d6 5 Kd4 Kc6 6 e4 Kd6 7 Kc4 Kc6 8 Kb4 Resigns. After 8...Kb6 the attack on the outside passed pawn is decisive: 9 Ka4 Kc6 10 Ka5 Kc7 11 Ka6 Kb8 12 c5, or 9...Kc5 10 Ka5 Kd4 11 c5. The protected passed e-pawn proves to be the decisive factor.
In his commentary Tarrasch wrote: 'As analysis after the game showed, the win would have been the most difficult after 1... Kc8. First White has to advance his h-pawn.' But he did not give any variations demonstrating the win.

Many analysts sought this demonstration, but each time came to the conclusion that a win is impossible. We give the most recent analysis by Botvinnik and Minev.

1... Kc8! 2 h4 Kd8 3 h5!? (3 Kd5 Kd7 4 Kx e4 Kxd6 5 Kd4 Ke6 6 e4 h6 7 Ke4 a5 8 Kb5 Kf6 9 Kxa5 g5) 3... Kc8 (3... g x h5? 4 f5) 4 h6 a5!

At just the right time! Only in this way can Black save the game. Bad is 4... Kd8 5 Kd5 Kd7 6 Kx e4 Kx d6 7 Kd4 Ke6 8 e4 Kd6 9 Ke4 Kc6 10 Kb4 Kb6 11 Ka4 Kc5 12 Ka5 Kd4 13 e5 Kd5 14 Ka6 Ke6 15 Kx a7 Kf5 16 Kb6 g5 17 f x g5 Kx g5 18 Ke6, and White wins.

5 Kb5 Kd7 6 Kx a5 Kx d6 7 Kb6 Ke6 8 Kc6 Kf5 9 Kd5! g5 10 f x g5 Kx g5 11 Kx e4 Kx h6. Draw.

The undermining manoeuvre is an important way of battling against a protected passed pawn, and in such situations it must always be taken into account.

A protected passed pawn may prove stronger even than two enemy passed pawns, provided, of course, that these pawns are not far advanced.

634. The attempt by White to break through to the c7 pawn with his king does not succeed: 1 Kg4 Ke7 2 Kf3 Kd6 3 Kg6 Kx d5 4 Kx g7 Kc4 5 a4 Kb4 6 Kf7 Kx a4 7 Ke7 Kx b5 8 Kd7 a5 9 Kx c7 a4 10 Kb7 a3, and the pawns queen simultaneously. Defending the d-pawn, so as to gain time for a3-a4, also does not work: 3 Ke4 g5 4 a4 g4 5 Kf4 Kx d5 6 Kx g4 Ke4! 7 Kg5 Ke5, and Black's king completely neutralizes its opponent.

The only way to win is by the sacrifice of a pawn, with the aim of creating a protected passed pawn: 1 d6! c x d6 2 Kg4 Kd8 3 Kf5 Ke7 4 a4!

Black's misfortune is that he has no reserve tempo. He is forced to retreat his king.

4... Kd8 (no better is 4... g6 + 5 Kx g6 Ke6 6 Kg5 d5 7 Kf4 Kd6 8 Kf5) 5 Ke6 Kc7 6 Kd5 g5 7 Ke4 Kd8 8 Kf5 Ke8 9 Kx g5 Ke7 10 Kf5, and White wins.

The following example is highly instructive.

Bottlik, 1952

635. White’s positional advantage lies in his protected passed pawn and the active position of his king, enabling him easily to invade the opponent’s position. In spite of
Black's extra pawn, this proves sufficient for a win.

1 Kd4 f4 2 Ke5 a5!

Black aims for counter-play. If, for example, 3 b x a5?, then 3 ... K x e5, with equal chances.

3 a3 a4 4 Kf5 d4 5 Ke4 d3 6 K x d3 Kd5.

By giving up a pawn, Black has returned the opponent's king to its initial position and has markedly activated his own. Now White's problem is to attack the K-side pawns with his king, but if Black succeeds in playing ... g4, this will be impossible, e.g. 7 Ke2? g4! 8 Kd3 Ke5 9 c6 Kd6 10 Ke4 K x c6 11 K x f4 Kd5 12 K x g4 Kc4 etc.

7 g3!

The only way to win. By undermining the pawns, White creates in the opponent's position a weak pawn which can then be successfully attacked by the king.

7 ... f x g3.

There is nothing better. If 7 ... Ke5, then 8 g x f4 + g x f4 9 Ke2 Kd5 10 Kf3 Ke5 11 c6 Kd6 12 K x f4 K x c6 13 Ke5, and White wins.

8 Ke2 (8 Ke3 g4 9 Ke2 is also possible) 8 ... Ke5 (8 ... g4 9 Kf1 Ke5 10 Kg2 etc.) 9 Kf3 Ke6 10 K x g3 Kf5 11 Kf3.

11 c6? would be over-hasty: 11 ... Ke6 12 Kg4 Kd6 13 K x g5 K x c6 14 Kf5 Kd5, with a draw.

11 ... Ke5 12 Kg4 Kf6 13 c6 Ke6 14 K x g5 Kd6 15 Kf5 K x c6 16 Ke6, and White wins.

Problems over exploiting a protected passed pawn usually arise when it is difficult to penetrate with the king into the enemy position, and one then has to resort to various ruses in order to make a breach.

636. White carried out a typical plan: by 1 h5! he tied-down Black's K-side pawns. Now 1 ... e5 is met by 2 g4, and 1 ... f5 by 2 f4, when White's king penetrates into the opponent's position. It only remains to create a zugzwang situation, and this is easily done: 1 ... Kb7 2 Kf1 Ke8 3 Ke2 Kd7 4 Kd3 Kc8 5 Kc4 Kb7 6 Kb4 Ka6 7 Ka4 Kb7 8 Ka5 Kb8 9 Ka6 e5 10 g4 Resigns.

But what if it had been Black to move, when he himself could have played 1 ... h5, not allowing this plan to be carried out? The resulting position deserves special consideration.

637. Now little is achieved by taking the king to the Q-side, to create a zugzwang situation: 1 Kf1 e5 2 Ke2 f5 3 Kd3 Ka8 4 Kc4 Kb8 5 Kb4 Ka8 6 Ka5 Kd7 7 g3 g6 8 f3 Kb8, and 9 Ka6 is not possible due to 9 ... e4 10 f x e4 f x e4 11 b7 e3 12 Kb6 e2, when it is Black who wins.

Correct is 1 f4!, preparing to make a breach in Black's defences, a plan which he has no
way of opposing. A possible continuation is 1 ... Kb7 2 Kf2 Ka8 3 Kf3 Kb8 4 g4! g6 (4 ... h×g4+ 5 K×g4 g6 h5 etc.) 5 g5! f5 6 Ke3, and White’s king penetrates via e5 into the enemy position.

Incidentally, after 1 Kf1 e5, instead of the incorrect 2 Ke2? it is still possible to play 2 f4! e×f4 3 Kf2 g5 4 Kf3 Kb7 5 Ke4 Kb8 6 Kf5! f3! 7 g×f3 g×h4 8 Kf4 f5 9 Ke3, and White wins by eliminating the dangerous passed pawns on the h-file.

Thus a player with a protected passed pawn should strive to penetrate with his king into the opponent’s position. If this invasion can be prevented, the game usually ends in a draw.

The following example illustrates well the various possibilities of defence against an invasion.

Thomas-Sämisch
Baden Baden, 1925

A desperate attempt to create play on the K-side. If 9 b4, then 9 ... a×b4 10 a×b4 b5! 11 c×b5 c×d5+ 12 K×d5 Ke7, and Black wins easily.

9 ... b×g5 10 Kf3 b5 11 a4 b×a4 12 b×a4 c×d5 13 c×d5 Ke7 (the breach in the defences has been made, and Black’s king heads into the enemy position) 14 Kg4 e4! White resigns.

In 1929 Sozin subjected this ending to a thorough analysis, but this was published by Maizelis only in 1956. Sozin established that 8 Ke4? was the decisive mistake, and that by 8 b4! White could have drawn, e.g. 8 ... a×b4 9 a×b4 b5! (otherwise after 10 b5! Black can no longer break through with his king) 10 c×b5 Kc8 11 b6! Kb7 12 b×c7 K×c7 13 Ke4 Kb6 14 Kd3 Kb5 15 Ke3 e4 16 Kd4 K×b4 17 K×e4 Kc5 18 h4 Kc4 19 h5 g5 20 Kf5! K×d5 21 Kg6 Ke5.

The best plan. If Black plays to queen his d-pawn, only he can lose. He is saved by the fact that he can stop the white king moving off the h-file.

22 K×h6 Kf6 23 Kh7 Kf7 24 K×h6 Kf6 with a draw.

However, in this complicated ending Black too did not play the best. Instead of losing a tempo with 4 ... g6, it was correct, as shown by Sozin, to move his king immediately to the Q-side: 4 ... Kf6! 5 Kf3 (5 b3 is too late due to 5 ... b5! 6 c×b5 e4 7 Kf4 e3! 8 K×e3 Ke5 9 a4 K×d5 10 b4 a×b4 11 a5 Ke5 12 a6 K×b6 13 Kd4 c5+) 5 ... Ke7 6 b3 Kd7 7 a3 c6! 8 b4 a×b4 9 a×b4 b5!, and Black must win.

On the basis of this analysis, Sozin concluded that White could nevertheless have saved this ending, but he went wrong on the 4th move by playing 4 Kg3?! He should have continued 4 b3!, and if 4 ... Kf6, then 5 a3 Ke7 6 b4, as already considered in the note to White’s 8th move. If Black answers 4 b3 with 4 ... e4+, then 5 Ke3! K×g4 6 K×e4, again with a draw.

In reply to 4 b3, the move suggested by Zinar, 4 ... Kb4, comes into consideration.

Black tries to invade with his king on the
Protected Passed Pawn

opposite wing. Now 5 h3 is bad due to 5 ... g6 6 a3 e4+! 7 K×e4 Kg3 8 b4 a×b4 9 a×b4 K×g2 10 Kf4 K×h3 11 g5 h5 12 Kf3 h4 13 b5 Kh2 14 Kf2 h3, and Black wins.

The most precise defence is 5 a3! g6! 6 b4 a4! 7 b5, and only after 7 ... g5—8 h3! Now on 8 ... e4+ 9 K×e4 Kg3 White can play 10 Kf5! K×g2 11 Ke6! (11 Kg5? K×h3) 11 ... K×h3 12 Kd7 K×g4 13 K×e7, when he cannot lose.

However, as was correctly pointed out by Zinar, the safest move order for White is 1 ... Kh7 2 b3 Kg6 3 Kf2!, when 3 ... Kg5 can be met by 4 Kg3!, and 3 ... Kf6 by 4 Kf3!

In the following examples, due to certain features of the position which we can rightfully call exceptional, the protected passed pawn does not provide a win. Each time the opponent’s king finds a stalemate shelter.

639. The typical plan—the exchange of passed pawns and the breakthrough of the king into the enemy position—runs into an unexpected refutation: 1 Kh3 Kf5 2 Kh4 Kg6 3 d6 Kf6 4 K×h5 Ke6 5 Kg5 K×d6 6 Kf5 Kc6 7 Ke5 Kb6 8 Kd5 Ka5! This, it turns out, is the point—after 9 K×c5 Black is stalemated! Therefore—draw.

The following study by Chekhover is extremely interesting, a stalemate finish arising as a result of subtle manoeuvring.

Chekhover, 1956

640. At first sight White stands badly, since Black can create by force a protected passed pawn, and then set off with his king to eliminate the opponent’s passed pawn...

1 Ke4 c6 2 Kf5 d5 3 Ke5 d4 4 Ke4 Kg7 5 Kd3!

The attempt to hold on to the h-pawn leads to defeat: 5 Kf4? Kg6 6 Kg4 Kh6 7 a3 Kg6 8 h5+ Kh6 9 a4 d3! 10 Kf3 K×h5 11 Ke3 Kg4 12 K×d3 Kf3, and by penetrating into the opponent’s position, Black wins.

5 ... Kg6 6 Kc2 Kf5.

If 6 ... Kh5 7 Kb3 K×h4, White reveals his cards: 8 Ka4! d3 9 Ka5! d2 10 a4 d1 =Q—stalemate. Therefore Black is not in a hurry to capture the pawn.

7 h5! Kg5 8 Kb3!

If 8 h6?, then 8 ... K×h6 9 Kb3 d3! 10 Kc3 Kg5 11 K×d3 Kf4, and Black wins.

8 ... Kh6! 9 a3!! (of course, not 9 Kc2? K×h5) 9 ... Kg7 10 Kc2 (or 10 Kb2) 10 ... Kh7 11 Kb2 Kh6 12 Kb3 K×h5 13 Ka4! d3 14 Ka5! d2 15 a4 d1 =Q—stalemate.

Zinar (1979) established that Black can manoeuvre more cunningly: 5 ... Kh6 6 Kc2 Kg6!, setting White serious problems.

Nikolayevsky-Taimanov
Tbilisi, 1967
It is bad, for example, to play 7 Kb3 (or 7 a3) 7 ... Kh5 8 a3 (8 Ka4 d3 9 Ka5 d2 10 a4 Kg4, or 9 Kb3 K×h4 10 Kc3 Kg4 11 K×d3 Kf3, and Black wins) 8 ... Kg6! 9 Kb2 (9 h5 + Kh6! 10 Kc2 K×h5) 9 ... Kf5! 10 Kb3 (10 h5 Kg5 11 Kb3 Kh6! or 10 Kc2 Kg4! 11 Kb3 Kh5! etc.) 10 ... Kf4! 11 h5 Ke3 12 h6 d3 13 h7 d2 14 h8 = Q d1 = Q+, and Black easily wins this queen ending.

White should play 7 Kb2!! Kf5 8 h5! (8 Kb3? Kf4! 9 h5 Ke3) 8 ... Kg5 9 Kh3! Kh6 10 a3 K×h5 11 Ka4! d3 12 Ka5 d2 13 a4 d1 = Q—stalemate.

Zinar’s suggested improvement for Black improves the study significantly.

In certain cases the creation of a protected passed pawn can be a highly effective method of defense, neutralizing a positional or material advantage.

Troyanska-Nedelcheva
Bulgaria, 1974

9.4 BREAKTHROUGH

The possibility of carrying out a pawn breakthrough—the sacrifice of one or several pawns, to create a passed pawn—is a highly important, and often decisive feature of a position. In the course of play it is very important to foresee the possibility of a breakthrough for oneself, and to forestall in time the threat of a breakthrough by the opponent.

The following example is typical.

Averbakh-Bebchuk
Moscow, 1964

641. White’s king is threatening to penetrate into the enemy position via c4 and b5. How can Black forestall this plan?

The game went 1 ... f6 2 f4 g5 3 g3! h5 4 h4 g×f4 5 g×f4 f×c5 6 f×e5 e6 7 d6, and here Black risked a breakthrough: 7 ... b5 8 a×b5! c4 9 Ke3 c×b3 10 Kd3 a4 11 Kc3 Kd8 12 b6 Ke8 13 Kb2. As a result, a familiar position (620) arose with pawns on the sixth rank, in which Black inevitably ends up in zugzwang.

Correct was 1 ... h5!! 2 Ke3 (now 2 a×b5 c4! 3 Ke3 c×b3 4 Kd3 does not succeed, due to 4 ... Kc7! 5 Kc3 a4 6 Kb2 Kb6—6 ... Kd7 7 b6—7 d6 e×d6 8 e×d6 Kd7 9 Ka3 Kb6) 2 ... c4! 3 b×c4 b4! 4 c5 (4 Kd4 c6) 4 ... e6 5 d6 Kc6 6 Kd4 Kd7 7 Ke4 Ke6 8 f4 Ka7 with a draw (variation by Minev).

642. Black has an outside passed pawn, and if his king were on the K-side or in the centre, the chances would be only on his side. But his king is at some distance from the main mass of pawns on the K-side, and this allows White to carry out a pawn break-
through. True, it is a very close thing. Were the black king a move nearer to the pawns, say at e6, the breakthrough would not be dangerous.

1 e4 Ke6 2 e5! f × e5.

After 2 ... Kd7 3 e6+ White sets up a protected passed pawn, and then sends his king over to the b7 pawn (630).

3 g5! h × g5.

No better is 3 ... Kd6 4 f6 Ke5 5 f × g7 Kf7 6 g × h6 b5 7 Ke4 b4 8 Kd3!, when White manages to eliminate both pawns.

4 f6! Resigns: after 4 ... g × f6 5 h5 the lone white pawn cannot be prevented from queening.

Some curious subtleties associated with a breakthrough are shown in the following practical example.

Zubarev-Grigoriev
Leningrad, 1925

643

643. After 1 ... b5! 2 a × b5+ Black made the at first sight incomprehensible move 2 ... Kb6!!

But the explanation is quite simple: to carry out the breakthrough he will have to play 3 ... a4 (or 3 ... c4), and if his king is at b5 he will lose a second tempo. For example: 2 ... K × b5 3 Ke6 c4 4 b × c4+ K × c4 5 f4, and the queens appear simultaneously.

But after the move played, Black wins.

3 Ke6 a4! (had White played 3 f4 or 3 Kg8, 3 ... c4! would have been correct, but here this is wrong in view of 4 b × c4 a4 5 Kd6! a3 6 c5+ with a draw) 4 b × a4 c4 5 f4 d3, and Black won quickly.

More tenacious was 3 Ke7! a4! 4 b × a4 c4 5 f4 d3 6 c × d3, and if 6 ... c × d3, then 7 a5+!

In this case, as shown by Knyazev, the only correct move is 7 ... K × a5! (after 7 ... K × b5 8 a6 K × a6 9 f5 d2 10 f6 d1 = Q 11 f7 it is a draw, since the black king is outside the winning zone) 8 f5 d2 9 f6 d1 = Q 10 f7, and thanks to his b5 pawn Black wins, e.g. 10 ... Qe2+ 11 Kd7 Qd3+ 12 Ke7 Qe4+ 13 Kd7 Qd5+ 14 Ke7 Qe5+ 15 Kd7 Qf6 16 Ke8 Qe6+ 17 Kf8 Qd7! 18 b6 Qd8+ 19 Kg7 Qg5+ 20 Kh7 Qf6 21 Kg8 Qg6+ 22 Kf8 K × b6 23 Kc7 Qg7 24 Ke8 Kc6(c7) 25 f8 = Q Qd7 mate.

In the above examples a pawn majority on one wing has suggested the idea of a possible breakthrough. It may happen that there is no pawn majority, but the pawn formation itself allows a breakthrough to be made.

644

644. In this amusing position White wins by the advance of any of his pawns: in each case one of the pawns breaks through to queen. For example: 1 a6 b × a6 (1 ... b6 2 d6) 2 d6 c × d6 3 b6 or 3 c6; 1 b6 a × b6 (1 ... c × b6 2 a6 b × a6 3 c6) 2 c6 d × c6 3 d × c6 b × c6 4 a6; 1 c6 d × c6 (1 ... b × c6 2 d6 c × d6
3 b6) 2 b6 a×b6 3 a×b6 c×b6 4 d6; 1 d6
c×d6 (1 ... c6 2 a6) 2 a6 b×a6 3 b6 or 3 c6.
Here Black has no way of preventing the
breakthrough.

Another form of pawn quartet is also preg-
nant with the threat of a breakthrough.

645. Here 1 b5 (or 1 c5) is possible, then 2
c5 d×c5 3 b6 c×b6 4 d6, and the d-pawn
queens. Incidentally, even if it is Black to
move, he cannot avert the threat of a
breakthrough. Therefore in such situa-
tions the black king cannot move outside the
d-pawn's 'square', which reflects strongly on its
activity.

The following example is highly instruc-
tive.

analysis by Em. Lasker, 1914

646. Black to move continues 1 ... a4,
creating the threat of a breakthrough, so that
White cannot play 2 Kh5 c4 (or 2 ... b4). He
is forced to retreat: 2 Kf3 Kg5 3 Ke2 c4! 4 Kd2
Kf4 5 Ke2 c3! 6 b×c3 d×c3 7 Kd1 b4! 8 Kc1
b3 9 c×b3 a×b3 10 a4 Ke3, and Black wins.

If White begins, he is able to avert the
threat of a breakthrough by 1 b3, and after 1
... Kg6 the draw is obvious. But by 1 a4! he
can attempt to play for a win. Now 1 ... b4
loses after 2 Kh5 c4 3 b3! The correct contin-
uation is 1 ... b×a4! 2 Kh5 Ke6 3 Kg6 Kd6 4
Kf6 c4! 5 d×c4 Kc5 6 K×c4 K×c4 7 Kd6 d3
8 c×d3 + Kb3 9 e5 K×b2 10 b6 a3 11 e7 a2 12
e8 = Q a1 = Q 13 Qe5 ÷ Kb1 14 Q×a1 +
K×a1 15 Ke5 Kb2 16 Kb5 Kc3. Draw.

Where there is the threat of a break-
through, events must be watched very closely,
since any inaccuracy may prove fatal.

Ed. Lasker-Mohle
Berlin, 1904

647. This example is interesting for the
mistakes made by both sides. White is threat-
ening by g4–g5 to create the threat of a
breakthrough, so Black played 1 ... h6? and
after 2 f4? f6 3 g5 Kd4 he went on to win. But
the incorrect 1 ... h6 could have been met by
2 f6! g×f6 3 f4 Kd4 4 g5 f×g5 5 f×g5 Ke5 6
g×h6 Kf6 7 Kc2, when it is White who wins.
The correct continuation was $1 \ldots f6!$, and if $2 \, b6 \, g \times h6 \, 3 \, f4$, then $3 \ldots \text{Kd}5$ etc.

Weinstein-Rohde
Lone Pine, 1977

648. White has an active king and the possibility of creating an outside passed pawn. If Black does not find some way of countering this, he will lose. The game went $1 \ldots h4? \, 2 \, g \times h4 \, g \times h4 \, 3 \, \text{Kd}4 \, \text{Ke}6 \, 4 \, a5 \, \text{b} \times a5 \, 5 \, b \times a5 \, \text{Kd}6 \, 6 \, a6 \, \text{Kc}6 \, 7 \, \text{Ke}5$, and the play took a standard course; after $7 \ldots \text{Kb}6 \, 8 \, \text{K} \times f5 \, \text{K} \times a6 \, 9 \, \text{K} \times e4$ Black resigned. And yet he could have won the game by a spectacular breakthrough!

$1 \ldots f4! \, 2 \, g \times f4 \, (2 \, \text{Kd}4 \, e3! \text{ comes to the same thing}) \, 2 \ldots g \times f4 \, 3 \, \text{Kd}4 \, e3! \, 4 \, f \times e3$.

$4 \, \text{Kd}3$ also fails to save the game: $4 \ldots f3! \, 5 \, g \times f3 \, h4 \, 6 \, \text{Ke}2 \, h3 \, 7 \, \text{Kf}1 \, e2+ \text{ or } 7 \ldots h2 \, 8 \, \text{Kg}2 \, e \times f2$.

$4 \ldots f3! \, 5 \, g \times f3 \, h4$, and the black pawn cannot be stopped.

A breakthrough is especially dangerous when a passed pawn is also present.

649. A protected passed pawn is normally stronger than an isolated one, but here the decisive factor is Black's defective pawn formation, which allows the possibility of a breakthrough.

White wins by force: $1 \, e5! \, d \times e5 \, 2 \, a5! \, b \times a5 \, 3 \, b6! \, c \times b6 \, 4 \, d6$ (by the sacrifice of three pawns, White has created another passed pawn) $4 \ldots \text{Kf}6 \, 5 \, g5+ \text{ (this gain of tempo is the whole point)} \, 5 \ldots \text{Ke}6 \, 6 \, g6 \, a4 \, 7 \, g7 \, \text{Kf}7 \, 8 \, d7$, and wins.

In the above positions one of the sides has possessed various advantages, but they have all faded before the threat of a breakthrough. Therefore it can rightfully be said that the breakthrough is perhaps the strongest tactical weapon in a pawn ending.

Here are some further instances of a breakthrough, arising due to various defects in the pawn formation.

Barasz-Schönmann
1927

650. The game went: $1 \, b6 \, \text{Ka}6 \, 2 \, h3? \, e3 \, 3 \, \text{Kd}3 \, \text{Kb}5 \, 4 \, \text{K} \times e3 \, \text{K} \times c5 \, 5 \, h4 \, \text{Kd}5 \, 6 \, \text{Kd}3$
Endings with Several Pawns (positional advantage)

Kc5 7 g4 f×g4 8 f5 Kd6 9 f×g6 Ke6 10 h5 g3
11 Ke3 f5 with a draw.

But by 1 h4! White could have created the irresistible threat of a breakthrough, e.g. 1 ... e3 2 Kd3 K×b5 3 g4! f×g4 4 f5 g×f5 5 K×e3 K×c5 6 h5, and the h-pawn queens.

Kuznetsoz-Zelenskikh
correspondence, 1971

651

B

K×f3 8 g5 e4 9 g6 e3 10 g7 e2 11 g8=Q e1=Q +.

Thanks to the fact that Black queens his pawn with check, he is able to exchange queens immediately: 12 Kh5 Qh1+ 13 Kg6 Qg1+ 14 Kf7 Q×g8+ 15 K×g8 Ke4 16 Kf7 Kd5, and Black won.

You should watch particularly carefully for the possibility of a breakthrough, if you take your king deep into the enemy position.

Pomar-Cuadras
Olot, 1974

653

B

653. White's defective pawn formation should have suggested to him the possibility
of a breakthrough, but he has just serenely played his king to d6, expecting to win after 1 ... Kg6 2 Ke6 Kg5 3 Kf7 h4 4 g x h4 + K x h4 5 g3 + Kh3 6 Kf6 Kg2 7 K x f5 K x f2 8 Kf4!

But like a bolt from the blue there followed 1 ... f4! 2 Kd5 (totally bad is 2 e x f4 h4! 3 g x h4 g3 4 f x g3 e3) 2 ... h4! 3 K x e4 (there is no way of saving the game; if 3 g x f4 then 3 ... h3) 3 ... f3! (3 ... h3?? 4 g x h3 g x h3 5 Kf3) 4 g x f3 h3 White resigns.

When transposing into a pawn ending, one should always bear in mind that unusual pawn formations often conceal the threat of a breakthrough.

Post, 1941

654. White forced the transition into this ending, intending by g2-g4 to block the K-side, and then to take his king across to Black’s Q-side pawns. He had in mind the following variation: 1 g4 Kf7 2 Ke2 Ke6 3 Kd3 Kd6 4 Kc4 a5 5 f3 Kd7 6 Kc5 Kc7 7 c3 b x c3 8 b x c3 Kc7 9 Kd6 Kd6 10 c4 Kd7 11 c5.

But the game went 1 g4 h x g3 2 f x g3 g4! 3 h4.

White has acquired a protected passed pawn, but the special feature of the position is that his king cannot move a long way across to the Q-side, due to the danger of the ... f5 breakthrough.

3 ... c5 4 Ke2 Kh7 5 Kd3 Kh6.

It transpires that now 6 Ke4 even loses: 6 ... f5! 7 e x f5 (bringing the king back no longer saves the game: 7 Kd3 f4 8 g x f4 e x f4, and the white pawns fall) 7 ... e4! 8 c3 a5 9 K x c5 c3 etc.

Therefore after 6 c3 a5 7 c x b4 a x b4 the players agreed a draw.

In conclusion, an example where the entire play is based on the avertion of a breakthrough.

655. 1 ... b5 2 c x b5 c4 is threatened, so White must take his king inside the ‘square’ of the c-pawn. At the same time he must think how he can exploit his winning chances, resulting from the creation of an outside passed pawn by f3-f4-f5 (and, in addition, this pawn must not prevent its king from covering the breakthrough of the black c-pawn).

But how should White continue—1 Kg4 or 1 Kg5?

Let us consider the first continuation: 1 Kg4 Kb8! 2 f4 Kc7 3 f5 (3 Kg5 b5!) 3 ... g x f5 + 4 e x f5 Kd7 5 f6! (5 Kg5 loses to 5 ... d5! 6 c x d5 c4 7 Kf4 h5! 8 a x b5 a4 9 b6 c3 10 Ke3 a3 11 f6 c2! 12 Kd2 a2 etc.) 5 ... Ke6 6
656. This instructional example is an attempt to demonstrate that, in the Exchange Variation of the Ruy Lopez, Black’s poor Q-side pawn formation (doubled pawns) leads to a loss, if White should succeed in exchanging all the pieces. Although the demonstration reduces to only a sample variation, it is unlikely that Black can do anything to oppose White’s standard plan, which is to create a passed pawn on the K-side, advance it, and then invade the enemy position with the king.

1 Ke2 Ke7 2 Ke3 Ke6 3 f4 c5 (to deprive the white king of the d4 square) 4 c4! c6 5 a4 b5 6 b3!

Of course, it would be absurd to exchange on b5, relieving Black of his main defect—his doubled pawns.

6 ... f6 7 a5 b4 8 g4 g5 9 e5!

A typical procedure: by sacrificing a pawn, White creates an outside passed pawn, which is then sacrificed to divert the enemy king and clear the way for his own king into the opponent’s position.

9 ... g × f4 + 10 K × f4 f × e5 + 11 Ke4 h6
12 h4 Kf6 13 g5+! h × g5 14 h × g5 + K × g5
(14 ... Ke6 15 g6) 15 K × e5 Kg4 16 Kd6 Kg4
17 K × c6 Ke4 18 K × c5 Kd3 19 K × b4 Kd4 20
Ka3 (20 e5? Kd5) 20 ... Ke5 21 Ka4 Kd4 22
Kb4 Ke5 23 Ke5, and White wins without difficulty.

It may seem that 8 ... g5 accelerated Black’s defeat, but 8 ... h6 would have been no better due to 9 f5 + Ke5 10 h3 Kd6 11 Kf4 followed by 12 e5, nor 8 ... g6 due to 9 f5 +
and 9 f5 + g × f5 (9 ... Kf7 10 f × g6 + h × g6 11 h4) 10 e × f5 + Ke5 11 h3 h6 12 h4 Kd6 13 Kf4 and
14 g5.

It should be pointed out that, if a king is forced to defend weaknesses, it begins playing a passive role, loses its activity, and may quickly end up in a zugzwang position.

The following example is typical.

657. White has not only weak pawns at a2 and d4, but also another highly significant weakness—the invasion square e4, which also has to be defended.

1 ... Ke7 2 c3.

Kg5 Kf7 7 Kf5 b6! (thanks to this tempo, Black nevertheless succeeds in making his breakthrough) 8 Kg5 d5! 9 c × d5 c4 10 d6 c3
11 d7 c2 12 d8 = Q c1 = Q +. Draw.

White wins by 1 Kg5!, creating an unusual zugzwang position: Black has no useful move. Indeed, if 1 ... Kb8, then 2 f4, and the pawn queens with check, and if 1 ... Ka6, then again 2 f4, and on 2 ... b5 comes 3 c × b5 +, gaining a decisive tempo. There only remains 1 ... b6, but the Black loses the reserve tempo which was so necessary to him in the previous variation. There can follow: 2 Kg4! Kb7 3 f4 Ke7 4 f5 g × f5 - 5 e × f5 Kd7
(5 ... d5 6 f6 Kd6 7 c × d5) 6 f6 Ke6 7 Kg5
Kf7 8 Kf5 Kf8 (8 ... d5 9 c × d5 c4 10 d6 c3
11 d7, and the new queen mates) 9 Ke6 Ke8 10
f7 + Kf8 11 Kf6, and the game ends in mate.

9.5 DEFECTS IN THE PAWN FORMATION

Defects in the pawn formation can be of various types:

(a) backward or doubled pawns;

(b) isolated or breakaway pawns, which, on becoming weaknesses, require defending;

(c) 'holes'—weak squares, which have to be defended by the king against invasion by the opponent.

Euwe, 1940

656
Defects in the Pawn Formation

Hansen-Nimzowitsch
Randers, 1925

657

B

Good advice here is hard to come by. 2 c4 can be met by 2 ... Kb6 3 c × d5 c × d5 4 Kc2 Ka5! 5 Kb2 Ka4 or 5 Kb3 Kb5, when Black wins.

2 ... Kb6! 3 c × b4 (if 3 c4, then 3 ... Ka6 4 c × d5 c × d5 5 Kd2 Kb5 etc.) 3 ... Kb5 4 Kc3 Ka4. The king has broken through onto the critical squares, and so White resigned.

Sometimes the struggle for the invasion squares may become very complicated, and to find the solution it may be necessary to resort to the theory of corresponding squares.

658. Here the key squares will be c7 and c8—to win, White must occupy them.

The diagram indicates the corresponding squares. The square corresponding to d6 is b6, and we will mark them by the number '1', d7-b7 ('2'), and d8-b8 ('3'). Continuing the analysis, we find a further series of corresponding squares: e6-a6 ('4'), e7-a7 ('5') and e8-a8 ('6').

By analogy we can also establish certain subsidiary squares: f6-b6 ('1'), f7-b7 ('2') and f8-b8 ('3').

On its next move, the black king will have to step onto one of the squares on the 6th rank ('4' or '1'). On noticing this, it is not hard to guess that the simple waiting move 1 Kf5! will be decisive, since, irrespective of where the black king goes, White seizes the correspondence.

1 ... Kb6 (1 ... Ka6 2 Ke6!) 2 Kf6! Kb7 3 Kf7!

Now, depending on the side to which the opponent's king moves, White embarks on a by-pass.

3 ... Kb8 4 Ke6! (on 3 ... Kb6 there would have followed 4 Ke8) 4 ... Kb7 5 Kd7! Kb6 6 Kc8, and White wins.

Sackmann, 1913

Botvinnik, 1939

659

+ 8

6 3 × 3 6 3

7 2 × 2 5 2

6 1 × 1 4 1

5 2 5

4

3

2

1

a b c d e f g h

659. This position is curious for the fact that the plausible attack on the d5 pawn leads
only to a draw: 1 Kf5 Kb6 2 Ke5 Ke7! 3 K×d5 Kd7 4 Ke5 Ke7 5 d5 Kd7, and while White is eliminating the a6 and a5 pawns, the black king has time to reach c8.

It turns out that, to win, White must first capture the a6 pawn, and such a deep raid into the black position requires very subtle manoeuvring with the king, using the attack on the d5 pawn only as a subsidiary threat.

The diagram again indicates both the key squares, and the corresponding squares. And it immediately becomes apparent that by 1 Kf5 White seizes the correspondence.

1 Kf5! Kb6 2 Kf6! Kb7 3 Kf7! Kb8 4 Ke6 (the by-pass can be commenced) 4 ... Ke7 5 Ke6! Kd6 6 Kc8 Kc6 8 Kd8 Kd6 9 Ka8!, and White wins.

It should be noted that 1 Kh5 fails to win. After 1 ... Kc6 2 Kg6 Kd6 3 Kf6 Kd7 4 Kf7 Kd6 5 Ke8 Black can play 5 ... Ke6, launching an attack on the d4 pawn.

The use of corresponding squares in the playing of complex pawn endings is covered in more detail in chapter 10.

Cohn-Rubinstein
St Petersburg, 1909

660. White’s pawns are weakened on both wings, and this allows Black to carry out a clear-cut winning plan. He breaks through with his king to the h2 pawn, ties down the opponent’s king to the defence of this pawn, then advances his K-side pawns and, by exchanging them, clears the way for his king to White’s weakened pawns on the opposite wing.

1 ... Kf6 2 Kd2 Kg5 3 Ke2.

The counter-attack on the Q-side is too slow: Black has time to pick up the h2 pawn and queen his own h-pawn. Only, in countering this plan it is important to be able to meet Kc7 with ... b5.

3 ... Kh4 4 Kf1 Kh3 5 Kg1 e5 6 Kh1.

White simplifies somewhat his opponent’s task. Of course, 6 a4 b6 7 b5 would have been more tenacious, although Black again wins by advancing his pawns: 7 ... f5 8 Kh1 g5 9 Kg1 h5 10 Kh1 h4 11 Kg1 e4 12 f×e4 f×e4 13 Kh1 (or f3 e×f3 14 e4 g4 15 e5 g3 etc.) 13 ... Kg4 14 Kg2 h3+ 15 Kg1 Kf3 16 Kf1 g4 etc.

6 ... h5 (now Black also has an extra tempo, which may prove useful) 7 Kg1 f5 8 Khl g5 9 Kg1 h5 10 Kh1 g4.

If now 11 f×g4, the simplest is 11 ... f×g4 12 Kg1 e4 followed by 13 ... h4 and 14 ... g3. Black’s task is made more complicated by 11 ... h×g4 12 Kg1 f4 13 e×f4 e×f4 14 Kh1, when Maizelis’s recommendation of 14 ... g3? 15 f×g3 f×g3 16 h×g3 K×g3 leads to a draw after 17 Kg1! Kf3 18 Kg1 Ke3 19 Kd1 Kd3 20 Kd1 Ke3 21 a4!

Instead of 14 ... g3, Speelman has shown that Black still wins after 14 ... f3! 15 Kg1 Kh4—thanks precisely to the additional tempo! For example: 16 Kh1 Kg5 17 h3 g×h3 18 Kh2 Kg4 19 Kg1 Kf4 20 Kh2 Ke4 21 K×h3 (21 Kg3 h2?) 21 ... Kd3 22 Kg4 Ke2 23 Kg3 a6. If the white king heads to the centre, the play develops more simply: 16 Kf1 Kh5! 17 Ke1 Kg5 18 Kf1 (18 Kd2 Kh4) 18 ... Kg4 19 Ke1 Ke4 20 Kd2 Kd4 21 Kc2 Kc4 22 Kd2 Kb3 23 Ke3 K×a3 24 Kf4 K×b4 25 K×g4 a5, and Black wins.

11 e4 f×e4! 12 f×e4 (or 12 f×g4 h×g4 13 Kg1 e3 14 f×e3 e4 15 Kh1 g3) 12 ... h4 13 Kg1 g3 14 b×g3 h×g3 White resigns.

We have already established that, if the
Defects in the Pawn Formation

opponent has weaknesses, the basic plan is to invade the opponent’s position with the king. But this takes the king away from its own pawns, and a careful watch must be kept on the possibility of a breakthrough.

Liebert-Onat
Skopje, 1972

661

662

661. Black’s pawn formation is compromised. White’s king is ready to break into the opponent’s position, and in addition he has reserve tempi. All this adds up to a big positional advantage for White.

The game took the following course: 1 f3 Ke6? 2 f4 e×f4 3 g×f4 c5! (Black finds a way of creating counter-play) 4 f×e5 g×f4! 5 h×g4 h×g6 6 Ke6 7 g7 Kf7 8 e6+ K×g7 9 Kd6 h5 10 e×h5 h×g4 11 e5+ Qh1 = Q 12 Qe5+ Kf8? 13 Ke×g4, and White managed to win this ending.

Botvinnik showed that Black could have played better on at least two occasions. Thus instead of 12 ... Kf8, more accurate was 12 ... Kg6 13 Qf5+ Kg7, when White has only practical chances of success.

And instead of 4 ... g5, stronger was 4 ... Ke6? 5 Kd5 (5 Kd4 Ke6), and only now 5 ... Ke6, e.g. 6 h×g5 h×b4 7 g×h6 8 g7 Kf7 9 e6+ K×g6 10 Kd6 h2 11 e7 h1 = Q 12 e8 = Q. White’s chances are altogether insignificant.

But it turns out that the entire plan with 2 f4, allowing Black to make a breakthrough, was wrong. Correct was 2 Kb6! Kd6! 3 g4! K×f3 4 Kb7 Kd6 5 Ke8!, and by carrying out a bypassing manoeuvre, White invades the opponent’s position with decisive success, e.g. 5 ... Ke5 (5 ... Kc6 6 Kd8 Kd6 7 Ke8) 6 g×h5 g×h5 7 Kd7 Kd4 8 K×e6 Ke3 9 K×e5 K×f3 10 Kf5, and White wins.

The following two examples show the drawbacks to isolated pawns.

Balogh-Ertel
Hungary, 1973

662. By the precise move 1 ... g5! Black not only deprives the opponent’s pawns of their mobility, but also prepares to create a zugzwang position. Bad now is 2 h×g6 f×g6, since after ... h5 Black’s passed pawn is decisive. The attempt at a breakthrough also proves unsuccessful: 2 f4 f6! 3 f×g5 f×g5 4 Ke3 e5! 5 d×e5 K×e5 6 Kd3 Kf4 7 Kc4 K×g4 8 K×b4 Kf3, and Black’s pawn queens first.

2 Ke3 f5 3 g×f5 (3 Kd3 f4) 3 ... e×f5 4 Kd3 f4 White resigns.

In this example White had no possibility at all of creating counter-play, since his king was severely restricted, and Black’s task turned out to be very simple.

The following practical example is much more complicated.
663. White has a clear positional advantage, since the opponent has to defend not only the isolated d-pawn, but also the invasion squares e5 and e5. The question is only one of tempi, and also of whether or not Black can create counter-play.

The game continued: 1 Kd4 Kd6 2 b4 b6 3 h4 h5 4 a4 a5!

Black aims for counter-play. 5 a5 cannot be allowed, and besides, at a5 the black pawn is closer to the queening square.

5 b x a5 b x a5 6 f4 g5 7 e3.

White has attained a zugzwang situation, and now Black must allow the opponent’s king into his position. But the struggle is far from over.

7 ... Kc6 8 Ke5 Kc5 9 Kf6 Ke4!

Here it is, the desired counter-play! The king makes not for the a4 pawn, but the e3 pawn, to obtain a passed pawn on the d-file as quickly as possible.

10 K x g6 Kd3 11 K x f5 K x e3 12 Kg5 d4 13 f5 d3 14 f6 d2 15 f7 d1 = Q 16 f8 = Q Q x a4.

Now the question is: who will be the first to obtain a second queen?

17 Qf5 Qb4 18 K x h5 a4 19 g4 a3 20 g5 Qb2 21 g6 a2 22 Qg5 + Ke4 23 Qg4 + Ke3 24 Qg5 + Ke4 25 g7 a1 = Q 26 Qg6 + Kf4 27 g8 = Q.

There are four queens on the board!

27 ... Qh8 + 28 Qh6 + (or 28 Q x h8 Q x h8 + 29 Q x h6 + Q x h6 + 30 K x h6 Kf5 with a draw) 28 ... Q x h6 + 29 K x h6 Qf6 + 30 Kh5 Ke5, and a draw was agreed.

As was shown by Bondarevsky (1966), instead of 1 Kd4? White should first have played 1 Ke3!, threatening an immediate invasion by 2 Kf4. Black then has two main continuations (analysis by Bondarevsky):

(a) 1 ... g5 2 b4 h6 (weaker is 2 ... g x h4 3 g x h4 Kd6 4 Kf4 Ke6 5 Kg5, when the king moves to the h7 pawn; no better is 2 ... f4 + 3 g x f4 g x h4 Kf3 or immediately 3 Kf3) 3 h x g5 h x g5 4 f4 g4 5 Kd4.

Now that Black has been given another weakness—the f5 pawn, the attack on the d5 pawn can be begun.

5 ... Kd6 6 b4 b6 7 a4, and Black has no satisfactory defence.

(b) 1 ... Kd6 2 Kf4 Ke6 3 Kg5 Ke5.

On 3 ... Kd4 there could have followed 4 f4 g6 5 Kh6 Kd5 6 K x h7 Ke4 7 K x g6 Ke3 8 h4 K x e2 9 h5 d3 10 h6 d2 11 h7 d1 = Q 12 h8 = Q, with a won queen ending.

4 a3 b5 5 b4 Ke4 6 h4 d4.

If 6 ... Ke5, then 7 h5, e.g. 7 ... Ke4 8 h6, or 7 ... d4 8 f3 h6 + 9 Kg6 f4 10 g4, or, finally, 7 ... Ke6 8 e3! (but not 8 f4 d4) 8 ... Ke5 9 f3 etc.

7 h5 h6 + 8 Kg6 f4 9 g x f4 + K x f4 10 K x g7 K g5 11 f3! Kf4 (11 ... K x h5 12 Kg6 Kh4 13 Ke5 etc.) 12 K x h6 Ke3 13 Kg6 K x e2 14 h6 d3 15 h7 d2 16 h8 = Q d1 = Q 17 Qe5 + Kf2 18 f4, and White must win this ending.

664. The defects in Black’s pawn formation and the remotesness of his king from the main battlefield allow White to hope for success. He immediately began active play by 1 g4, and the continuation was: 1 ... f4 + 2 Kc4 h6 3 h4 Kb6 4 g5 f x g5 5 h x g5 h x g5 6 K x e5 g4 7 K x f4 g x f3 8 K x f3 Kc7 9 Ke4 Kd6 10 Kf5, and White won.

Later analysis showed that 1 ... f4? was a decisive mistake. Correct was 1 ... f x g4! 2
White must open lines on the Q-side and penetrate with his king into the opponent’s position.

In the game, events developed as follows: 1 Kd5 Ke7 2 a4 a5 3 Kd4 Kf7 4 Kc3 Ke7 5 b4 Kf7 6 b × a5 b × a5 7 Kd4 Ke8 8 Kd5 Kf7, and Black conceded defeat. Indeed, after 9 c5 d × c5 10 K × c5 Ke7 11 Kd5 (of course, this is the simplest, but 11 Kb5 Kd6 12 K × a5 K × c6 13 Ka6! was also possible) 11 ... Kf7 12 Kc4 Ke7 13 Kc5 h6 14 Kd5 Kf7 15 Kc4 Ke7 16 Kc5 Kf7 17 Kb5 White wins.

After the publication of this ending in the first Russian edition of this book, Pavlovichev (1958) drew attention to the inaccurate play of both sides. Thus, after 4 Ke3 the correct reply was 4 ... d5! with an immediate draw, e.g. 5 c × d5 Ke7 6 Kc4 Kd6 7 Kd4 Ke7, and the white king has no way of breaking through.

Instead of the incorrect 4 Ke3, White could have won by 4 b3! Ke7 5 Kd4 Ke7 6 Kd3 Ke7 (6 ... d5 is not dangerous; 7 Kd4 d × c4 8 b × c4!) 7 Kd4 h6 (now White again has to gain a tempo) 8 Ke4 Kf7 9 Kd3 Ke7 10 Kd4 Kf7, and only now 11 Ke3, and if 11 ... d5 12 Kd4.

But, as shown by Maizelis (1962), a clearly won position can be attained much more simply, by playing 1 Kd4!, e.g. 1 ... Ke7 2 b4! b5! (otherwise 3 a4 and 4 c5) 3 Kc3 h6 4 a3 (if 4 Kb3, then 4 ... d5! 5 c × b5 a × b5 6 a4 b × a4 + 7 K × a4 Kd6, and Black has sufficient counter-chances) 4 ... Kf7 5 Kb3! Ke7 6 a4, and White wins.

If Black avoids e7 before the white king goes to c3, the play can develop as follows: 1 ... Ke8(8) 2 b3 Kf7(d8) 3 a4! a5 4 Ke3! Ke7 5 b4 a × b4 + (incidentally, this is stronger than 5 ... Kf7 6 b × a5, as occurred in the game) 6 K × b4 d5! (the most tenacious; if 6 ... Kd8, then 7 a5 Ke8! 8 Kb5 b × a5 9 K × a5 Kb8 10 Kb4! Ka7 11 Kb5 Kb8 12 c5, and White wins) 7 c5! d4 8 c × b6 c × b6 9 Kb5! d3 (9 ... Kd8 is weaker: 10 K × b6 Kc8 11 a5) 10 K × b6 d2 11 c7 d1 = Q 12 c8 = Q.

In the resulting queen ending 12 ... Q × a4
Endings with Several Pawns (positional advantage)

is of course bad because of 13 Qe6+ Kf8 14 Qxf6+ Kg8 15 Qe6+ with the inevitable exchange of queens. But even after the better 12...Qb3+ 13 Kc7 Qc4+ 14 Kb8 Qxa4 15 Qe6+ and 16 Qxf6+ White must gradually be able to win.

9.6 RESERVE TEMPI

In pawn endings it is hard to overestimate the importance of extra tempi. As a rule, they should be preserved, and not used without necessity, since they can have a great influence on the course of events. Tempi allow a king to occupy invasion squares and penetrate into the enemy position, and in critical situations, in zugzwang positions, each tempo is in general worth its weight in gold. In a struggle for critical or key squares, even one extra tempo may prove decisive.

We have already seen several times that the struggle for tempi—or 'tempo' play, in other words—occurs in all forms of pawn ending. Here we will be considering examples where the entire content of the play, the entire strategic struggle, is determined by the presence or absence of reserve tempi.

We will begin with some positions where the familiar 'untouchable' pawns are present.

Schweda-Sika
Brno, 1929

666. In such situations it is important to make an accurate count of the two sides' tempi. At first sight it appears that this number is equal: on the Q-side Black has a tempo less, but on the K-side he has one more. But this is not so. Whichever player begins, he is able to repair the position on the 'unfavourable' wing, and his extra tempo on the opposite wing brings victory.

Suppose White begins: 1 Kf3! Ke5 2 h4! a5 (2...h5 3 b4 b5 4 a3) 3 h5 a4 4 h6! b6 5 b4! a×b3 6 a×b3 b5 7 b4, and White wins.

With Black to move: 1...Ke5! 2 Kf3 a5! 3 b3 (if 3 h4, then 3...a4 4 h5 h6) 3...b5! 4 a3 (4...b4 was threatened) 4...a4 5 b×a4 (5 b4 h6!) 5...b×a4 6 h4 h5 etc.

Here are some further examples of pure 'tempo' play in positions with 'untouchable' pawns.

Paoli-Michel
Vienna, 1930/31

667. On the K-side White has an extra tempo, therefore his problem is to stabilize the position on the Q-side: 1 a4!

Only in this way is White able to win the battle for the tempi. 1 h4?, for example, is bad because of 1...a6, as is 1 h3 g5 2 b3 (2 a4 a5 or 2 h4 b5) 2...b5 3 a4 b×a4 4 b×a4 a5,
when Black wins. No better is 1 b3 g5! 2 h3 a6
3 a4 a5 with the same result.

1... b6.

1... a5 is met by 2 h4, and 1... h5 by 2
\text{g} \times \text{h5} \text{g} \times \text{h5} 3 \text{b4} \text{h4} 4 \text{h3}.

2 b4 a6 3 h4 a5 4 b5! h5 5 g5 Resigns.

N. Popov-Dankov

Albena, 1978

668. 1... h5 is threatened, so that White's
first move is simply forced, but at the same
time it immediately puts everything in its
place.

1 g4! a6.

There is no satisfactory defence. If, for
example, 1... g5, then 2 g3, while if 1... b6,
then 2 b5 g6 3 g5! h \times g5 4 g4.

2 a5 g6 3 g5! h \times g5 4 g4 Kd6 5 K \times e4 Ke6 6
d5+ Resigns.

If there are pawns on one or two files,
counting the reserve tempi is not too difficult
a business. But if there are pawns on three
files, this complicates the problem.

669. In this position the game was
adjourned, and Alekhine sealed his next
move. In order to determine correctly
whether the black king can be maintained at
g5, accurate calculation is required.

It is immediately apparent that it is bad to
play 1... d5 2 e \times d5 c \times d5 3 b4. Analysis
shows that Black also loses after 1... c5 2 a4!
670. After 1 Kf4 b×c4 2 g4! it becomes obvious that Black is bound to lose the battle for the f5 square, since White has a large store of reserve tempi.

2 ... a5 3 a4 h6 4 h3 f5 (desperation, but 4 ... c3 5 b×c3 c4 was no better due to 6 h4 Kd6 7 Kf5 Ke7 8 h5) 5 g×f5 + Kd6 6 f6 Ke6 7 e5 Kd5 8 Kf5 h5 9 h4 Resigns.

In many cases even one reserve tempo may be sufficient to win the game.

Filip-Barcza
Sofia, 1957

672. Since the black c-pawn cannot move, it means that in this position White has at least two extra tempi (moves with his c-pawn). His problem is to exploit these tempi favourably.

The game concluded as follows: 1 g4 Kd7 2 g5.

White's plan is perfectly clear: by the advance of his h-pawn he intends to make a breach in Black's K-side defences, and then, exploiting his extra tempi, create a zugzwang position.

2 ... Ke7 3 h4 Kd7 (3 ... g6 4 a4 a5 5 c4, and White wins) 4 h5 Ke7 5 a4 a5 (5 ... a6 6 a5 Kd7 7 e4 Ke7 8 h6 etc.) 6 c4 Kd7 7 h6 g6 (7 ... g×h5 8 h×h7 Ke7 9 c5) 8 Kf6 Kd6 9 Kg7 Resigns. If 9 ... Ke7 10 K×h7 Kf7, then 11 c5.

As we see, White fulfilled his task fairly easily, but it appears that Black did not exploit all his chances. Instead of 1 ... Kd7, more tenacious was 1 ... h6, nipping in the bud White's plan involving the seizure of the f6 square.

In the Yugoslav Encyclopaedia of Chess Endings, Botvinnik suggests in this case 2 h4 g6 3 g5, but 2 ... g6 is a blunder, and correct is 2 ... Kd7, for the moment adhering to waiting tactics. Then on 3 g5 comes 3 ... h×g5 4 h×g5 Ke7 5 a4 Kd7! (5 ... a5 loses: 6
c4 Kd7 7 Kf5 Ke7 8 Kg6 Kf8 9 Kh7 Kf7 10 c5, and White attains a decisive zugzwang position) 6 a5 (now 6 Kf5 Ke7 7 Kg6 Kf8 8 Kh?? does not succeed: after 8 ... Kf7 9 c4 c5 10 a5 a6 it is White who is in zugzwang) 6 ... Ke7 7 a6 Kd7 8 c4 Ke7 9 c5 Kd7, and White has not made any particular achievements: he has used up all his tempi, and not obtained any advantage from them.

But perhaps the correct path lies in the cunning move 2 h3? Let us check: 2 ... Kd7 3 h4 Ke7 4 a4 Kd7 5 a5 Ke7 (5 ... a6 6 g5 h × g5 7 h × g5 Ke7 8 c4) 6 a6 Kd7 7 g5 h × g5 8 h × g5 Kd7 9 c4 Kd7 10 Kf5 Ke7 11 Kg6 Kf8 12 Kh7 Kf7 13 c5, and we reach a decisive zugzwang position.

But here too Black's defence can be improved: instead of 4 ... Kd7 he should continue 4 ... a5!, e.g. 5 g5 h × g5 6 h × g5 Kd7! 7 c4 Ke7, when White has not achieved anything. The attempt to break through with the king on the Q-side—5 Kd4 Kd6 6 Kc4 is also unsuccessful, since Black has the strong reply 6 ... Ke5 7 Kc5 Kf4 8 K × c6 K × g4 9 c4 K × h4 10 c5 g5, when the pawns queen simultaneously.

So, does this mean that White cannot win? No, it turns out that he nevertheless can! The truth lies in a precise order of moves. The "tempo" play should be begun with the advance of the a-pawn—2 a4! If Black plays 2 ... Kd7, then 3 a5! Ke7 4 h3! Kd7 5 h4 Ke7 6 a6 Kd7 7 g5 h × g5 h × g5 Ke7 9 c4 and White wins, while on 2 ... a5 he has the correspondingly decisive 3 h4! Kd7 4 g5 h × g5 5 h × g5 Ke7 6 c4.

The "tempo" method of play, employed here by White after 1 ... h6, is typical of such endings. White stabilizes the position in the place where he has no advantage in tempi, and where he has reserve tempi he varies them, so that each time a decisive zugzwang position arises.

673. Here White has one extra tempo on the K-side, and two on the Q-side. A zugzwang position could have most simply been created as follows: 1 h4 Ke8.

673. White has a tempo in reserve, and on 9 ... Kc7 he

If 1 ... h5, then 2 g3 Kc8 3 Ka7 Kc7 4 a3! Kc6 5 a4 Kc7 6 b5! a5 (6 ... a × b5 7 a × b5 Kc8 8 h6) 7 Kc8 Kc6 8 Kc8, or 1 ... h6 2 g4 Kc8 3 Ka7 Kc7 4 a3 Kc6 5 a4 Kc7 6 b5 etc.

2 Kc7 Kc7 3 a3! Kc6 4 a4 Kc7 5 b5 a × b5 6 a × b5 h6 7 g4 g5 8 h5, and White wins.

Note that the winning plan is identical to that carried out by White in example 672 after 1 ... h6.

In the game, events developed rather differently: 1 a3.

In the Yugoslav Encyclopaedia of Chess Endings an exclamation mark is attached to this move, and it is suggested that 1 a4 does not succeed due to 1 ... Kc8 2 b5 a × b5 3 a × b5 Kc6 4 g5 2 h4 h5 5 h4 Kc8. But White also wins after 1 a4 Kc8, by continuing 2 Ka7! Kc7 3 b5, when the extra tempo on the K-side is decisive.

1 ... Kc8 2 a4 g5 (Black's position is completely hopeless, and this desperate move does not spoil anything) 3 h3 Kb8 4 g3 Kc8 5 h4 g × h4 6 g × h4 Kb8.

As a result of the opponent's indecisive manoeuvring, Black has managed to prolong the resistance, but the win is still there, since White has much the more active king.

7 Kc5 Kc7 8 Kd5 Kd7 9 Ke5 Ke6 (White still has a tempo in reserve, and on 9 ... Kc7 he
has the decisive 10 a5) 10 h5 a5 (10... b5 11 a5) 11 b × a5 Kc5 12 h6 Kb4 13 Kf6 Resigns.

Objectively speaking, Black's 8th move was not the best, and 8... h5 would have set his opponent more problems. For example: 9 a5 Kd7 10 Ke5 Kc6 11 Kf5 Kb5 12 Kg5 K × b4 13 K × h5 K × a5 14 Kg5 b5, and the pawns queen simultaneously. Correct is 9 Ke5 Kc6 10 Ke6! (10 Kf5 b5! 11 a5 Kd5 12 Kg5 Kc4 13 K × h5 K × b4 does not promise White anything). If now 10... b5, then 11 a5 leads to a win, while if 10... b6, then 11 Ke5! a5 12 b × a5 b × a5 13 Kf5 Kc5 14 Kg5 Kb4 15 K × h5 K × a4 16 Kg5, and White's pawn queens first; finally, if 10... a5, then 11 b5+! Kc5 12 Kd7! Kb4 13 Kc7 K × a4 14 b6, and White wins.

It is curious that, although this ending has many times been published, the commentators have failed to notice all of its nuances.

How an advantage in tempi can be created is strikingly shown by the following example.

**Thomas-Maroczy**
Nice, 1916

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674. To win, White must break through with his king at b3. But to do this he needs at least two reserve tempi, since the direct 1 Ke4 Kd6 2 Kd3 Kd5 3 b3 f5 does not succeed.

White obtains the tempi needed on the K-side, by advancing his h-pawn: 1 h4! Ke7 2 h5

**675.** This position contains a highly interesting 'zugzwang' position, in which his extra tempo will bring him success. If, for example, the white king moves to d3, then after 1... b4! 2 a4 Kb6 3 Kc4 Ka5 4 Kb3 g6 or 4... h5 5 h3 g5 Black immediately decides the game in his favour.

The attempt to transpose into a queen ending also loses in interesting fashion: 1 Kb3 Kd4 2 Kb4 K × e4 3 K × b5 Kd4 4 a4 e4 5 a5 e3 6 a6 e2 7 a7 e1 = Q 8 a8 = Q Qb1 +, and by exchanging queens, Black wins.

The game continued as follows: 1 h3 Kc6!

Having a tempo in reserve, Black aims to reach a5 with his king. In this case the position Ka5/Kb3 will be one of zugzwang, and Black's extra tempo will enable him to penetrate to a4 and win. Weaker was 1... Kb6 2 Kb4, when Black is forced to return a tempo.
Better King Position

2 Kb3! (2 Kb4 Kb6) 2 ... Kb7! 3 Kc3 (3 a4 Kb6!) 3 ... Ka6 4 Kb2 Ka5 5 Kb3 g5 (now the game is decided) 6 g4 (if 6 h4, then 6 ... g x h4 7 g x h4 h5) 6 ... h6 7 Kc3 Ka4 8 Kb2 b4 9 a x b4 K x b4, and Black won.

But White could have saved the game, by correcting his position on the 'unfavourable' wing with 1 g4!, e.g. 1 ... g5 (2 g5 was threatened, and if 1 ... h6, then 2 h4! Kc6 3 h5) 2 Kd3! (now this is possible) 2 ... Kb6 (if 2 ... b4 3 a4, when 3 ... Kb6 in fact loses, but a draw is given by 3 ... b3 4 Kc3 b2) 3 Kc3 Ka6 4 Kb2 Ka5 5 Kb3 h6 6 b3, with a draw.

There is also some original play in the following position.

Jovicic-M. Vukovic
Yugoslavia, 1965

676. Black's position looks critical. If 1 ... K x d7, then 2 K x e5 Ke7 3 Kd5 Kd7 4 b3, and White wins. But Black is not obliged to take the pawn, and he finds a splendid possibility of avoiding zugzwang.
1 ... Kd8! 2 Ke4!
A fully worthy reply. 2 K x e5 could now have been met by 2 ... K x d7 3 Kd5 a4, with a draw.
2 ... Ke7 3 Kd5 Kd8.
It appears that all resources have been exhausted and that a draw can be agreed, but White finds an interesting possibility of continuing the struggle.
4 Ke6! e4 5 K x b6 e3 6 Ka7! e2 7 b6 e1 = Q 8 b7 K x d7 9 b8 = Q Qf2 10 Qb5 + Kd6 11 Kb6 Ke6, and here White could have decided the game in his favour by 12 Qe8+, since 12 ... Kd6 13 Qc6+ leads to the exchange of queens. Of course, instead of 10 ... Kd6, more tenacious is 10 ... Ke6 11 Kb6 Qd4, but even then after 12 K x a5 White has good winning chances.

But Black could have saved the game by 9 ... Qb4!, exploiting the fact that the exchange 10 Q x b4 c x b4 11 Kb7 a4 12 c5 a3 leads only to a draw, while 10 Qb5 + Kd6 11 b3 can be met by 11 ... Ke5! 12 Kb6 Kd4 13 Q x b4 a x b4 14 Kb5 Kc3 with the same result.

9.7 BETTER KING POSITION

In pawn endings the king is the main piece, in both the direct and the figurative sense, with a great diversity of tasks to perform. As the well known saying puts it, the king is the 'jack of all trades'. It has to battle with the enemy king for invasion squares, and for the key and critical squares. It 'personally' prevents the advance of enemy pawns and itself attacks them, defends its own pawns and assists their advance ... In many of the examples considered, when exploiting this or that positional factor, the degree of activity of the king was also extremely important.

In the present chapter we will be mainly considering examples where the difference in activity of the kings determines the character of the struggle and the final result. Thus in the following instructional example a quite insignificant difference in the position of the kings (just a single tempo) decides the game in favour of one side or the other. The winner is the side that begins, since he is the first to attack the opponent's pawns.

677. With White to move: 1 Kb6 Kb3 2 Kc6 Kc3 3 Kd6 Kd3 4 K x e6 K x e3 5 Ke5!
This last move is very important, and is the one which decides the fate of the game. Now
5 ... Kd3 is met by 6 K × d5, and 5 ... Kf3 by 6 K × f5, winning.

If Black begins, the situation is repeated literally in a mirror reflection: 1 ... Kb3 2 Kb6 Kc3 3 Kc6 Kd3 4 Kd6 K × e3 5 K × e6 Ke4!, and it is Black who wins.

Example 678 demonstrates a procedure which is sometimes called 'widening the bridgehead'. A pawn storm is undertaken with the aim of eliminating certain enemy pawns, so as to penetrate onto the critical squares of others.

Alapin-Reti
Vienna, 1908

678. With his king more active, White has the possibility of making a breach in the enemy position by the advance of his rook's pawn.

The winning path is not difficult: 1 a4 Ke8 2 Kb6 Kb8 3 a5 Ke8 4 a6 Kb8 5 a7+.

Of course, 5 a × b7 would have led to stalemate, but simpler was 5 Ka5! Ka7 6 a × b7 K × b7 (6 ... Kb8 7 Ka6) 7 Kb5, when Black can resign.

5 ... Ka8 6 Kc7 b5 7 K × d6 b4 8 Kc6! b3 9 d6 b2 10 d7 b1 = Q 11 d8 = Q + K × a7 12 Qa5 + Kb8 13 Qb6 + Resigns.

A similar procedure is used in example 679, but the penetration of the king onto the critical squares becomes possible due to the reaching of a zugzwang position.

Eliskases-Skalicka
Podebrady, 1936

679. Were it White to move, he would block the enemy king's path into his position by 1 g4, and the game would end in a draw. But it is Black to move, and he succeeds in opening lines on the K-side.

1 ... g5! 2 Kd2 (2 h5 g4) 2 ... Kf6 3 Ke3 Kg6 (of course, not 3 ... g × h4 4 g × h4 Kg6 5 Kf3 Kh5 6 Kg3 with a draw) 4 Kf2 Kh5 5 h × g5 K × g5 6 Kf3 h5 7 a3 h4 8 a4 h × g3 9 K × g3 a5 (zugzwang) White resigns.

In certain cases a poor king position may become the cause of defeat.
680. The black king has penetrated into the opponent’s position, but due to the absence of reserve tempi it has become trapped. By a temporary pawn sacrifice White creates a zugzwang position, and gains a decisive material advantage.

1. g3+ f×g3+ 2. Kg2 g4 3. h×g4 Kg5 (desperation; if 3...h×g4, then 4.f4 Kh5 5.K×g3 etc.) 4. g×h5 Resigns.

In the following example, where White’s king is a long way from his remaining forces, the difference in the kings’ positions is especially marked. Nevertheless the path to victory is not simple, and consists in breaking up the pawn barrier which arises in the path of the black king, and then penetrating into the opponent’s position. This must be done quickly, before White has time to bring up his king.

681. In striving to invade the opponent’s position, Black must play accurately. Thus only a draw results from 1...Kf5 2.f3 g5 3.h3 (3.g4+ Ke4 4.Kb4 f5 5.g×f5 is also possible) 3...h5 4.g4+ h×g4 5.h×g4+ Ke5 6.Kb4 f5 7.g×f5 e×f5 8.Kc3.

1...Ke5! 2.f3 g5 3.Kb4 g4 4.f4+ Kd5!


Had White prevented...g4 by 3.h3, Black would still have won: 3...f5! 4.Kb4 h5!

The only move. If 4...f4, then 5.g×f4 g×f4 6.e×f4+ K×f4 7.Kc5 K×f3 8.Kd6, while if 4...g4, then 5.h×g4 f×g4 6.f×g4 Ke5 7.Kc5 K×c3 8.Kd6 Kf3 9.K×e6, with a draw.

5.Kc3 (there is nothing better; on 5.Kc5 comes 5...g4 6.h×g4 f×g4 7.f×g4 h×g4, winning) 5...g4 6.f×g4 f×g4 7.h×g4 h×g4 8.Kd3 Kd5! 9.e4+ Kc5 10.Kc3 e5, and Black wins.

We have already mentioned that, when taking the king into the opponent’s position, one should watch carefully for threats of a pawn breakthrough. This is especially important in positions with a defective pawn structure.

682. 1...Ka2 2.h3 h5 3.h4 f6.

White has used up all his reserve tempi, and is now forced to allow the opponent’s king into his position.


682. B

\[d3\ 12\ Ke7\ d2\ 13\ f6\ d1=Q\ 14\ f7.\ True,\ it immediately\ has\ to\ be\ exchanged,\ but\ after\ 14\ ...\ Qd4\ 15\ f8=Q\ Qc5+\ 16\ Kf7\ Q\ \times\ f8+\ 17\ K\ \times\ f8\ Kf3\ 18\ Kf7\ K\ \times\ g3\ 19\ Kg6\ K\ \times\ h4\ 20\ Kf5\ Kg3\ Black\ wins.\]

5 ... Ke1 6 Ke2 Ke2.

The opponent's king has made deep inroads into White's position, so he now tries to effect a breakthrough.

7 e4 f6 e4.

7 ... dxe4 8 g4 hXg4 9 h5 g3 10 h6 g2 11 Kf2 e3= 12 Kxg2 e2 was also possible, but Black has no way of avoiding the breakthrough. If instead 7 ... Ke3?, then 8 Qg4 Kxh4 (8 ... fXg4 9 f5 eXf5 10 eXd5 g5 fXg5 10 hXg5 h4 11 g6 h3 12 Kf2, and it is White who wins.

8 g4 hXg4 9 h5 g3 10 h6 g2 11 Kf2 e3= 12 Kxg2 e2 13 h7 e1=Q 14 h8=Q Qe4+ 15 Kg3 QXd4 (the queen ending is easily won for Black) 16 Qa8 (16 Qc8+ Kd2 17 QXe6 Qe3+) 16 ... Qe3+ 17 Kd4 d4 18 Qc6+ Kd2 White resigns.

In this case the breakthrough allowed White to prolong the struggle somewhat, but did not affect the result. It more often happens that a breakthrough sharply changes the pawn formation, has a marked effect on the activity of the kings, and tells decisively on the result.

683. Black’s position is better, since he has the more active king and a reserve tempo (... b5). He played 1 ... g5, expecting the following continuation, favourable for him: 2 Kf3 gXh4 3 gXh4 f5 4 eXf5 KXf5 etc.

But this was met by the unforeseen 2 g4! hXg4 3 h5, and thanks to the breakthrough the situation changed sharply—White acquired a dangerous outside passed pawn. The black king had to set off to deal with it: 3 ... Ke6 4 Kf2 Kf7 5 Kg3 Kg7 6 Kxg4 Kh6 7 Kf5 KXh5.

Now both sides queen pawns, then they are exchanged by force, after which the more active position of White’s king decides the game in his favour.

7 ... Kg7 was no better due to 8 e5! fXe5 9 KXg5!, when again White has an outside passed pawn. It is true that, in the event of 7 ... b5 8 KXf6 g4 White queens his pawn without check—9 e5 g3 10 e6 g2 11 e7 g1=Q 12 e8=Q, but in view of the threat of 13 Qg6+, the right to move is of little consolation to Black, e.g. 12 ... Qg5+ 13 Ke6 Qe3+ 14 Kd7, and White must win the queen ending.

8 KXf6 g4 9 e5 g3 10 e6 g2 11 e7 g1=Q 12 e8=Q+ Kh4 13 Qh8+ Kg3 14 Qg7+ Kf2 15 QXg1+ KXg1 16 Ke5 Kf2 17 Kd5 Ke3
18 Kc6 Kd2 19 K x b6 Kc2 20 Ka5 K x b2 21 K x a4 Kc3 22 Kb5 Resigns.

We have already met a similar break-through (652), but it would be of interest to establish this: how should Black have played?

Of course, the plan involving ... g5 was wrong. As was shown by Vinokurov (Kiev), correct was 1 ... f5! 2 e x f5 K x f5 3 Kf3 g5! 4 h x g5 K x g5 (684), aiming to obtain a position in which the extra tempo could be of decisive significance. In this case, thanks to his active king, Black does indeed win.

684. 5 Kf2!

This is more tenacious than 5 Kg2 Kg4 6 Kf2 Kh3 7 Kf3 b5 8 Kf2 (no better is 8 Kf4 Kg2 9 g4 h x g4 10 K x g4 Kf2) 8 ... Kh2 9 Kf3 Kg1 10 Kf4 Kf2 etc.

5 ... Kf5!

Nothing is achieved by 5 ... Kg4 6 Kg2 b5 7 Kf2, and bad is 7 ... Kh3 8 Kf3 Kh2 9 g4!, when it is White who wins.

6 Kf3 (6 Ke3 Kg4 7 Kf2 Kh3 8 Kf3 b5) 6 ... Ke5 7 Ke3 b5! 8 Kd3 (White fails to save the game after 8 Kf3 Kd4 9 Kf4 Ke4 10 Kg5 Kb3 11 K x h5 K x b2 12 g4 b4) 8 ... Kd5 9 Kc3 Ke4 10 Kb4 Kf3 11 K x b5 K x g3 12 K x a4 h4, and Black wins.

Certain subtleties, involving king manoeuvring inside the enemy position, are shown by the following example.

685. In this situation the standard plan of creating a passed pawn on the K-side does not win, e.g. 1 Ke4 Kd6 2 f3 a6! 3 f4 Ke6 4 f5+ g x f5 5 Kf4 a5 (here the strength of doubled pawns is once again revealed; while not dangerous from the viewpoint of obtaining a new queen, they provide extremely important reserve tempi) 6 g6 Kf6 7 g7 K x g7 8 K x f5 Kf7 9 Kg5 Ke6 10 K x h5 Kf5 11 Kh6 Kf6 12 h5 Kf7 13 Kg5 Kg7 14 Kf5 Kh6, and, according to Bähr’s rule (209), Black gains a draw.

The correct plan is to exploit the more active position of the king and to invade the opponent’s position: 1 Ke5! Ke5.

The best defence. The seemingly dangerous 1 ... Kf5 leads by force to mate: 2 Kd6 Kg4 3 Ke5 K x h4 4 Kf6 Kg4 5 K x g6 h4 6 Kf6 h3 7 gh h2 8 g7 h1 = Q 9 g8 = Q+ Kf4 (9 ... Kf3 10 Qa8+) 10 Qb8+! Kg4 11 Qg3+ Kh5 12 Qg5 mate.

If instead Black plays 1 ... a6, possible is 2 f4 Kf5 3 Kd6 Kg4 4 Ke6 K x h4 5 f4!, but not 2 Kd4 Kf5! (2 ... Kd6 3 Ke4 Ke6 4 f4 a5! 5 f5+) 3 Ke3 Kg4 4 f4 K x h4 5 Kf3 a5.

2 Ke6 Ke6 (the attack on the a3 pawn is too slow: 2 ... Kd4 3 Kd6 Ke4 4 f4 Kb3 5 f5) 3 Kc7 Kc7 4 f3! a6 5 Kc8 Ke8 6 Kb7, and White won (5 f4! a5 6 Kb6 etc. was even simpler).

We will now examine a number of posi-
tions in which, despite one side having the advantage of a better king position, against correct defence it cannot be realized.

an ancient position, 1843

686. This position, from the 1st edition of Bilguer’s Handbuch, was for a long time considered drawn. As a demonstration of this, Novotny later (in the 4th edition) gave the following variations: 1 e5 f × e5 2 f × e5 Kd7 3 Kd5 g5 4 e6 + Ke7 5 Ke5 h5 6 Kf5 g4 7 Kg5 K × e6 8 K × h5 Kf5, or 1 h4 g6! 2 Kd5 Kd7 3 e5 f × e5 4 K × e5 Ke7 etc. with a draw.

Fine (1941) tried to show a win here by 1 Kd5 Kd7 2 e5, e.g. 2 ... f × e5 3 K × e5 Ke7 4 Kd5 Kd7 (if 4 ... Kf6, then 5 Ke6 Kf5 6 Kb6 K × f4 7 K × a6 g5 8 K × b5 h5 9 K a6 g4 10 b5, winning). 5 Kc5 Kc7 f5! h6 7 h5 h5 h5 h4 Kd7 9 Kb6 Kd6 10 K × a6 Ke5 11 K × b5 K × f5 12 a4 g5 13 a5 g4 14 a6 g3 15 a7 g2 16 a8 = Q g1 = Q 17 Qd5 + Kf6 18 Q × h5 Qf1 + 19 Kb6 Qf2 + 20 Ka6 Qa2 + 21 Kb5, and White should realize his advantage.

But, as was shown by readers of the magazine Shakhmaty v SSR, against correct defence White’s plan is unrealizable. By 2 ... g5! Black obtains essential counter-play and forces a draw, e.g. 3 e6 + (3 e × f6 g × f6 4 K e4 Ke6) 3 ... Ke7 4 f5 (4 f × g5 f × g5 5 Ke5 h5 6 Kf5 g4 etc.) 4 ... h5 5 Kc6 h4! 6 Kd5 Ke8 7 Ke4 Kf8 8 Kf5 Kg7 9 Kg4 Kf8 10 Kh5 Kg7 11 h3 Kf8! (11 ... Kg8? 12 Kg6) 12 Kg6 Ke7. Draw.

The construction of a ‘fortress’ is the most effective way of countering an active enemy king. In fact, if the king cannot penetrate into the opponent’s position, all its activity proves quite useless.

Mieses-Schlechter
Coburg, 1904

687. Here everything depends on whether the black king can invade the enemy position, using the h3 pawn for diversionary means.

1 h5 Kd5 2 a4! This sacrifice is the whole point. Since nothing is promised by 2 ... b2 3 Ke2 b × a4 4 K × b2 Kc4 5 Ka3 K × c3 6 K × a4, when 6 ... Kb2 is strongly met by 7 b5 (the same pawn advance follows on 6 ... Kd4 7 Ka5 c5), Black has to reply 2 ... b × a4. But then 3 c4+ drives back the king, and closes all its paths to the white pawns. After 3 ... Kd6 4 Kc3 Kc7 5 Kb2 Kd6 6 Ka3 Black can try to open lines by 6 ... c5, but 7 b5 Ka5 8 Kb2 is not dangerous for White (if 8 g3, Black can gain the necessary tempo by ‘triangulation’, by returning his king to b7, a7 and b6). Bad, for example, is 8 ... Kb4 9 b6 a3 + 10 Kb1 Kc3 11 b7, when it is White who wins.

The following example is much more complicated.
688. White’s position looks hopeless: Black has a protected passed pawn, and his king has the open b8–h2 diagonal to the opponent’s K-side pawns. And yet White manages to save the game.

1 a4!!

All other moves are simply bad, e.g. 1 a3? Kc7 2 b4 c×b4 3 a×b4 d3! 4 Kb2 a4! 5 h4 d2 6 Kc2 a3, or 1 Kb1? Kc7 2 Kc2 Kd6 3 Kd3 Ke5 4 a3 h5. Black is also able to forestall the threat of a breakthrough: 1 h4 Kc7 2 g5 h×g5 3 h×g5 (3 f6 g×f6 4 h5 g4) 3 ... Kd6 4 f6 g×f6 5 g6 Ke7 etc.

1 ... Kc7 2 b4!! c×b4 3 c5! Kd7 4 h4 Ke7 5 g5 h×g5 6 h×g5. Drawn.

It is useful to know that the penetration of the king into the enemy position by no means always brings success. The king must be able to make an effective attack on the enemy weaknesses.

689. The black king is very strongly placed, and its opponent has not managed to occupy its best square at f1. This is therefore the most favourable moment for a breakthrough: 1 ... f4.

White cannot now play 2 c×f4?, since after 2 ... e3 3 f×e3 K×e3 his g-pawn cannot be defended.

2 g×f4 Kg4 3 Ke2 h5! (not immediately 3 ... K×h4 because of 4 f3, with a draw) 4 Kf1 (if 4 f3+, then 4 ... e×f3 + 5 Kf2 g6) 4 ... K×h4 5 Kg2 Kg4 6 Kh2 Kf3 7 Kg1 h4, and thanks to his passed pawn, Black won.

In 1926 Reti showed that 3 Ke2 was a mistake, and that by 3 Kf1! White could have gained a draw. For example: 3 ... K×h4 4 f3! e×f3 5 e4 Kg4 6 e5 Kf5 7 Kf2 etc., while after 3 ... h5? 4 Kg2 Black himself is too late with the advance of his pawn: 4 ... g6 (4 ... K×h4 5 f3) 5 f3 + e×f3 6 Kf2 K×h4 7 K×f3 g5 8 e4!, and it is White who wins.

In the following example Black’s king penetrates into the opponent’s position, but fails to make an effective attack on the weak pawn, since, by successfully manoeuvring with his king, White creates counter-threats.

690. The attempt to defend the h-pawn leads to defeat: 1 Kg2? Kf5! 2 Kg3 g6. The correct plan of defence is a counter-attack on the c-pawn: 1 Kf1! Kh5 2 Ke2 K×h4 3 Ke3 Kf4 4 K×e4 Kg2.

The game now continued: 5 f4 Kf4 6 f5! Kg2 7 Kf4 Kf2 8 Ke4 Kg2, and the players agreed a draw. Later Canal showed that, instead of the inaccurate 5 ... Kf2, Black could have won by 5 ... Kg3! 6 f5 (6 Ke3 g6 7 Ke4 Kf2) 6 ... Kg4 7 Ke5 Kf3 etc.

However, White too played inaccurately. Correct, as established by Knyazev, was 5 Ke3!, when White is able to maintain the
balance, e.g. 5 ... Kf1 (5 ... g5 achieves nothing due to 6 f4!) 6 Kf3! Ke1 (6 ... g5 7 Kg3 Ke2 8 f3 Ke3 9 Kg2 Ke2 10 Kg3 Kf1 11 Kh3! Kf2 12 Kg4, and 12 ... Kg2 is bad due to 13 f4!) 7 Kg3 Ke2 8 f4 Kd3 9 f5 Ke3 10 Kg4 Ke4 11 Kg5 Kf3 12 Kh5!
This flank manoeuvre by the king saves the game, as in the variation with 6 ... g5 just examined (cf. 11 Kh3!).
12 ... Kf4 13 Kg6 Ke4 (13 ... Kg4? is still bad in view of 14 f6!) 14 Kg5 with a draw.
But perhaps Black should have immediately sent his king to the support of his e-pawn: 1 ... Kf5 2 Ke2 Kf4? In this case White is rescued by 3 h5!, and if 3 ... e3, then 4 f3!, not allowing the creation of an outside passed pawn. For example: 4 ... Kg5 5 K × e3 K × h5 6 Kf2 Kg5 (6 ... Kh4 7 Kg2 g5 8 Kh2) 7 Kg3 Kf5 8 f4 Ke4 9 Kg4 g6 (there is nothing better) 10 Kg3! Ke3 11 Kg4 Kf2 12 Kh4! Kf3 13 Kg5, with a draw.
In the event of the king penetrating, one of the methods of defence is to block it in, i.e. restrict the king to a limited space, usually in the corner of the board.
691. White has three reserve tempi, and by 1 Ka3 he intends to invade the opponent's position with his king.
Passive waiting is fatal for Black, e.g. 1 ... Kc7 2 Ka4 Kc6 3 a3 Kc7 4 Kb5 Kb7 5 h4 Ke7 6 Ka6 Kc6 7 a4 Kc7 8 Ka7 Kc6 9 Kb8 etc.
But there is a way to save the game: 1 ... g5! (the threat of a breakthrough forces White to use up one tempo, but it is his most precious one) 2 h4 (2 f × g5? h4 3 g6 Kc6) 2 ... g × f4 3 g × f4 Kc7 4 Ka4 Kc6 5 a3 Kc7 6 Kb5 Kb7 7 a4 Kc7 8 Ka6 Kc6 9 Ka7 Kc7 10 Ka8 Kc8. Drawn.

Alekhine-Bogoljubow
World Championship Match, 1929
(variation from the game)
692. The white king has broken into the opponent's position, and Black is bound to lose his a-pawn. But this leads only to a draw, since the king finds itself shut in the corner. True, the play is not without subtleties, since Black also has to reckon with the e4-e5 break.

1 ... Kf7!

A precise move. Bad is 1 ... Ke7 2 Kc5 Kd7 (2 ... Ke6 3 Kc6) 3 Kd5 Ke7 4 e5, when White wins.

2 Ke5 Ke7!

Now 3 Kd5 Kd7 4 e5 fxe5 5 Kxe5 does not hold any special promise due to 5 ... Kc6, e.g. 6 Kf5 Kb6 7 Kxg5 Kxa6 8 Kxf4 Kb5 9 Kg5 (9 Ke3 Kc4 10 Kd2 Kd4 11 Kc2 a5 12 Kb3 Ke3 13 Ka4 Kf4! 14 Kxa5 Kg3 15 Kh5 Kxg2 16 f4 Kxh3) 9 ... Ke5! 10 f4 Kd6 11 Kg6 Ke7 12 Ke7 Ke6, with a draw.

3 Ke6 Ke6 4 Ke7 Ke7! 5 Ke8 Ke8 6 Kb8 Kd8! (a necessary finesse: 6 ... Kd7? is bad due to 7 Kd6 Ke8) 7 Ka8 Ke8 8 Kxa7 Kc7. Draw (analysis by Grigoriev).

Interesting drawing possibilities are shown by the following practical example, all the subtleties of which were revealed only very recently.

Horowitz-Denker
Philadelphia, 1936

693. Positions of this type (672 and 673), in which the king has already invaded the enemy position, have already been considered in the section on extra tempi. But we have included this example here, on account of the special features of the pawn formation which create drawing possibilities.

The game took the following course: 1 h4 Kg8 2 g4 Kf8 3 Kh7 Kf7 4 Kh8. But why not the standard 4 g5? This move does not win, on account of the unexpected 4 ... h5! 5 Kh8 Kg8—stalemate. (Black answered 4 Kh8 with 4 ... Kf8, but after 5 g5 he resigned).

Zinari (1974) found that after 3 Kh7 Black could have forced a draw by 3 ... h5!, e.g. 4 g x h5 Kf7 5 h6 g6! 6 Kh8 Kf8, and the white king is imprisoned, or 4 g5 Kf7 5 Kh8 Kg8 with stalemate. In this last variation 5 g6+ (instead of 5 Kh8) does not win, in view of 5 ... Kf6 6 Kg8 Kxg6 7 Kf8 Kf6 8 Ke8 g5 9 h x g5 + K x g5 10 Ke7 h4 11 K x d6 h3 12 Kc7 h2 13 d6 h1 = Q 14 d7 Qh7 15 Kc8 Qh3 etc.

But does this mean that the ending is drawn? Of course not. It turns out that the plan chosen by White was wrong.

The correct continuation was 3 g5! h x g5 4 K x g5 Kf7 5 h5 Ke7 6 Kg6 Kf8, although the winning path is difficult and requires deep calculation.

7 h6!

By exchanging the K-side pawns, White aims to approach his opponent's main weakness—the d6 pawn. If now 7 ... g x h6 8 K x h6 Kf7, then 9 Kh7 Kf6 10 Kg8! Kg5 11 Kf7 Kg4 12 Kf6 Kg3 13 Kg6 Kf4 14 K x c5 e4 15 d6 e3 16 d7 e2 17 d8 = Q e1 = Q 18 Qf6+ Kg4 19 Kc6, and White easily realizes his material advantage.

But Black finds an unexpected resource.

7 ... Kg8! 8 Kh5!

Of course, not 8 h x g7 with stalemate, but possible is 8 h7+ Kh8 9 Kf5 K x h7 10 K6 g5 11 K x d6 g4 12 Kc7 g3 13 d6 g2 14 d7 g1 = Q 15 d8 = Q with winning chances. Nevertheless 8 Kh5! is stronger, since it transpose into a continuation already examined.

8 ... g x h6.

There is nothing better. If 8 ... Kh7, then 9
h × g7 K × g7 10 Kg5 Kf7 11 Kf5, and White wins very easily.

9 K × h6 Kf7 10 Kh7 Kf6 11 Kg8!, and White wins.

And now a few further improvements found by Zinar. After 3 Kh7 Kf7 4 Kh8, instead of the incorrect 4 . . . Kf8 there was still a draw by 4 . . . h5! 5 g × h5 Kf8 or 5 g5 Kg6 6 Kg8, with stalemate. The correct continuation was 4 h5! Kf6 (4 . . . Kf8 5 Kg6 Kg8 6 Kf5 Kf7 7 g5 leads to variations already studied) 5 Kg8 g6 (5 . . . g5 6 Kh7) 6 Kf8! g × h5 7 g × h5 Kg5 8 Ke7 K × h5 9 K × d6 Kg4 10 K × e5, winning. Finally, Black also fails to save the game by 4 . . . Kg6 (instead of 4 . . . Kf8) 5 Kg8 Kf6 6 h5 g6 7 Kf8!, when White transposes into the continuation just considered.

As we see, in this example White wins not by the standard method of creating a zugzwang position, but by exchanging all the King's side pawns followed by penetrating to Black's main weakness—the d6 pawn.

In conclusion, here is a position with a slightly different pawn configuration.

694. Here Black's weakness at c7 is rather further away than in example 693.

Let us try employing the plan of widening the bridgehead. In this case the play can take the following course: 1 h5 Kg8 2 g5 h × g5 3 K × g5 Kf7 4 Kf5 Ke7 5 Kg6 Kf8 6 h6 g × h6 7 K × h6 Kf7 8 Kh7, and the simplest way to draw is by 8 . . . Kf8 9 Kg6 Kg8 10 Kf6 Kf8 11 Ke6 Ke8. However, a counter-attack is also possible: 8 . . . Kf6 9 Kg8 Kg5 10 Kf7 Kf4 11 Ke6 K × e4 12 Kd7 K × d5 13 K × c7 e4 14 Kd7 e3 15 c7 e2 16 c8 = Q e1 = Q 17 Qc6+ with a draw.

But nevertheless White can obtain winning chances, by again changing plan: 1 Kh7 Kf7 2 g5 h5! 3 g6 + ! (3 Kh8 Kg6! with a draw) 3 . . . Kf6 4 Kg8 K × g6 5 Kf8 Kf6 6 Ke8 g5 7 h × g5 + K × g5 8 Kd7 h4 9 K × c7 h3 10 K × d6 h2 11 c7 h1 = Q 12 c8 = Q Q × e4 (in view of the threat of 13 Qf5+), it is unlikely that Black has anything better) 13 Qh8!, and it would appear that Black is unable to save this ending.

Thus various nuances in the pawn formation can significantly affect both the choice of plan, and the final result.
10. The Theory of Corresponding Square Systems

In the preceding chapters the reader will on many occasions have encountered positions of varying complexity, in which the correct path could be found by using corresponding squares. We have met some of the most common instances of corresponding squares (for example, such as ‘triangulation’), but other systems are also known, as will be described in the present chapter. The author of it is chess composer M. Zinar, a major specialist in the field of pawn endings.

It is worth recalling that the theory of corresponding squares began to be developed only in the 20th century. The basis of it was a well known publication by Emanuel Lasker (1901). Much attention has been paid to this question by Grigoriev, Bianchetti, Ebersz, Halberstadt, Duchamp, Bähr, Maizelis, and also a number of other theorists.

It has been established that, in spite of the great variety of examples, the number of different corresponding square systems is comparatively small, and in each individual case the manoeuvring of the kings is of a definite nature, typical of the given system.

In this chapter an attempt is made to generalize everything of value that has been devised in the theory of corresponding square systems, so as to give practical players appropriate recommendations, which should help them to find their way in certain complex pawn endgame positions. Therefore the present chapter is intended mainly for players of high standard.

What is required for the correct handling of pawn endings with extensive zones of corresponding squares? Firstly, one must be able to determine quickly the system of corresponding squares; secondly, it is essential to understand its structure, and thirdly, one must be able to find one’s bearings in the given system.

We should mention that, while the second and third questions have been studied more or less fully by theorists, the first question, the most important one for practical players, has until now remained practically untouched. In the present work the author will attempt, at least partially, to fill this gap.

On examining a position with a complicated system of corresponding squares, an experienced player will immediately pick out the front-line corresponding squares, if the struggle is for adjoining critical and key squares, or the shortest paths, if the play revolves around two invasion points. It is from the front line or the shortest path that we will proceed.

We will first introduce the concept of base squares—this is the name we will give to two adjoining, strictly unambiguous squares of the front line or the shortest path, which without fail are adjacent to the rear squares. Depending on the arrangement of the base squares, there may be either one, or two, such rear squares. If there is one such square linking the base squares, we will have a base triangle, if there are two, we will have a $2 \times 2$ base square.

Using these three concepts we have introduced, it is easy to determine what will be the system of corresponding squares.

10.1 Eight-Square System

If the base triangles can be amplified to a
The Theory of Corresponding Square Systems

3 × 3 square, we have in operation an eight-square system.

Locock, 1892

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\[
\begin{array}{cccccccc}
& a & b & c & d & e & f & g & h \\
8 & & & & & & \text{\textcolor{red}{1}} & & \\
7 & & & & & & 4 & 5 & 6 \\
6 & & & & & & 1 & 3 & 7 \\
5 & & & & & & 2 & \text{\textcolor{red}{8}} & \\
4 & & & & & & 1 & 4 & 1 & \\
3 & & & & & 3 & 5 & 3 & 2 \\
2 & & & & & 7 & 6 & 7 & 8 \\
1 & & & & & 3 & 5 & 3 & 2 \\
\end{array}
\]

695. In view of the threat of e4–e5, the first position of mutual zugzwang is Kd4/Kf6. The second, Ke3/Kg5, arises from the struggle for the key f4 square. We thus have two strictly unambiguous front-line squares. They are linked by one rear square: d3 for White and g6 for Black. The base triangles d4–e3–d3 and f6–g5–g6 can be amplified for each side to a 3 × 3 square (with the e4 square inaccessible for White, and f5 for Black).

We thus come to the conclusion that we have here an eight-square system.

We will designate the base squares by the numbers ‘1’ and ‘2’ respectively, and the linking square by ‘3’. On the sides of the base triangle we successively fill in the numbers from ‘4’ to ‘8’ (we write ‘4’ alongside ‘1’). As a result we obtain the main zone of the eight-square system. The main zone is needed to reduce the designations to a minimum.

Rear squares receive the same designations as those squares of the main zone which are one square away horizontally (or vertically). Corresponding squares are marked until an obstruction or counter-play appears. The correspondence does not extend forward beyond the sides of the base triangle. Downwards squares are marked as far as the edge of the board, and to the left—as far as, and including, the b-file. The a-file is excluded due to the possibility of counter-play by Black. Incidentally, every case, where the opponent has counter-play, requires specific checking by the calculation of variations.

To draw, the defender’s king must occupy a square of the same designation as that occupied by the opponent’s king. If the stronger side’s king is to the rear, the defender can occupy a square of equivalent designation either in the rear, or in the main zone. But if the stronger side’s king is in the main zone, the opponent’s king must also be there.

To win, on each move one must occupy a square of equivalent designation, or a square left unattended, endeavouring at the same time to approach the base squares. If this is not possible, one must continue manoeuvring, at least not moving away from the base squares.

The solution of the position is as follows: 1 Kb1! (occupying a ‘3’ square, and approaching the base square ‘2’) 1 ... Kg7 2 Ke1 (also to a ‘5’, approaching ‘2’) 2 ... Kg6 3 Kd1!

It is not possible to approach the base squares, so White continues manoeuvring, at least not moving away from them.

You must never return to a square where you have already been! For this reason White does not play his king to b1, although it too is designated with a number ‘3’.

3 ... Kg5 4 Ke2!

Here White had a choice: should he occupy a ‘2’ square (4 Ke1), or move to a ‘6’ square, which has been left unattended by the black king? In the event of 4 Ke1 White does not approach the base squares, whereas with 4 Ke2 he approaches the base square ‘1’. This means that 4 Ke2! is correct.

4 ... Kh5 3 Ke3.

White cannot occupy an ‘8’ square, but he goes to a ‘5’ square which has been left unattended, while approaching the base square ‘1’.

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5 ... Kg5 6 Ke4.

White cannot approach any closer to the base squares, so he continues his king manoeuvre, by not moving away from them, and places it on a '4' square which has been left unattended.

6 ... Kg6 7 Kd3 (also to '3', approaching the base square '2'), and now 7 ... Kg5 is decisively met by 8 Ke3, and 7 ... Kf6 by 8 Kd4.

Bähr, 1936
from Locock

696. This is a similar position to 695, but moved to the left. It is of interest for the fact that, due to the obstruction on the h-file, Black loses the correspondence.

1 Kbl! Kh7 2 Ke1 Kh6 3 Kd1 (the h5 square is inaccessible, and Black cannot defend the '7' and '8' squares) 3 ... Kh7 4 Kd2 Kg6 5 Ke2 Kg5 6 Kb3 Kf5 7 Kb4 Kf6 8 Ke3, and White wins.

697. Here one end of the shortest path is Kc5/Kc7, while the other is Kf4/Kh5. The shortest path consists of 4 squares.

Which two squares should be taken as the base pair? Not the c3–f4 pair: there is no linking square. If one takes c5–d4 as the base squares with one linking square, in reply Black will have the e7–f7 pair with two linking squares, i.e. White will have a base triangle, and Black a 2 × 2 base square.

A 2 × 2 base square gives an excess of correspondence in comparison with a base triangle. In this case Black will not be forced to adhere to strict correspondence on the rear squares. So we should choose a system which forces both sides to adopt the strictest manoeuvring. Therefore we take as the base squares d4–e3 for White and f7–g6 for Black. The base triangles can be amplified to 3 × 3 squares, so we again have an eight-square system.

The markings, the zone structures and the play are all analogous to the previous examples. Black has no counter-play, and so the zone extends to the edge of the board.

1 Ka2! Kg8 2 Ka3 Kf8 3 Kb2 Kf7 4 Kc2 Kf8 5 Kd2 Kf7 6 Ke2 Kg7 7 Kd3 Kf7 8 Kd4, and White breaks through on one of the wings.

698. The shortest distances a5–b4–c3–d2–e3 and b7–c7(c8)–d7–e6–f5 consist of five squares. For the base squares we take the c3–d2 pair, since the d2–e3 pair has no linking square, while with White's king at b4 the black king can be either at c7, or at c8, i.e. the correspondence is not unambiguous. The course of the further reasoning is the same as in the preceding examples.

1 Ka2! Kd7 2 Kbl!
The Theory of Corresponding Square Systems

Brogi, 1965
from Bianchetti

But not 2 Ka3? Kc7!, since the a3 square is removed from the correspondence by counter-play: 3 Kb2 Kb7 4 Kc2 Ka6.
2...Kd8 3 Kc1 Kd7 4 Kd1 Ke7 5 Kc2 Kd7 6 Kc3, and wins.

Bähr, 1936

The shortest distances d4–e4–f3–g4–h5 and c6–d6–e7–f7(f8)–g7 consist of five squares. The g4 square does not have unambiguous correspondence, and cannot be taken as a base square. For the base squares we will take e4–f3 for White and d6–e7 for Black—an eight-square system.

The question arises: why did we not take d4–e4 as the base squares? As will become clear from what follows, we would then have obtained a quadratic system, the main zone of which consists of four squares, whereas, as was mentioned in the analysis of example 697, we are obliged to choose the system which leads to the most strict maneuvering. Therefore preference is given to the eight-square system, the main zone of which consists of 8 squares.

The markings are given in the diagram.
The correspondence does not extend to the b-file, since on the c-file there is an inaccessible square at c4.
1...Kc1! Kc7 2 Kd1 Kd7 3 Ke1 Kd7 4 Kd2 Kd8 5 Ke2 Ke8 6 Kd3 Kd7 7 Ke3 Kd6 8 Ke4 Kc6.

Now White breaks through on the K-side, but the game is not yet over.
9 Kf3 Kd5 10 Kg3 Kd4 11 Kh4! Kc3 (11...Ke4 12 Kg4 Ke3 13 Kh5 K×f4 14 K×g6 Ke5 15 a5!—this is why White needs his tempo) 12 a×b5 a×b5 13 Kh5 K×b4 14 K×g6 K×b3 15 K×f6 Kc3! 16 Kc5! b4 17 h6 b3 18 h7 b2 19 f8=Q b1=Q 20 Qc5+ Kd2 21 Qd4+, exchanging queens.

700. With the white king at g4, Black must defend his pawn from h6, so that he can meet 1 g4 with 1...Kg6! 2 g5 b5! with a draw.

From the necessity to defend the key squares c4 and d4, the second end of the shortest path of six squares, h4–h3–g2–f2 e3–d3 and h6–g6(g7)–f6(f7)–e6–d5–c5, will be Kd3/Kc5. The correspondence on the K-side is not unambiguous, and so, as in example 699, we take e3–f2 as the base squares, giving preference to the eight-square system.

1 Kc1! Kc7 2 Kd1 Kd7 3 Ke1 Ke7 4 Kf1 Kf7 5 Ke2! Ke6 6 Kf2! Kd6 (the counter-attack 6...Kd5 7 Kg2 Kd4 fails to 8 g4) 7 g4 h×g4 8...
f × g4 f × g4 9 Kg3 Ke6 10 K × g4, and White wins: 10 ... Kf6 11 f5 Kf7 12 Kf4 Kf6 13 Ke4 Ke7 14 Kd5 Kf6 15 Ke6.

Compare the relative placing of the main zones in the above examples. Everywhere it is different. But, whatever the placing of the zones relative to each other, the determination, the structure and the play all follow the same principle.

10.2 QUADRATIC SYSTEM

If an eight-square system cannot be formed, but on the side of the base triangle a square can be constructed, we have in operation a quadratic system.

If both sides have 2 × 2 base squares, the quadratic system again operates.

701. Compared with example 697, here the e2 square is inaccessible to White. The base triangle cannot be amplified to a 3 × 3 square, but on the d4–d3 side a 2 × 2 square can be constructed. We conclude that here the quadratic system operates.

We designate the apex of the triangle by the letter ‘a’ (this square is unsystematic and does not have any related squares to the rear, and so we designate it with a letter). We designate the second base square with the number ‘1’, and the linking square with the number ‘2’. Alongside ‘1’ we write ‘3’, and alongside ‘2’ we write ‘4’.

We have obtained the main zone of the quadratic system, which consists of four squares. The rear squares receive the same designations as the squares of the main zone which are one square away horizontally (or vertically).

In width the quadratic system extends for the same number of lines as have an outlet through the triangle apex ‘a’. In our example this is the 4th, 3rd and 2nd ranks.

In depth the quadratic system extends until the appearance of counter-play or some obstruction, or as long as there is an unequal distance from adjacent rear squares to the nearest square of the shortest path. In example 701, from b4 to the nearest square of the shortest path c5 it is one move, and from b3 it is two moves. But the correspondence does not extend to the a-file, since from a4 and a3 to c5 the distance is the same (two moves).

Here we will employ the same approach as in the eight-square system.

1 Kb2! (to a ‘1’ square, on which the black king stands, and approaching the base squares) 1 ... Kh8 2 Kc2 (also to ‘3’, and approaching the base square ‘a’) 2 ... Kh7
Kd2 (to a ‘1’ square, approaching the ‘a’ square) 3 ... Kh6 4 Ke3 (to a ‘4’ square, which has been left unattended, and approaching the ‘1’ square) 4 ... Kg6 5 Ke4! (it is not possible to approach ‘1’ or ‘a’, so we continue the manoeuvre to a ‘3’ square, which has been left unattended) 5 ... Kf7 6 Kd4!, breaking through on one of the wings.

Kd6 5 Ke4 Ke6 6 Kf3 Kd5 7 Kg4 Kd4 8 Kh5 Ke3 9 Kg6 Kh3 × b3 10 K × f6, and White wins.

The correspondence does not extend to the 1st rank, since from d1 and e1 to f3 the distance is the same; nor does it extend to the c-file, since there is no outlet to the key squares c5 and d5.

Em. Lasker, 1901

702. In comparison with example 699, here the f2 square is inaccessible to White, and therefore an eight-square system cannot be formed. White cannot use his pawn tempo, since after f2–f3 this lengths the shortest path d4–e4–f3–g4–h5, and c6–d6–e7–f7(f8)–g7.

Which should be taken as the base squares? If the d4–e4 pair, we obtain a 2 × 2 base square—a quadratic system; if the e4–f3 pair, we obtain a base triangle—also a quadratic system. The main zones in both cases coincide.

In such situations there is a choice available. We have chosen the d4–e4 pair, but we could have also taken the e4–f3 pair, designating the apex (f3) by the letter ‘a’, e4 by the number ‘1’, and so on. This would not have changed anything.

1 Kd2! Kd8 2 Ke2 Ke8 3 Kd3 Kd7 4 Ke3!

703. On the K-side the correspondence here is not unambiguous. On the c3–d3 side of the base triangle a square can be constructed. The marking is made in analogy with previous examples.

The shortest paths are c4–d3–c3(e2)–f3(f2)–h4 and b6–c7–d7–e7–f7–g7. In width the zone occupies three files, from which there is an outlet to the ‘a’ square. In depth it extends to the edge of the board, since the distances from c1 and d1 to the line of the shortest path are different.

1 Kb1! Kb7 2 Kc1 Kc7 3 Kd1 Kd7 4 Ke2 Ke8 5 Kd2 Kd7 6 Ke3 Kc7 7 Kb4, and wins.

Compare the following example with position 700.

704. 1 Kd1! Kd7 2 Ke1 Ke7 3 Kf1 Kd7 4 Kg2.

The king emerges from the correspondence zone onto the line of the shortest path e4–f3–g3–h4–h5 and d6–e7–f7–g7–g8–h7.
705. For the base squares we will take c4—c3 with the linking square b3, since further to the right the correspondence is not unambiguous. And although the base triangle has a different appearance than in the earlier examples, this does not affect the determination of the system. On the c3—b3 side we construct a square.

1 Kb1! Kb7 2 Kc1 Kc7 3 Kb2 Kb8 4 Ke2 Kc8 5 Kb3 Kb7 6 Kc3 Kc7 7 Kc4 Kb8 8 Kb5 Kb7 9 a3, and White wins.

706. Here the shortest paths consist of eight squares: c4—d3—e2—f1—g2—h3—h4—h5 and a5—a6—b7—c8—d7(d8)—e7(e8)—f8—g7.

The pairs c4—d3 and h4—h5 cannot be taken as the base squares, since there is no linking square. We can take d3—e2 (a6—b7 for Black) or e2—f1 (b7—c8 for Black). In both cases we obtain a quadratic system with main zones which coincide.

1 Kb1! now Black is forced to step onto a ‘1’ or ‘3’ square) 1 ... Kb8 2 Kc1 Ka8 3 Kd1 Kb8 4 Ke1 Kb7 5 Ke2, and White wins.

707. An insignificant change in the posi-
tion, and Black's main zone has moved to the right here in comparison with example 706.

1 Ke1! Kb8 2 Kd1 Kb7 3 Kd2 Kc7 4 Ke2, and wins.

Zinar, 1976

708. Here d4, e4, f4 are the key squares, and the front-line squares are d3, e3, f3 and d5, e5, f5. Both pairs can be taken as base squares, since in both cases this leads to a quadratic system with coinciding main zones. White succeeds in winning the correspondence at the distant approaches, by exploiting the unfortunate position of the black king.

1 Ka2! Kg6! 2 Ka3!! Kf5 3 Kb3 Ke6 4 Kc2 Kd6 (4 ... Ke5 5 Kc3 Ke6 6 Kd4 Kd6 7 g6!, winning) 5 Kd2 Ke6 6 Ke2! Kd6 7 Kf3! Ke5 8 Ke3 Kd5 9 Kf4 Ke6 10 Ke4, and White wins.

709. Apart from passive defence, Black also has the possibility of counter-play against the f2 pawn. This results in the non-systematic zugzwang position Ke5/Kg4.

For passive defence the squares of the shortest path are d5–d4–e3 and f6–g5–g4. Either pair can be taken as the base squares, and, as in examples 701, 706 and 708 we obtain a quadratic system with coinciding main zones.

1 Ke2! Kh6 2 Ke3 Kg6 3 Ke4 Kg5 4 Kd4 Kh5 5 Kd5 Kg4 6 Ke5 Kg5 (6 ... Kh3 7 Kf4!) 7 Ke6, and White wins.

Zinar, 1983

Let us compare the arrangement of the corresponding squares in the examples considered: in 703 they are on adjacent files, on squares of opposite colour; in 706 they are on adjacent files, on squares of the same colour; in 707 they are on the same file, on squares of opposite colour; and in 701 and 709 the main zones are altogether rotated relative to each other.

As we see, it is not possible to employ any method other than the theory of corresponding squares. Note also that the quadratic system occurs more often than the eight-square system, since it requires less space.

We have met several examples where any pair of three adjoining squares of the shortest path (or the front line) can be taken as the base pair. Let us generalize these cases:

(a) The number of possible base squares must always be reduced to the minimum.

(b) With coinciding main zones, if the systems are different, then preference should be given to the one with the larger main zone; if the systems are identical, either pair of squares can be taken as the base pair.

10.3 QUADRATIC SYSTEM WITH NON-UNAMBIGUOUS REAR

This is a transitional system between the quadratic and triangular systems.
Triangular System

If the distances from the '3' and '4' squares of the main zone of a quadratic system to the nearest square of the shortest path are equal, we have in operation a quadratic system with non-unambiguous rear.

**Zinar, 1983**

**710**

```
    8 7 6 5 4 3 2 1
8     3 3 3       3 3 3
7     1 2 1       1 2 1
6     a           a
5     3           3
4     1 2 1       1 2 1
3     3 3 3       3 3 3
2     a           a
1     b c d e f g h
```

5 Ke3 K × h5 6 Kd2! (only now does the king make for the saving a4 square) 6 ... Kg4 7 Kc2(c1) h5 8 Kb2 h4 9 Ka3 h3 10 Ka4 h2 11 a3 h1 = Q — stalemate.

**Zinar, 1983**

from Bianchetti

**712**

```
    8 7 6 5 4 3 2 1
8     1 3       1 3
7     a 2 3     a 2 3
6     1 3       1 3
5     3           3
4     1 2 1     1 2 1
3     3 3 3     3 3 3
2     a           a
1     b c d e f g h
```

712. Compare this example with positions 697 and 701. The distances from d2 and e2 to f4 are identical. That means we have a quadratic system with non-unambiguous rear.

1 Ke2! Kg8 2 Kc3 Kf8 3 Kd2 Kf7 4 Ke2 Kg7 5 Kd3 Kg6 6 Ke3, and White wins.

10.4 TRIANGULAR SYSTEM

If a 2 × 2 square cannot be constructed on
one side's base triangle, we have in operation a triangular system. The reader will already be familiar with certain cases of the triangular system ('triangulation'). Here we will describe certain features of this system.

Zinar, 1983

713. The base squares receive the designations '1' and '2', and the linking square '3'. This is the main zone. The rear squares are marked with the same numbers, such that in each new triangle there should be the numbers '1', '2' and '3'. The correspondence zone extends until it proves possible to construct a square on the side of the triangle.

Play reduces to the exploitation of rear squares. On them the correspondence is won, and play then follows the principles examined earlier. One must occupy a square of identical designation, or one left unattended, while approaching the base squares.

Due to Black's counter-play against the f6 pawn, the system in our example is irreversible. The correspondence must be won immediately by a king manoeuvre on the rear squares.

1 Ke3! Kg8 2 Kd3 Kh8 3 Kc4 Kh7 4 Kd4 Kh6 5 Kd5 Kh5 6 Kd6 Kh6 (6... Kh4 7 Ke7; 6... Kg4 7 Ke5) 7 Kd7! Kh5 8 Ke8! Kg6 9 Ke7, and wins.

Zinar, 1983

714. Here in the defence of the key d4 square the following positions of mutual zugzwang arise: Ke4/Kc5 and Ke3/Kd5.

A quadratic system cannot be formed. The e3 and f3 squares are designated by the numbers '2' and '3', which means that e2 receives the missing number '1'. Since e3 and e2 are designated '2' and '1', the d2 square receives the missing number '3'. The system does not extend further, since a square can be constructed.

1 Kd1! Ke6 2 Kd2 Ke5 3 Ke2 Ke6 4 Kf3 Kd5 5 Ke3 Kc5 6 Ke4 Ke6 7 Kd4 Kb6 8 Kxe4 Kc6 9 Kd4 Kb6 10 e4 Kc6 11 e5 Kc7 12 Kc4 Ke6 13 f3, and White wins.

Zinar, 1983
Two Systems

715. This rather unnatural position demonstrates the maximum zone of the triangular system.

For the base squares we take the b5–c4 pair, since the b5–c6 pair does not have a linking square. The pawn tempo a2–a3 must be retained.

1 Kh1 Kf6 2 Ke1! Ke7 3 Ke2 Kd6 4 Kd2 Ke5 5 Ka3 Kf6 6 Kb3 Kd6 7 Kb4 Kf6 8 Kd5 Ke7 9 Ke4! (not 9 Kc6 Ke6! 10 Kd7 Kd7 11 K × a7 Ke7! 12 a3 f6) 9 ... f6 10 a3, and wins.

The triangular system occurs frequently, with the most diverse pawn structures. We have considered the most complicated cases, since where there are only the main zones of a triangular system, the play does not present any difficulty. Thus, for example, in position 715 White has an excess of correspondence to the rear, in the absence of counter-play for the opponent. The play in such cases is of a reversible nature: in the event of a mistake one can return to the rear, and again begin the battle for the correspondence.

10.5 TWO SYSTEMS

Cases are possible where two systems exist on different parts of the board. This does not usually affect the marking of the squares, since the play takes place in one of the systems.

Leick, 1939

716. In the centre we have the base triangles d4–d3–c3 for White and c6–d7–c7 for Black, and on the K-side we have 2 × 2 base squares; i.e. two quadratic systems.

To draw, of course, both systems must be maintained, but to win, one is sufficient.

1 Ke1! Kd7 2 Kd1! Ke7 3 Ke2 Kd8 4 Kd2 Ke8 5 Ke3 Kd7 6 Kd3 Ke7 7 Ke2 Kd7 8 Kf3 Ke7 9 Kg4 Kf7 10 Kh5! Kg7 11 Kg5, and White wins.

Zinari, 1983
from Bähr

717. The tempo has to be preserved here, of course. On the base triangle d3–c4–c3 we form an eight-square system, while on the K-side a quadratic system operates.

1 Kh2! (Black is forced to stand on a ‘2’ or ‘4’ square) 1 ... Kf8 2 Kg2! Ke8 3 Kf2 Kd8 4 Kg3 Ke7 5 Kf3 Kd6 6 Kg4 Ke8 7 Kf4 Kd8 8 Kg5 Ke7 9 Kf5 Kf7 10 Ke4 Kg6 (10 ... Ke7 11 Kd3 Kd7 12 Kc4 Ke7 13 Kd7 Kd7 14 h5) 11 Kd3 K × h6 (11 ... Kf5 12 Ke3! Kf4 13 Kb4! Ke4 14 Kc4 Ke3 15 Kh5 K × d4 16 Ke6 Ke5 17 h5) 12 Ke4 Kh5 13 Kb5 K × h4 14 Kc6 Kg3 14 K × d6 h5 16 Ke5! h4 17 d6 h3 18 d7 h2 19 d8=Q h1=Q 20 Qg5+! Ke2 22 Qf4+, winning.

718. One end of the shortest path is clear: Kg3/Kf5. The second is determined from the
possibility of White playing 1 \( c4 \times c4 \) 2 \( b \times c4 \) 3 \( Kc3 \) — at this point Black must be able to play 3 \( \ldots \) \( Kb5 \).

On the Q-side a quadratic system operates, while in the struggle for the key f4 square on the K-side a quadratic system with non-ambiguous rear is formed. It is curious that on the left the systems are shifted relative to each other, while on the right they are rotated through 90° relative to each other.

1 \( Kb1! \) \( Kb7 \) 2 \( Kc1 \) \( Kc7 \) 3 \( Kd1 \) \( Kd7 \) 4 \( Kc2 \) \( Kc6 \) 5 \( Kd2 \) \( Kd6 \) 6 \( c4 \times c4 \) 7 \( b \times c4 \) 8 \( Kc3 \) \( Kd5 \) 9 \( b5 \) \( Kc5 \) 10 \( b6 \) \( K \times b6 \) 11 \( K \times c4 \), and White wins.

### 10.6 SIX-SQUARE (RECTANGULAR) SYSTEM

Up till now we have considered quadratic systems with coinciding rears or situated on different parts of the board. But cases are possible where adjoining quadratic systems combine, since they have common base and rear squares.

If both sides have adjoining quadratic systems, we have in operation a six-square system.

719. The key line here is the 5th rank. On the front-line squares \( d4-c4 \) is one quadratic system, and on the \( c4-f4 \) squares is a second, adjoining it. This means that a six-square system is in operation.

720. On the base squares \( c4-d3 \) is one quadratic system, and on the \( d3-c4 \) squares is an adjoining quadratic system.

721. On the base triangle \( d4-e3-d3 \) we have one quadratic system, and on the base square \( c3-f3-e2-f2 \) we have a second.

In all three cases, according to our definition, we have a six-square system.

We will designate ‘2’ the base square which is common to both quadratic systems, those on the same line as it — ‘1’ and ‘3’, and below them — ‘4’, ‘5’ and ‘6’. This is the main zone.

It has already been mentioned that, if a quadratic system is formed on a base square,
the apex of the triangle is marked by a letter and does not come into the main zone. In example 720 these are the c4 and e4 squares, and in example 721—d4. In the rear the squares receive the same designations as those one rank (or file) away in the main zone.

In depth the zone extends until the appearance of an obstruction or counter-play. In position 720 the 1st rank is excluded from the correspondence in view of counter-play.

In width the six-square system extends according to the same principles as the quadratic systems which form it: to the number of files from which there is an outlet to the triangle apex or to the key squares (719—3 files, 720—4 files, 721—5 files). As we see, in examples 719 and 720 the corresponding squares for both sides are on the same file, while in position 721 they are on adjacent files.

The play here follows familiar principles. To win, the white king must occupy squares of equivalent designation, or ones left unattended. At the first opportunity it must approach the base squares.

Now let us examine the solution of the positions.

719. 1 Kd1! Ke7 2 Ke1 Kd7 3 Kf2 (the '3' square has been left unattended, which means that there is a chance to approach) 3... Ke8 4 Ke2! Kf8 5 Kd3 Ke7 6 Ke3 Ke6 7 Ke4, and wins.

720. 1 Kb2! Ke8 2 Kc2! Kd8 3 Kd2 Ke8 4 Ke3 Kd7 5 Kd3 Ke7 6 Ke4 Ke6 7 Kd5 Kb5 8 Kb6 Kc4 9 Kxb7 Kxe5 10 Ka6 Kd4 11 Kxa5 Ke3 12 Kxb5 Kxf3 13 a5 e4 14 a6 e3 15 a7 e2 16 a8 = Q +, and White wins.

721. 1 Ke1! Kc7 2 Kd1 Kd7 3 Ke1 Kc7 4 Kd2 Kd8 5 Ke2 Ke8 6 Kd3 Kd7 7 Ke3 Kc7 8 f5, and wins.

Another typical position with a horizontal six-square system is shown in the following example.

722. The key squares here are b6,c6,d6. The correspondence does not extend to the rear or to the K-side, on account of Black's counter-play.

1 Kc2! Kb7 2 Kd3 Kc7 3 Ke4 Kd6! (3... Kd8 4 Kd4 Kc8 5 Ke5 Kd7 6 Kd5 Kc7 7 Kc5, winning) 4 Kd4! Ke6 5 Kc5 Kf5 6 Kb6 Kxg5 7 Kxa6 f5 8 Kb5! f4 9 Kc4 f3 10 Kd3 Kf4 11 a6 Kg3 12 a7 f2 13 Ke2 Kg2 14 a8 = Q +, and White wins.

In practice the vertical six-figure system is more common, but the play follows the same principles.

723. It is unfavourable for either the white or the black pawns to advance: this merely weakens the position.
Apart from the six-square system, Black has another method of defence: he can try to cut off the white king on the 7th rank, when at this point it cannot go to the 8th rank due to the threat of ... d3.

1 Kg5! (this sham attack on the d4 pawn leads to Black losing the correspondence) 1 ... Ka6 2 Kg6! Ka5 3 Kg7 Kb4 4 K'6 Kb5 5 Kf7 Ke4 6 Ke6 Ke5 (6 ... d3 7 c×d3 + K×d3 8 Ke5!) 7 Ke7! Ke6 8 Kd8 Kb5 9 Kd7 Ke5 10 Ke7 Kd5 11 Kb6 Ke4 12 Ke6 d3 13 c×d3+ K×d3 14 Kd5, and White wins.

As we see, the play here was much more complicated than in earlier examples where the six-square system was operating in pure form.

10.7 'T' SYSTEM

Adjoining quadratic systems against coinciding quadratic systems form a 'T' system.

The first study with this system was composed by Sacconi, but for its name it is indebted to Bähr. We begin our examination of the system with an already familiar example (409).

724. The shortest paths are e5–f5–g6 and e7–f7–g8. The 2×2 base square on the e5–f5 squares and the base triangle on the f5–g6 squares are adjoining. But Black's 2×2 base square on the e7–f7 squares and base triangle on the f7–g8 squares are rotated relative to each other, and their main zones coincide. This means that here the 'T' system is in operation.

As usual, we designate the triangle apex by the letter 'a' and do not include it in the main zone. The square which is common to both quadratic systems we designate with the number '2', those on the same line as it—'1' and '3', and under it—'4'.

The shape of White's main zone resembles a letter 'T', hence its name. Black's main zone is a square, but an unusual one. The '1', '2', '3' and '4' squares are arranged in a circle, in contrast to the quadratic system.

White's rear squares are marked such that in each 2×2 square there are all four numbers. Thus we designate e4 with the number '3', and it is located at the distance of a knight's move relative to the '3' square of the main zone.

The system extends in depth until the appearance of an obstruction or counterplay; here this is only as far as the 4th rank, since the g4 pawn is an obstruction. In width it extends according to the usual rules for quadratic systems: as long as there is an outlet to the key squares or to the triangle apex.

1 Ke4! Ke8 2 Kf4! Kf8 3 Kg5 Kf7 4 Kf5 Ke7
(4 ... g6 + 5 Ke5 Ke7 6 g5) 5 Kg6 Kf8 6 Kh7 Kf7 g5, and White wins.

Bähr, 1934

725

726. The pawn tempo c2–c3 has to be preserved. Therefore: 1 Ke1! Ke7 2 Kf1 Kf7 3 Kg2 Kf6 4 Kf2 Kf7 5 Ke3 Ke7 6 Kf3 Kf7 7 Kg4 Kf6 8 Kf4 Ke6 9 Kg5 Kf7 10 Kh6 Kf6 11 c3 Kf7 12 Kh7 Kf6 13 Kg8 etc.

Zinar, 1983
from Foltys

727

727. Here there is one quadratic system on the base squares c4–d3, and an adjoining one on the squares d3–e4. Black also has two quadratic systems, but with fully coinciding main zones.

As usual, we designate the triangle apexes by letters. The system extends over five files—from b– to f–, since from them there is an outlet to the triangle apexes. The alternation of the rear squares is normal for the ‘T’ system.

1 Kb1! Kg7 2 Kc1 Kg8 3 Kd1 Kg7 4 Ke2 Kf8 5 Kd2 Kf8 6 Ke3 Ke7 7 Kd3 etc.

Certain difficulties arise over the marking of the following position.

728. In defending the critical squares, Black must constantly reckon with the possible sacrifice of the f-pawn. Therefore Ke3/Ke5 is one position of mutual zugzwang, as are Kb3/Kc5 and Kc3/Kd5 on the Q-side. The shortest paths will be e3–d2–c3–b3 and e5–d6–d5–c5. But we see that these are not
altogether the 'shortest' paths: Black marks time on the d-file.

To construct a system it is essential that the base squares should be located on different files or ranks. If this cannot be done on the squares of the shortest path, we must take as a base square the nearest rear square which satisfies this demand.

From this reasoning we must take as the base squares e3–d2–c2 and e5–d6–c6. On these we construct for White two adjoining quadratic systems, while for Black they fully coincide. We again have a 'T' system, which we mark out as usual. Black's main zone is rotated with its rear towards White.

1 Kg2! Kd7 2 Kf1 Ke6 3 Ke1 Kd5 4 Kb1 Kc6 5 Kc2 Kc5 6 Kb3 Kd6 7 Kd4 Kd5 8 Kc3 Ke5 9 Kb4 Kd6 10 Kb5 Ke6 11 K×e4 K×f6 12 Kd5, and White wins.

729. The shortest paths here are similar to those in the previous example: c5–b6–c7–c8 and e6–f7–e7–e8. After carrying out the 'operation' to replace the square c7 of the shortest path (e7 for Black) with the nearest rear square b7 (f8 for Black), we obtain normal base squares and exactly the same 'T' system as in example 728, except that here White's zone is arranged vertically.

1 Ka4! Kf8 2 Ka5 Ke8 3 Ka6 Ke7 4 Kb5 Kf7 5 Kc6 Ke7 6 Kc7 Ke6 7 Kd8 d6 8 e×d6 K×d6 9 Ke8 Ke6 10 Kf8 Kf6 11 Kg8 g5 12 h5 etc. We carry out the same procedure in position 730, since the attack on the a7 pawn is not dangerous for Black.

Zinar, 1983
and a8 squares are removed from the correspondence by Black’s counter-play.

1 Ka4! Kh8 (1 ... Kg8 2 Kb4! Ke7 3 Kc5 Ke6 4 Kd6 Ke7 5 Kd5) 2 Ka5! Kg8 3 Ka6 Kg7
4 Kb5 Kh7 (4 ... f6 5 Kc6 f×g5 6 Kd6 Kg6 7 Ke6 Kh5 8 K×f5) 5 Kb6! Kg7.

Black fails to save the game by 5 ... Kg6 6 Kc6(c5) Kh5 7 Kd6 Kg4 8 Ke7 Kf3 9 K×f7;
Ke2 10 g6 K×d2 11 g7 K∞ 12 g8 = Q d2 13 Qd8 d1 = Q 14 Q×d1 + K×d1 15 Kf6 Ke2
16 K×f5 K×f2 17 K×e4, and wins.

6 Kc7! Kg8 7 Kc6 Kg7 8 Kd5 Kh7 9 Kd6 Kg7
10 Ke7 Kg6 11 Kf8 f6 12 Ke7 f×g5 13 Kf6
Kh5 14 K×f5, and White wins.

10.8 MULTI-SQUARE SYSTEMS

Although they occur rarely, cases are possible where four squares can be taken as base squares. In such positions it is hard to deduce any clear rules: we will regard them as hypothetical.

With adjoining six-square and quadratic systems, a rectangular eight-square system operates.

732. As usual, the marking is given in the diagram. The alternation of corresponding squares is normal, at an interval of one square. The main zones consist of eight squares, but they are arranged differently from the eight-square system examined at the start of the chapter.

1 Kf1! Ke7 2 Ke1 Kf7 3 Kd2! Ke6 4 Kc3 Kd5
5 b4 etc.

Grigoriev, 1934

733. In this example we see a vertically-arranged rectangular eight-square system, arising in the struggle for the key squares on the e-file.

The rear of Black’s system is located on the key squares, due to the obstruction on the e-file. The additional non-systematic zugzwang position. Ke4/Kc5 is associated with stalemate.
The Theory of Corresponding Square Systems

1 Kh4! Kd5 2 Kh5 Kd6 3 Kh6 Kd5 4 Kg7 Ke6 5 Kf8 Kd7 6 Kf7, and wins.

With adjoining eight-square and quadratic systems, a ten-square system operates.

Grigoriev, 1925

734. Here there are three base squares. On the base triangle c4-d3-c3 there is an eight-square system, and on the base squares d3-e3 there is an adjoining quadratic system. Thus a ten-square system arises, the marking of which is given in the diagram. The b4 square is removed by counter-play.

1 Kb1! Kh7 2 Kc1 Kh6 3 Kb2 Kg6 4 Kb3 Kf6 5 Kb4 Ke7 6 Kc4, and White wins.

There is the same superposition of systems in the following example.

Zinar, 1983

735. 1 Kb7! Kf8 (due to the threat to the d7 pawn, Black is forced to cede the correspondence) 2 Ka6 Ke8 3 Ka5 Kf8 4 Ka4 Kg8 5 Kb5 Kf7 6 Kb4 Ke7 7 Kc3 Kd6 8 Kb3! Kf7 9 Kc2 Kg6 10 Kc3 Kg5 11 Kb4 Kf5 12 Kb5 Ke6 13 Kc5 etc.

Zinar, 1983

736. 1 Kc1! Kc7 2 Kc2 Kd8 3 Kd2 Kc7 4 Ke2 Kd8 5 Kc3 Kc7 6 Kd3 Kc6 7 Kd4, and wins.

Ebersz, 1930

737. Here there are three base squares, but on each pair one can construct coinciding eight-square systems.

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Multi-square Systems

With coinciding eight-square systems, a thirteen-square system operates.

This is the largest of the known systems. To memorize all 13 squares of the main zone is extremely difficult. One can only hope that these last two systems will never occur in practice.

1 Kb1! Kh7 2 Ke1 Kh6 3 Kd1 Kh5 4 Ke2 Kg6 5 Kd2 Kf6 6 Kd3, and White wins.

In certain cases a superposition of systems gives an excess of correspondence.

Adjoining eight-square and quadratic systems give an excess of correspondence against coinciding systems.

Zinar, 1983

from Bianchetti

738. White has adjoining eight-square and quadratic systems, whereas Black's coincide. It turns out that Black's 'b' square is simultaneously the 'b' square of the quadratic system.

1 Ke3! Ke5 2 Kf3 Kf5 3 Kg3 Ke6 4 Kg4 Kf7 5 Kf3! Ke7 6 Kc3 Kf7 7 Kd4 Ke6 8 Ke4 Kd6 9 Kf4 Ke6 10 Kg5 Kf7 11 Kh6 Kf6 12 b4 etc.

There is the same superposition of systems in example 740.

Grigoriev, 1920

740. 1 Kb3! Kb7 2 Kc4 Kc7 3 Kd5 Kd7 4 Ke5 Ke7 5 Kf5 Kf7 6 Kg5! Kf8 7 Kf4 Ke8 8 Ke4 Kg8 9 Kf5 Ke7 10 Kg6 Kf8 11 Kh7 Kf7 12 g5 etc.

To consider all possible cases, where the
superposition of systems leads to an excess of correspondence, is hardly possible. The more so as it is difficult to give rules for all cases in life. What is important is that the play here is normally reversible, and one can always correct a mistake by again winning the correspondence, as we have seen in the analysis of the triangular system.

Chopyak, 1981

741. One should always aim to reduce sensibly the number of marked squares. Here, for example, f4–g5 (c6–d7 for Black) could be taken as the base squares, an eight-square system constructed on them, and its superposition on the triangular system considered.

But it is quite sufficient to consider the base triangles e3–f4–f3 and c5–c6–b6, to see that on the '2'–'3' side White can construct a square, whereas Black cannot.

The solution is therefore as follows: 1 Kg5 Ke7 2 Kh5! (the threat of ... f6 has to be parried) 2... Kf8 3 Kg4 Ke7 4 Kg5 Kd7 5 Kf4 Kc6 6 Ke3(f3,g3) oo 7 Kf2(e2) oo 8 Ke2(f2) Kb6 9 Kf3! Kc6 10 Kf4!, and White wins.

10.9 IRREGULAR SYSTEMS

Cases are possible in which the squares of the main zone have a disorderly arrangement. We will consider a few examples.

742. One end of the shortest path here is Kb4/Kd6, the other being Kc3/Kg4. It is, naturally, unfavourable for White to waste his pawn tempo: the opponent also has a tempo in reserve.

If an attempt is made to make at least some order of the corresponding squares, we find that, against Black's normal six-square system, White has a six-square system in which for some reason the '1' and '4' squares are shifted.

1 Ka2! Ke6 2 Ka3 Kd7 3 Kb2 Ke7 4 Kb3 Kd7 5 Kc2 Kc6 6 Kc3, and wins.

Zinar, 1983

743. One end of the shortest path is Ka3/
Second-order Systems

Ka5, while the second is Kg4/Kf6. For the base squares, a3-b3 (a5-b5 for Black) and c2-d1 (b6-c7 for Black) can be taken. As a result of the superposition of systems, White again obtains a six-square system with the '1' and '4' squares shifted.

1 Ka2! Ka6 2 Kb2 Kb5 3 Kb3 Ka5 4 Kc2 etc.

745. Here too White has the '4' and '10' squares shifted in a ten-square system: 1 Ka4! Kf7 2 Ka3 Kg7 3 Kb4 Kf6 4 Kc3 Ke7 5 Kc2 Kf7 6 Kc3 Ke7 7 Kd2 Kf6 8 Kd3 etc.

It is hard to give practical recommendations on the play in such irregular systems.

10.10 SECOND-ORDER SYSTEMS

If in the struggle for the key squares the stronger side has in reserve a pawn tempo, the base squares change their designations to adjoining ones and a second-order system is formed.

A second-order system by no means always meets those definitions that we made for first-order systems. Due to the instability of the pawn formation, both sides acquire new possibilities. Some squares fall out of the correspondence, but new ones appear. And the play itself in a second-order system is different: its aim is usually to transpose at a favourable moment into a first-order system.

The play in second-order systems demands a deep penetration into the position, and an understanding of its nuances.

746. The key squares here are e6, f6, g6. The front-line squares are e5–f5. We have a quadratic system with non-unambiguous rear, since there is no second invasion point. Black maintains the balance by 1 ... Ke8!

Let us move the white h-pawn back one
step. It is clear that Black cannot now hold out on the same defensive lines. The key squares move forward, and on the base squares f4,e4,d4 a six-square system is formed.

747. This is a first-order system, since in comparison with example 746 the struggle is for other key squares: 1 Kd1 Ke7 2 Ke1 Kd7 3 Kf2 Ke8 4 Ke2 Kf8 5 Kd3 Ke7 6 Ke3 Kd7 7 Kf4 Ke6 8 Ke4 Kf6 9 Kd5 and wins. Incidentally, this position can also be solved by the opposition method.

If we move the h-pawn one further step back, Black immediately acquires counterplay, associated, firstly, with the possibility of penetrating with his king to g5, and secondly, with playing ... h5.

748. Now only the occupation of f5 guarantees a win.

The base squares exchange designations with adjoining ones, as do the rear squares. A quadratic system with base squares d4-e4 (e6-f6 for Black) would operate, except that ... counter-play interferes. Earlier too we encountered it in small doses, but here it 'spoils' even the main zones. Of the quadratic systems, only the '1' and '3' squares remain. As we see, counter-play can remove corresponding squares in the most peculiar fashion.

A detailed solution of this example was given earlier (198).

As can be checked, in the second-order system the solution was: 1 Kd2! Ke7.

In the event of 1 ... Kf7 there follows 2 h4.

Here is a significant difference from the play in a first-order system. There it was unfavourable to move the pawns until the occupation of the key squares, whereas here it is a quite normal procedure—transposing under favourable conditions into a first-order system.

2 Kd3! Ke6 3 Kd4! Kd6(f6) 4 h4!, and wins.

In example 747 let us now move the g-pawn back.

749. Here the base squares exchange desig-
Second-order Systems

nations, but a quadratic system does not result, since Black has the possibility by ... h5 of transposing into example 746, for example in the position Ke3/Kf7. On the other hand, a new zugzwang position has appeared, associated with the stalemate possibility Kf4/Kh5 (and, linked to it, Kf3/Kg6). As we see, here there is no system.

1 ... Kg7! 2 Ke3 Kf7 3 Ke4 Kf6 with a draw.

Let us move the h-pawn back one step. We obtain a second-order system.

Grigoriev, 1930

750. Black's counter-play has disappeared, and the system has unexpectedly become ordered!

Not only the base squares, but also the rear squares have exchanged designations. On the base squares e4–f4 (e6–f6 for Black) we construct a quadratic system. To the left it extends to the edge of the board, since there is access to the key lines; the system does not extend onto the 2nd rank, since the distances from e2 and f2 to g4, the nearest square of the shortest path, are identical.

But why does zugzwang arise in the position Kh5/Kh7, and in connection with it a triangular system is formed on the K-side? This turns out to be due to the possibility of a transition to example 749. After all, we have here a second-order system.

1 Kh5! Kg7! 2 h4! Kh7 3 Kg4 Kgs 4 Kg3 (to a '3' square) 4 ... Kf6 (playing for stalemate does not work: 4 ... Kh5 5 Kf4 Kg6 6 Ke5 Kh5 7 Kf6) 5 Ke4 Kf6 6 g4, transposing into the winning position 747.

But the marking in the centre is inaccurate—we have forgotten about the possibility of transposing into example 748. The correct marking will be as shown in the next diagram.

Zinar, 1983

751. 1 Ka2! Kb8 2 Kb2 Kc8 3 Kc2 Kd8 4 Kd2 Ke8 5 g4! (precisely at this point, as is clear from example 748) 5 ... Kf7 6 h4 Kc8 7 Ke2, and White wins (747).

There are plenty of strange features in second-order systems. For example, let us make a further step back with the g-pawn in example 751.

752. In view of the possibility of transposing both to position 747, and to position 748, the key squares are shifted forward. On the base squares e3–f3 a quadratic system is obtained.

1 Ka2! Kb6 2 Kb2 Kc6 3 Kc2 Kd6 4 Kd2 Ke6 5 Ke2 Kf6 6 Kf2 Ke6 7 Kg3 Kf5 8 Kh4 etc.
Let us trace back one further chain in the formation of second-order systems.

753. This is a well known theoretical position. In the struggle for the key squares d6, e6, f6 two systems of corresponding squares are obtained: to the left of the f-pawn—a quadratic system, to the right—a base triangle against a 2×2 base square with an excess of correspondence on the f8–g8 squares for Black. In addition, Black can conduct a defence on the key squares. There is an additional zugzwang position Kg4/Kf6.

1 Kb4! Ke8 2 Ke4 Kd8 3 Kd4 Ke8 4 Ke4! Kf8 5 Kd5! Ke7 6 Ke5, and wins.

754. Here a step back has been made with the h-pawn. The base squares have changed to the adjoining ones, and Black's strong counter-play has removed from the correspondence the squares on the d-file. Zugzwang with the black king on the key squares has changed to Kg4/Kg6; a new zugzwang position Kh6/Kg8 has appeared, associated with the possibility of attacking the h-pawn. In this second-order system there is total confusion.

1 Ke4! Ke8 2 h5!, transposing into position 753 and seizing the correspondence.

755. A further step back by the h-pawn, and the position has stabilized. The designations of the base squares have changed to those adjoining; according to our definition, a quadratic system operates in the centre and on the Q-side.
1 Kb4! Kc8 2 Kc4 Kd8 3 Kd4 Ke8 4 Ke4 Kf8 5 h4 (5 Kd5 is bad due to 5... Kf7) 5... Ke8 6 h5! etc. (transposing into example 753).

On the K-side the front-line squares f5–g5–h6 (f7–g7–h8 for Black) give a ‘T’ system. The ‘4’ square has been removed by counteryplay, as a result of which the non-systematic zugzwang position Kg5/Ke6 has also appeared. Thus, with the white king at g4 and Black’s at f6(f8), a draw is given by 1... Ke7! 2 Kg5 Ke6!

757. Again the designations have changed to the adjoining ones, and we have obtained a quadratic system with rotated main zones, from which the d-file is removed by counterplay.

Unexpectedly, an elegant second-order system has arisen. The Kg4/Kf6 zugzwang position has also changed, and both key squares are now in correspondence. It is on this additional correspondence that White wins.

1 Kc2! Kh5! 2 Kd3 Kc6 3 Ke4 Kd6 4 Kf4 Ke6 5 Kg4 Kf6 6 f4, and wins.

756. 1 Kg4! Kg8 2 Kh5 Kf8 3 Kh6 Kg8 4 h4! (754) 4... Kh8 5 Kg5 Kg7 6 h5 etc.

And now in position 753 let us move the f-pawn back one step.

Zinar, 1983

758. 1 Kg4! Ke8 2 Kf4 Kf8 3 Ke4 Ke8 4 f4!, and so on as in example 753.

Zinar, 1983
759. Here we have also moved the h-pawn back a step. On the base squares e5–f5–g5–h6 (e7–f7–g7–h8 for Black) a superposition of systems arises, as in examples 739 and 740. For Black the ‘6’ square of the rectangular system will simultaneously be the ‘b’ square of the quadratic system. White has an excess of correspondence: the win is not difficult.

1 Kh5 Kg8 2 Kg4 Kf8 3 h5, transposing into example 758, but 3 Kf4 is also possible. Incidentally, White also wins by 1 Kg4 Kg8 (1 ... Kg6 2 f4—754) 2 Kh5 Kf8 3 Kh6 Kg8 4 f4 etc.

These series of examples show not only the origin of second-order systems, but also the possibility of studying pawn endings with the help of the theory of corresponding square systems.

Zinar, 1983

760. In comparison with position 700, where an eight-square system operated, here White has an additional tempo. The base squares change designations to the adjoining ones (d3–e3 for White and d5–c6 for Black).

What is this, a quadratic system? No. In view of the fact that the main zone of the first-order system extends for three ranks, the designations received by the d1 and e1 squares are not ‘1’ and ‘2’, but ‘5’ and ‘6’. The result is a strange second-order six-square system.

1 Kb1 Kc7 2 Kc1! Kc6 (2 ... Kd7 3 f3) 3 Ke2 Kc5 4 Kd1 Kd6 (4 ... Kd5 5 f3) 5 Kd2 Kc5 6 Ke3 Kd5 7 f3!, transposing into example 700 and seizing the correspondence.

As we see, in second-order systems the play is much more complicated.

10.11 CERTAIN COMPLICATED CASES

In our examination of multi-square, irregular and second-order systems we have already met with some complicated cases. Here we will examine a few more. That which is now considered complicated may possibly, with the passage of time and the development of theory, prove to be simple.

A typical example is 761, which authors of books have usually explained in lengthy and obscure terms.

Reti, 1929

761. The key squares here are those on the 4th rank, since the exchange after g3–g4 leads only to a draw. On the base squares d3–e3–f3 we have a normal rectangular system.

We have already seen on several occasions that Black can conduct a defence on the key squares. Here too he does not have to allow the white king to d3 and e3, since its emergence at f3 is not dangerous due to 1 ... Kc3
2 g4 h × g4 + with a draw. Zugzwang arises in connection with the possibility of g3–g4, the zugzwang positions being Kf2/Ke4 and Ke2/ Kd4. This is the extent of the complexity.

1 ... Kd4! 2 Kg2 (2 Kf2 Ke4! 3 Ke2 Kd4! 4 Kd2 Ke4! 5 Kc2 Kd4—defence on the key squares) 2 ... Ke5! 3 Kf3 Kf5 4 Ke2 Ke6!—defence in the basic system.

Zinar, 1983

762. Here it is not possible to construct a square on the side of the base triangle d4–e3–d3. A triangular system seems to be in operation, but White has the possibility of attacking the b7 pawn. Black must answer with a counter-attack on the f2 pawn. Thus with the white king at c3, Black's must stand only at g6, preparing to emerge at f6. Unexpectedly a quadratic system arises.

1 Ka2! Kg8 2 Ka3 Kf7 3 Kb2!

3 Ka4? is insufficient: 3 ... Kf6 4 Ka5 Ke5 5 Kb6 Kd4 6 Kc7 Kd3 7 K × d7 Ke2 8 Kc6 K × f2 9 d7 Kg2 10 d8= Q K × h2! with a draw.

3 ... Kg7 4 Kb3 Kf6 5 Kc2 Kg6 6 Kc3 Kg5.

Or 6 ... Kh6 7 Kb4 Kg5 8 Ka5 f4 (8 ... Kf6 9 Kb6 Ke5 10 Kc7 Kd4 11 K × d7 Kd3 12 Ke6 Ke2 13 d7 K × f2 14 d8= Q) 9 g × f4+ K × f4 10 Kb6 Ke4 11 Kc7 Kd3 12 K × d7 Ke2 13 Ke6 K × f2 14 d7 Kg2 15 d8= Q K × h2 16 Qd4 Kg2 17 Qg4+; and wins.

7 Kd3! Kh5 8 Kd4! Kg5 9 Ke5; and White wins.

Zinar, 1978

763

763. Here the 'T' system should not extend to the 3rd rank, since there is no outlet to the triangle apex 'a'. But, as in the previous example, White has the possibility of an attack on the h3 pawn. With the white king at b3, Black's must only be at f7, ready to go to e8. The system therefore extends to the 3rd and 2nd ranks.

1 Ka2!

Nothing is given by 1 Kb1? Kf7! 2 Kc1 Ke8 3 Kd2 Kd7 4 Kc2 Kd6 5 Kf2 Kc5 6 Kg3 Kb4 7 K × h3 Kc3 8 Kg2 K × d3 9 h4 Ke2! 10 h × g5 d3 11 f6 (11 g6 d2 12 g7 d1= Q 13 g8= Q Ke2+, with perpetual check, or 11 g × h6 d2 12 h7 d1= Q 13 h8= Q Q ∧ g4+ with a draw) 11 ... e × f6 12 g × f6 d2 13 f7 d1= Q 14 f8= Q Q × g4+ 15 Kf2 Qh4+, with a draw.

1 ... Kf8 2 Kb2! Ke8 3 Ka3 Kf7 4 Kb3 Kf8.

Black fails to save the game by 4 ... Kg8 5 Kc2! Kf7 6 Kd2 Ke8 7 Ke2 Kd7 8 Kf2 Kd6 9 Kg3 Kc5 10 K × h3 Kb4 11 Kg2 Ke3 12 h4 K × d3 13 h × g5 Kc2 (13 ... K × e4 14 Kf1; 13 ... Kc4 14 g6, and the pawn queens with check; 13 ... Ke2 14 f6 e × f6 15 g × f6 d3 16 f7 d2 17 f8= Q d1= Q 18 Qf3+, exchanging
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queens) 14 f6 e x f6 15 g x f6 d3 16 f7 d2 17 f8 = Q d1 = Q 18 Qf2 + Kc1 19 Qf1 etc.

5 Kb4! Kg8 6 Kb5 Kf8 7 Kb6 Kf7 8 Ke5 Kg7 9 Kc6 Kf7 10 Kd6 Ke8 11 Ke6 d2 x e6 K x e6 13 Kf8 Kf6 14 Kg8 Ke6 15 Kg7 Ke7 16 K x h6 Kf6 17 Kh7 Kf7 18 h6, and White wins.

Zinar, 1983

764. The quadratic system on the base squares d4-e3 should extend here only to three files. But in the position there is a third invasion point—b5, in view of which the b3 square is designated the number '2', and the system extends to the edge of the board.

1 Kb1! Kb7.

The result is unaffected by 1 ... Ka8 2 Kb2! Ka7 3 Kb3 Ka8 4 Ka4 Ka7 5 Kb5 Kd7 6 c5! b x c5 7 b x c5 d x c5 (7 ... Kc7 8 c6!) 8 e5! Kc7 9 e6! Kd6 10 e7! K x e7 11 K x c5 Kd7 12 Kd5 etc.

2 Kc1 Kc7 3 Kd1 Kd7 4 Ke1 Kc7 5 Kf2 Kd8 6 Ke2 Ke8 7 Kd3 Kd7 8 Ke3 Ke7 9 Kd4 Kd8 10 Kd5 Kd7 11 b5, and wins.

...765. In this example too a third invasion point strongly influences the play. Were the position on the Q-side blocked, a quadratic system with non-ambiguous rear would operate, as in example 711. But due to the necessity to defend the a4 pawn, a rectangular system has been formed on the adjoining quadratic systems. The shortest paths are peculiar, although it appears that Black is even marking time (c7-b7-b6).

Black loses in view of the excess of correspondence for White: c1 and c2 against b7.

1 Ke1! Kc7.

Or 1 ... Kb8 2 Kb2 Kb7 3 Ka3 Kc6 4 K x a4 Kb6 5 Ka3 Kc7 (5 ... Ka5 6 Kh3 Kb6 7 Kc2 Ka5 8 Kd3 Ka4 9 Ke4 Ka3 10 Kd5 K x a2 11 K x c5, with a won queen ending) 6 Kb2 Kd7 7 Ke2 Kd8 8 Kd3 Kc7 9 Ke4 Kd6 10 a3! etc.

2 Kd1! Kd7 3 Ke1 Kc7 4 Kf2 Kd8 5 Ke2 Kc8 6 Kd3 Kd7 7 Ke3 Kd6 8 Ke4 (forcing the pawn to advance) 8 ... a3 9 Kd3 a5 10 Ke2! a4.

The position on the Q-side is blocked; a quadratic system with non-ambiguous rear (711) now operates. White exploits the unfortunate position of the enemy king.

11 Kc2! Kc7 12 Kd3 Kc6 13 Ke2 Kd6 14 Kf2 Kd7 15 Kc3 Kf3, and wins.

766. The concluding phase is more clearly seen in this example. After ... a4 the system depicted in the diagram will operate.

White must deprive the opponent of his reserve tempo by playing his king to c2 with the black king at c7, c6 or d6. As is apparent from the marking, this leads to the seizure of the correspondence. This can be achieved only by reaching f3, but White must move
there carefully, so as not to give Black the chance to use his ... a4 tempo.

1 Kd2! Ke8 2 Kd3 Kd7 3 Ke4 Kd6 4 Kf3!

Ke7 5 Ke3 Kd6 6 Kd2 Kd7 7 Kd3 Kd6 8 Ke2 a4
9 Kd2 Ke7 10 Kd3 Kc6 11 Ke2 Kd6 12 Kf2
Kd7 13 Ke3 Ke7 14 Kf3, and White wins.

As you see, with three invasion points there are plenty of complications.

Let us sum up. It is sufficient to remember a few straightforward definitions, in order to recognize which system is operating. Then the squares must be marked out according to the ready-made prescription. This method is much quicker, and much more reliable and understandable than that which existed earlier, by which, square by square, one had to seek the corresponding squares over the entire board.

This chapter has also posed certain new questions, the answers to which we may obtain as the theory of corresponding square systems develops.